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Prof. **D. Kikuchi, M. A.** Director of the College. *(ex officio.)*

Prof. **K. Mitsukuri, Ph. D.**

Prof. **C. G. Knott, D. Sc., F. R. S. E.**

Prof. **S. Sekiya.**

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ERRATA.

Vol. I. Pt. I.

P. 55. 2nd line from bottom read "*ventral*" for "*dorsal.*"

Vol. I. Pt. II.

S. 113 Z. 4 *v. u.* lese man für nus uns.
— 114 — 13 lese man für auf's neue auf's Neue.
— 120 — 17 setze man statt Fig. I (Tafel) fig. 1 (Tafel XIII).
— 120 — 12 *v. u.* lese man für Erdoberflöche Erdoberfläche.
— 130 — 13 *v. u.* lese man für sind ist.
— 136 — 12 lese man für schwierige Aufgabe, eine schwierige Frage.
— 137 — 3 setze man für $\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y}$, $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$.
— 139 — 7 *v. u.* ersetze $\left(\frac{\partial \Phi}{\partial n}\right)^2$ durch $\frac{\partial \Phi}{\partial n}$.
— 139 — 5 *v. u.* ersetze $\sqrt{\left(\frac{\partial \Phi}{\partial n}\right)^2 - R}$ durch $\sqrt{\left(\frac{\partial \Phi}{\partial n}\right)^2 - R^2}$.
— 142 — 2 lese man für Salz Satz.
— 142 — 7 *v. u.* lese man für Raumsirpale Raumspirale.
— 145 — 11 zwischen zu und während „thun" einzuschalten.
— 146 — 6 lese man für grasser grosser.
— 149 — 4 schreibe man für $\sqrt{1 - \frac{R^2}{4\lambda^2\omega^2} \cdot tag^2 i}$ $\sqrt{\left(1 - \frac{R^2}{4\lambda^2\omega^2} tag^2 i.\right)}$
Ebenso weiter unten S. 150.
— 150 — 9 lese man für Werk Werth.
— 150 — 15 lese man für der Azimuth das Azimuth.
— 152 — 1 lese man für Greuz Grenz.
— 152 — 2 lese man für gauzen ganzen.
— 166 — 11 setze man für $\frac{\partial W}{\partial x}$, $-\frac{\partial W}{\partial x}$
— 174 — 5 *v. u.* $\frac{1}{\rho^2}\frac{\partial^2 \varphi}{\partial X^2} \sin^2 X$ zu ersetzen durch $\frac{1}{\rho^2}\frac{\partial^2 \varphi}{\partial X^2} \cos^2 X$.

S. 181 Z. 14 u. 15 man ersetze $\frac{k^2}{\rho^2}$ durch $\frac{R^2}{\rho^2}$

— 191 — 15 $\frac{\partial \Delta W}{\partial x}\frac{\partial \varphi}{\partial y}+\frac{\partial \Delta W}{\partial y}\frac{\partial \varphi}{\partial x}$ zu ersetzen durch $\frac{\partial \Delta W}{\partial y}\frac{\partial \varphi}{\partial x}-\frac{\partial \Delta W}{\partial x}\frac{\partial \varphi}{\partial y}$.

— 197 — 12 zu lesen: deren horizontale Projection eine transcendente Spirale ist.

— 202 — 9 $\frac{\gamma}{4}[R^2(1-2\log R)+\rho^2]$ zu ersetzen durch $\frac{\gamma}{4}[R^2(2\log R-1)+\rho^2]$.

— 203 — 5 setze man für $\Delta W_i=\frac{2\lambda.\sin\theta}{(\kappa+\gamma)}$, $\Delta W_i=\frac{2\lambda.\sin\theta}{(\kappa+\gamma)}.\gamma$.

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PREFACE.

The Journal of the Science College of the Imperial University of Japan, the first part of which now makes its appearance, may be regarded under some aspects as a continuation of the Scientific Memoirs which had been from time to time published by the Tōkyō University. For this reason the Journal, in shape and size, is made uniform with these.

Outside the mere fact, however, of both being scientific publications of the highest existing Science School in Japan, there is little or no similarity. Each Scientific Memoir was, in itself, a distinct monograph on a particular scientific subject. The Journal, on the other hand, in each of its Volumes or Parts, is quite unrestricted us to the length and character of the papers or notes therein published. Many an interesting fact, in itself too small to make a formal memoir, may thus be saved from the oblivion that would almost certainly be its fate, were no ready means of publication at hand.

As the name itself all but implies, this Journal is intended to be the channel through which the world at large may receive Japan's own contributions to the progress of Science. A glance at the

contents of the present Part will speak more for the nature and scope of the project than pages of preface.

One unique feature, which will be apparent at once, has regard to the language or languages in which the various papers are to be presented. Each contribution must be written in one of the three languages, English, French, or German—the choice being left entirely to the author. The necessity for this tri-lingual character springs, of course, from the very peculiar but well-known conditions under which Science has been cultivated in Japan and by the Japanese.

It is intended to publish the Journal in annual volumes,—the number and size of the separate Parts of each volume being dependent upon the nature and extent of the contributions.

On the Life-History of Ugimya Sericaria, Rondani.

By

C. Sasaki, *Rigakushi.*

Professor in the Agricultural and Dendrological College, Tōkyō.

With Plates I—VI.

In Japan, the silkworm is reared at four different times of year, viz:—in May, July, August and September. It is however reared on a large scale only in the month of May, comparatively much fewer in number being raised at other times.

The so called "Uji" disease caused by the larva of a diptera Ugimya sericaria, Rond. plays terrible havoc among the silkworms reared in May and July. When the silkworm is once infested by this parasite it dies either before or after it spins a cocoon ; in the latter case the maggot eats its way out of the cocoon thus leaving a round hole in it with the consequence of making it unfit for reeling. In the spring or May breed of silkworms some 50 to 70 % or in extreme cases 80 % are attacked by the parasite, and the damage done is correspondingly great.

Fatal as the "Uji" disease is to the silkworm, no systematic observations have hitherto been made on the habits and life history of the maggot. The fragmentary observations recorded are very imperfect or erroneous. The first who gave a brief account of this parasite

was Adams,[1] and more recently Rondani,[2] Guerin Meneville,[3] Coronalia,[4] have published notes on the subject. Haberlandt also gives a few facts in his book "Der Seidenspinner." Pryer in his catalogue of Lepidoptera of Japan says under the heading of Bombyx mori, "I have noticed that the Uji, a diptera which is parasitical upon it and causes an immense amount of damage, deposits its eggs about the larva on the leaves, and not on the insect."

In 1873, my father Mr. N. Sasaki attempted to study the maggot and finding it in the main tracheal trunk of the silkworm directly beneath a stigma, concluded that the maggot gains this place by entering the stigma from outside, and is introduced into the breeding room by means of the eggs which are deposited on mulberry leaves.

In the Japan Times of May 4th 1878, Mr. G. A. Greeven makes the following statement:—"My experiments have now shown me that the hatching of the Uji takes place in the stomach of the caterpillar, and that it immediately forces its way through the membrane of the stomach, and makes a path for itself to a stigma * * *."

On account of the immense loss sustained every year by the sericulturist from the ravages of this parasite, the Principal of Komaba Agricultural College suggested to me the idea of making a thorough study of the habits and life-history of the fly, with the purpose of discovering, if possible, some means of protecting the silkworm from it.

My investigations on this subject were begun in 1883 and continued during two following years, and are principally on the develop-

(1) Adams—Deuxieme rapport sur la sericulture au Japon—V. Revue universelle de sericulture Lyons, No. 36, Avril 1870.

(2) Rondani Prof. Camillo—L'Ugi; estratto del Bolletino del Comizio Agrario del mese di Aprile 1870, No. 4.

(3) Guerin Méneville F. E.—Observations sur la nature de l'Oudji, parasite des vers à soie au Japon, presentés a l'academie des science dans sa Seance du 18 Avril 1870. V. Comptes-Rendus et revue universelle de sériculture, No. 3, Avril 1870.

(4) Dott. E. Coronalia-L'Ugi o il Parasita del filugello al Giappone.

ment of the maggot into the mature insect, and on the manner in which the maggot becomes parasitic on the silkworm. And as the result of these investigations I also venture to suggest various measures which, it is hoped, will prove efficient in protecting the silkworm from this most destructive parasite. I am much indebted to Mr. Zengo Tsuge, Vice-principal of Komaba Agricultural College who has allowed me to use every instrument necessary for the present investigation. My thanks are especially due to Dr. Kakichi Mitsukuri, Professor in the Imperial University of Tōkyō, who has helped me in various ways.

I.

External Characters and Habits of the Adult Fly.

The mature insect of Ugimya Sericaria, Rondani (*fig. 1*, *fig. 1 a fig. 2*) has a large stout body covered with black stiff hairs. The length of the body and the expansion of the wings differ with sex ; thus, the measurement of twelve large males gives an average of 15 mm. for length, and of 30 mm. for the expansion of the wings, while in an equal number of females the averages of the same dimensions are 14 mm. and 28 mm. respectively.

The head is nearly triangular in shape. The large compound eyes which it bears, are placed somewhat apart from each other so as to leave a space between them on the dorsal aspect of the head. In this space, three small simple eyes are situated. The antennæ (*Pl. I*, *fig. 4*) are club-shaped and consist of three joints, of which the terminal is by far the largest. A little distal to the junction of the terminal with the middle joint is attached a long stiff black bristle (*fig. 4 a*) about twice as long as the antenna itself. This consists of two joints: the proximal is

very short, but is broader than the distal which is long and tapering. The terminal joint of the antennæ as well as the bristle are covered with fine hairs.

The thorax is somewhat angular in front but rounded off behind. Its three segments are very distinct. The pro- and meso-thorax which are large in size and similar in color are marked with five dark longitudinal bands. The space between every two of these bands is again divided into two halves by a longitudinal row of stiff hairs. The metathorax which is much smaller than the two preceding segments assumes a hemispherical form, and has a few long stiff hairs scattered over its surface and along its hinder margin. It is further marked off from the other two segments by its color which is reddish brown. The abdomen is composed of six segments, four of which are apparent, while the other two are modified into a protrusible organ which ordinarily remains retracted in the abdomen (*Pl. I, fig. 10*). Of the two, the proximal one is chitinous and the distal membranous. The abdomen is thickly covered all over with stiff hairs and has on the junctions of its segments a few long bristles. In addition to the difference in size of the two sexes mentioned above, a marked difference also exists in the form and appearance of the abdomen. On one hand, the abdomen of the male is almost triangular in outline and is ornamented on each side with a large semi-circular dark brown patch. The abdomen of the female, on the other hand, has an almost oval figure and is destitute of the patches.

The mouth parts (*Pl. I, figs. 5, 6, 7*) are so modified as to form a tube-like proboscis, which is divided into two portions—the basi-proboscis (*A*) and the proboscis proper (*B*). The basi-proboscis is composed mainly of a pharynx (*a*) which is converted into a tubular body. At its posterior end there arise two pairs of processes directed backward:—one pair (*l*) on the upper, and the other (*m*) on the lower edge. Each of

these processes is provided with a muscle: those (*b*) on the upper processes serve to project the basi-proboscis forwards, while those (*b'*) on the lower tend to retract it into a cavity of the head provided for its reception. The upper surface of the pharynx is partly hard and chitinous, and partly membranous. On either side of the membranous portion lies a chitinous oval disc (*c*) beset with a few hairs. Each of these discs bears a single jointed palp (*d*) covered also with hairs, and from its insertion on the disc there extends laterally a fine hairy chitinous rod (*r*), so that the disc seems to be a modified form of a maxilla. The proboscis proper (*B*) is composed mainly of the labium (*j*) which assumes a form of short wide trough, and on its upper side in the median line there is found a lancet-like arched chitinous body—the modified labrum (*e*)—under which lies a jointed stylet.

On each side of the base of the labrum is inserted a long chitinous rod (*f*) which runs posteriorly to the side of the basi-proboscis. This rod seems to correspond with the rudimentary first maxilla which Kraepelin (5) has observed on the proboscis of Musca.

The free end of the labium is furnished with a somewhat triangular fleshy disc—the labellæ (*g*). It is divided into two flaps by two air tubes (*n*), which pass upwards in the middle line of the fleshy disc to its top, and then bending laterally at right angles extend to each side of it. From these air-tubes numerous fine branches of air-tubes—pseudo-tracheæ—pass regularly towards the periphery of the flaps.

On the outer surface (*i*) of the labellæ where two pointed processes of the labium lie, is situated a T-shaped chitinous piece (*k*). The axis of this piece extends longitudinally between the two flaps of the labellæ, and the two arms on its top extend towards the margin of each flap respectively. By the action of these, the flaps when not in use

(5) Zeit fur Wiss. Zool. Band 39. 1883.

may be folded on each other with their inner surfaces in contact.

The wings (*Pl. I, fig. 3*) are long and narrow becoming broader towards the base. They are transparent and greyish in color; the proximal portion is shaded dull brown. The entire surface is thickly covered with fine short hairs. The costal vein (*a a*) which runs along the costa of the wing and ends at the apex (*q*) is strong and rigid, and its portion near the base of the wing is thickened and covered with long bristles, while the rest is provided with a single series of short strong bristles. The fourth longitudinal vein (*g*) after reaching near the outer oblique edge (*r*) suddenly turns up and ends at a portion a little above the apex (*q*). The fifth vein (*h*) reaches the oblique outer edge; but the sixth (*i*) does not. The hinder margin of a wing near its base is incised into three alar appendages (*allulæ*); two (*n*, *o*) of them are supported each with a chitinous rod (*s* and *t*) and the third alula (*p* and *Pl. I, fig. 1 al*) which is somewhat triangular in form covers the upper corner of the abdomen, and conceals the balancers from sight. The two allulæ (*o* and *p*) are colored greyish yellow, and covered with a few fine hairs at their free margin.

The legs are of a moderate size, black in color, and covered with black hairs. There are eight pairs of spiracles on the body of the fly, two of which open on the side of the thorax and the rest (*Pl. I, fig. 10 a*) on the under surface of the abdomen. Of the two pairs of the spiracles on the thorax, one (*Pl. I, fig. 2 a*) opens on the pro- and the other on the meta-thorax. They are nearly spindle-like in form, with their margin hardened by the deposition of a chitinous matter, and their entrance protected by a series of finely branched hairs which grow along the margin (*Pl. I, fig. 8*). On the abdomen, there are six pairs of spiracles (*Pl. I, fig. 10*), one in each segment placed near the median ventral line. The abdominal spiracles (*Pl. I, fig. 9*) differ in shape from those on the thorax, each being a hollow chitinous cup.

The bottom of this is membranous and leaves only a narrow slit through which air is allowed to pass in. Again, the bottom as well as the whole inside walls of the cup is thickly covered with minute hairs.

Habitat.—The adult flies generally begin to appear in the middle of April. They are very active, and their power of flight is so strong that a peculiar sharp sound is heard when they are on wing. When confined in large cages of fine wire gauze they strike their heads so energetically against the cage walls that they die in two or three weeks. Of those which I tried to rear from the pupa stage not one attained sexual maturity. The flies which are hatched out-of-doors are still more active and stronger in their flight. From the time of their first appearance to the beginning of June, they frequent mulberry bushes preferring those which are planted in shady or moist ground or those which are old and languid and bear leaves clustered on their shortened stems or branches. It is a very strange fact that those which thus frequent the bushes are always females, and I have never obtained even a single specimen of the male. Besides mulberry bushes, female flies may sometimes be found on grassy grounds which are overshadowed by thickly branched trees; but here also I have not found a single male fly. Where the male fly does find its home remains unknown to me up to the present time.

Usually the time when the female flies appear is contemporaneous with the breeding season of the silkworm. Although the flies then occur in large numbers, they would not ordinarily be seen by a person visiting localities frequented by them, unless he has a clear idea of their form and habits. They have a very acute sense of hearing, and a very slight motion or sound in the bushes in which they are resting, causes them to fly away instantly. If, however, persons well acquainted with their form and habits wait in silence or walk as quietly as possible in the bushes, they will then perceive the female flies resting on the

leaves or approaching the bushes from some other places. In the latter case, after wandering about a while on the upper surface of leaves, they pass on to the under surface, and there deposit their eggs in close contact with the ramified veins, enveloping and gluing them with a transparent glutinous substance (*Pl. I, fig. 11*). The habit of thus depositing the eggs on the under surface of the leaves was first noticed by my father. That it is not accidental, but is a confirmed habit of the species is positively proved by the fact that those eggs deposited on the under surface of the leaves, not receiving the direct rays of the sun are kept moist, and remain alive many weeks after they are deposited, while the eggs not in this position receiving the direct rays of the sun, soon dry up, and die. There is also the further advantage that their position on the under surface of the leaf protects them from the effects of rain. If the eggs are deposited on spots on which the rain beats directly, the glutinous substance which attaches them to the leaf softens or liquefies by the action of the rain-water, and the eggs drop to the ground.

II.

Sexual and Other Internal organs of the Adult Fly.

The male organ is composed of testes, vasa deferentia, seminal reservoirs and penis (*Pl. II, fig. 1*). The testes (*a*) which are two in number are pear-shaped, and white in color. From each testis extends a single delicate slender tube, the vas deferens (*b*), which extending posteriorly a short distance unites with its fellow of the opposite side, and opens into a common tube (*b*). This passing straight backwards ends in a penis (*c*). At the junction of the vasa deferentia of two sides, there are placed side by side two seminal reservoirs (*k*) having

the form of tiny tubercles. The penis (*Pl. II*, *fig. 2 c*) is in the form of a membranous tube, and its distal half is protected by a dark horny sheath which dividing into two teeth reaches the free end of the penis. At the base of the intromittant organ, and generally overlying the penis as a protection, is a four toothed clasper (*b*) usually dark brown in color, and of a chitinous nature.

The female reproductive organ consists of ovaries, oviducts, seminal receptacles, cement glands and vagina (*Pl. II*, *fig. 3*). The ovaries (*a*) are two in number. Each of them is composed of about two hundred and ten ovarian tubes, thus making about 420 tubes in all for the two ovaries. As each of these tubes contains about fourteen ovarian eggs of various sizes, the eggs contained in the two ovaries at any given time amount to 5880 in all, although it by no means follows that all these develope into the young.

An ovarian tube (*Pl. II*, *fig. 6*) is long and slender, and contains throughout its length, eggs of various sizes and stages of development, their presence being shown externally by a series of swellings which gives to the tube the appearance of a rosary. The tube itself is enveloped by two membranes—Tunica propria (*a*), and epithelial layer (*b*). The former is thin and delicate, and provided with a few oblong nuclei (*c*), while the latter is composed of small nucleated cells. The length of the tube is about 3 mm. It is broad at its attached end but tapers gradually toward the free end, and may be divided into three portions :—1. the terminal thread (*d*), 2. the terminal chamber or Germogen (*e*) and 3. the egg-tube proper (*f*).

All the ovarian tubes belonging to one ovary, lie in close contact with one another by their terminal threads, and open into a common oviduct (*Pl. II*, *fig. 3 b*) with the broader end of the egg-tube proper, thus giving each ovary a globular form (*Pl. II*, *fig. 3 a*).

The terminal thread (*Pl. II*, *fig. 6 d*) is transparent and very deli-

cate in its texture.

The germogen (*Pl. II, fig. 6 e*) is in the form of a thin walled tube, and is filled up with a number of nucleated cells. These cells have each a large nucleus and have a comparatively small amount of protoplasm; in some cases, a few of these cells near the broad end of the germogen, are seen to be sorrounded by a part of the epithelial layer which envelopes the germogen, and are thus in the process of being differentiated into an egg (*Pl. II, fig 10 a*).

The first few eggs (*Pl. II, fig. 6 g*) in the slender end of the egg tube proper, contain also a number of nucleated cells which lie close to one another. Each of these cells-groups is surrounded by columnar epithelial cells arranged regularly side by side. A's these eggs grow, the nucleated cells contained in them increase also in size.

In the next stage (*Pl. II, fig. 6 h,h*) some nucleated cells which occupy the lower portion of the egg seem to lose their membranes, and their contents except the nuclei are converted into a mass of granulated yolk (*Pl. II, fig. 6 i*). Around this mass, there is found a vitelline membrane which is probably formed of a peripheral finely-granulated layer of it. The dissolution of the nucleated cells in an egg, and a conversion of their contents into granulated yolk are very similar to those studied by Dr. A. Weismann [(6)] on Musca vomitaria. The later stages of the development of the egg, I have not had an opportunity to examine; but the same author says in regard to this point:—"Diess ist dann das stadium, in welchem die Zellmembranen schwinden, der in den Zellen gebildete Dotter zusammentritt zu einer Masse und die kerne bis auf einen, welcher zum keimbläschen wird, zu grunde gehen. Es scheint, dass immer der Kern derjenigen eibildenden Zelle das Keimbläschen liefert, welcher am ausführungsgange der Kammer liegt,***"

Around the mature egg (*Pl. II, fig. 6 j*) which lies at the broad

(6) Zeit. f. Wiss. Zool. Bd. 14. p. 294.

end of the egg-tube proper, there is formed a chorion (*k*). It is of a somewhat chitinous nature, and bears hexagonal markings. Later it hardens and forms an egg-shell. At this stage, the egg assumes an almost elliptical form tapering slightly at one end, and rounded at the other, the longest and shortest diameters being respectively 0.25 mm. and 0.15 mm. A micropyle (*Pl. II, fig. 3 l*) is found at its tapering end.

It is very difficult to make out the mode of formation of the chorion. I believe that it is formed by a process somewhat like delamination from the epithelial cells round the eggs.

When the eggs in the egg tube proper are perfectly ripe, they are carried down into the oviduct, and are then introduced into the anterior portion of the vagina (*Pl II, fig. 3 c*) where they are fertilized by the spermatozoa from the seminal receptacles, (*d*) and are enveloped by a transparent gelatinous sticky substance which is secreted by the tubular glands (*e*). In this stage, the chorion assumes a light yellow color. The largest eggs found in the ovarian tubes are 0.25 mm. and 0.15 mm. in length and breadth respectively; but when they descend into the vagina, they grow larger, their length and breadth becoming respectively 0.3 mm. and 0.2 mm. Generally the eggs seem to complete their development in the anterior portion of the vagina, for I have always found that the eggs in the posterior portion already contain tiny maggots. The eggs which are found in the posterior portion of the vagina have the shell which becomes somewhat brittle tinged brownish black. When examined under a microscope, the convex side of the shell is colored deep brownish-black and is thick in texture, while the opposite flattened side is much lighter in color and thinner. If we inspect the flattened side with transmitted light there may be seen through the egg shell a tiny maggot developed in its interior (*Pl. II, fig. 7*). The contained maggot always lies with its dorsal

surface towards the convex side of the shell, and its ventral surface towards the flattened side. The anterior end of the maggot lies very close to the pointed end of the egg-shell where the micropyle (*Pl. II, fig. 7 m*) opens, and its posterior broader end lies in the rounded end of the egg-shell. The anterior end is at times projected through the opening of the micropyle, and it is possible that the maggot in this manner takes the sticky envelope as a food. If we examine the tiny maggot (*Pl. II, fig. 8*) taken out of the egg-shell it will be seen that each of its segments is provided at its posterior border with a single transverse row of brown setæ.

On the anterior portion of the vagina (*Pl. II, fig. 3 c*) near the junction of the two oviducts, are attached two kinds of appendages—seminal receptacles and tubular appendages or cement glands. The seminal receptacles (*Pl. II, fig. 3 d ; fig. 5*) are three in all. Each of them is a club-shaped blind tube, its free broader end which forms a sort of sac, being oval in shape, and dark brown in color on account of its chitinous nature (*Pl. II, fig. 5 a*). The entire surface of this oval sac is marked with fine wrinkles. One end of this sac opens into a long chitinous tube (*Pl. II, fig. 5 b*) which in its turn opens into the anterior portion of the vagina. The sac together with the slender tube is enveloped with a delicate cellular layer (*Pl. II, fig.* 5 *c*).

The tubular glands or cement glands (*Pl. II, fig. 3 e*) are two thin-walled slender tubes placed a little further down the vagina than the seminal receptacles. They are about 16 mm. in length, being thickly marked on their surface with irregular wrinkles, and are enveloped with a thick layer of glandular cells each containing a large nucleus.

The posterior portion of the vagina is usually widened and very extensible in nature. It is covered with numerous branches of trachea which are given off from the spiracles opening on the 4th, 5th and 6th

segments of the abdomen.

I may here add some observations on the digestive canal, and respiratory and nervous systems. Of the digestive canal (*Pl. II, fig. 11*), the proventriculus (*b*) is an oval muscular walled sac, and at the part where the œsophagus (*a*) opens into it, there is inserted the duct of the sucking stomach (*c*). The latter is a long slender tube having a bilobate thin-walled bladder-like sac at its end. The chylific stomach (*d*) is a long convoluted tube, and at the junction of it with the ileum open two malpighian vessels (*e*), of which each is divided into two long twisted branches. The chylific stomach is immediately followed by the ileum (*f*) and rectum (*re*). The rectum consists of a so-called "Rectaltasche" (*h*) and a short "Ausmündungstück" as A. Weismann [7] has called them in Musca vomitaria. The "Rectaltasche" is a thick-walled chamber, on each side of which there are found two horny white tubercles (*Rectalpapillen*, *g*). Each of these is slightly depressed at the tip, and from this depression there arise several fine air-tubes which are really branches of a trachea which opens at the spiracle on the third segment of the abdomen. The base of the tubercles, which is converted into a long pointed process, projects into the interior of the "Rectaltasche." Finally the rectum, passing posteriorly along with the common tube of vasa deferentia or vagina, ends at the anus. This opens on the dorsal side of the clasper in the male (*Pl. II, fig. 1 g*; *fig. 2 a*), while in the female it (*Pl. II, fig. 4 b*) opens on the ovipositor, separated however from the opening of the vagina (*Pl. II, fig. 4 a*) by two chitinous plates (*Pl. II, fig. 4 c*) placed side by side.

Respiratory system (*Pl II, fig. 12*) consists of a number of air-sacs and tubes. There are four pairs of air-sacs in the head, two in the thorax and one in the abdomen. Of the four pairs in the head one (*a a'*) occupies the base of the antennæ, two (*bb'*, *cc'*) the base of the

(7) A. Weismann loc. cit. p. 300.

compound eyes, and the remaining pair (*d d'*) lies in the proboscis. All these air-sacs communicate with one another by air-tubes, while the tubes sent posteriorly from the air-sacs marked *c*, *c'* put those of the head in communication with the air-sacs in the thorax and abdomen. The two pairs of air-sacs in the thorax (*e e'*, *f f'*) open directly into the spiracles on its lateral side, and the single pair of sacs in the abdomen (*g g'*) which occupy the greater portion of its cavity open into the spiracle of only the first and second segments by means of air-tubes (*Pl. I, fig. 10*). The air-tubes arising from the spiracles on the third abdominal segment have their ends attached to the tubercles resting on the "rectal-tasche," while those arising from the spiracles on the fourth, fifth, and sixth segments have most of their branches distributed on the vagina (*Pl. II, fig. 3 c*).

With regard to the nervous system (*Pl. II, fig. 13*) it will be found that the arrangements of the ganglia (Supra-and infra-œsuphageal and optic) in the head are very similar to those of the pupa of Musca vom. studied by A. Weismann, but there are two thoracic ganglia which are connected with each other by two commissures. From the posterior one of these ganglia there are given out again two long commissures, each of which, passing into the abdomen, bifurcates into two at its free end.

III.

Development of the Maggot.

As I have stated before, the flies deposit eggs during May and June, and the greater portion of them towards the end of May, this being about the time when the silkworm attains the third or fourth stage

in its development. In June, the flies decrease in number, and consequently deposit a fewer total number of eggs. This is easily seen from the small number of flies which frequent mulberry bushes, and from the scarcity of eggs attached on the leaves sprouted after the month of May.

Though the eggs are firmly attached on the under side of the leaves, they can be easily removed by wetting them with a few drops of water, for the water softens the sticky substance with which the eggs are attached. The eggs do not differ from those contained in the vagina in their form and size. Both the surfaces of the egg are marked out into hexagonal areas, which on the flat surface are however almost always rather indistinct. Its convex surface, colored dark brown and of a chitinous nature, has a lustre, while the opposite or flat surface, colored greyish brown, is membranous and destitute of a lustre, so that the contained maggot can be distinguished only when looked at from this flat surface (*Pl. II, fig. 7*).

As a general thing, eggs laid on the leaves sprouted during the month of May remain alive during the month of June, but later they are destroyed by the severe summer rays of the sun. Thus even where leaves from the same bushes are given to the summer and autumnal breeds of the silkworm, which appear respectively in the months of July and August, they are much less affected by the "Uji" than the spring breed. And, for the same reason, the winter breed is entirely free from this parasite.

As stated in the previous section, the deposition of the eggs of the fly takes place most abundantly in the last part of May viz :—at the time when the silkworm is in its third or fourth moult. Accordingly, at this period, the silkworm takes the eggs of the fly most frequently into its body, and it does this by taking them along with the mulberry leaves which serve as food. At first thought it might seem as if the

eggs would be crushed with the strong jaws of the silkworm, when the leaves are chewed, but in reality this does not take place. Owing to their hard chitinous covering and to their diminutive size, the eggs enter uninjured and whole into the digestive canal of the silkworm. That the eggs do not get hurt in passing into the body of the worm is further confirmed by comparing the size of the pieces of the leaves contained in the digestive canal with that of the egg. It will be found that the size of the former is many times that of the latter.

In one to nine hours after the eggs are introduced into the digestive canal (*Pl. III, fig. 1 a*) of the silkworm, their shell breaks open by a longitudinal slit appearing on their flat surface (*Pl. III, fig. 2*), and then a young maggot hatches out into the contents of the canal. It is however still enveloped by another thick transparent oval sac (*vetelline membrane, Pl. III, fig. 1 k*), colored light yellow. This, in its turn, soon opens at one end, and the imprisoned tiny maggot (*Pl. III, fig. 1 i; fig. 2A*) now becomes free. The empty egg shells and oval sacs mingling with fæces pass out later from the body of the silkworm. The shells and sacs thus voided by the silkworm are easily seen under a low power of microscope, by pressing fæces on a glass plate with a cover glass and by applying a few drops of water. The maggot (*Pl. III, fig. 1 i*) thus hatched out in the digestive canal measures 0.3 and 0.2 mm. in length and breadth respectively. It is colorless and transparent; its smaller anterior end is provided with a horny hooked jaw while its broader posterior end has two spiracles, and each segment of the body is covered with a transverse row of setæ.

After remaining in the digestive canal from one to eight hours, the maggot passes out through the lining wall of the canal probably with the aid of its hooked jaw, and enters directly into the ganglia which lie close beneath the canal, generally leaving those ganglia free (*Pl*

III, fig. 1 m m) which are separated from the canal by the interposition of the silk glands (*Pl. III, fig. 1 b*).

A single silkworm has usually one or two of its ganglia infested by the maggots; but sometimes more are found: in one case I found five ganglia thus infested by the parasite (*Pl. III, fig. 7 a, b, c, d, e*). Furthermore, a single ganglion may have more than one parasite in its interior. Nevertheless, it is true that almost invariably only one mature maggot crawls out of a single silkworm or cocoon. This is due to two reasons: 1st, the silkworm, when infested by more than one maggot, dies from not being able to endure the injuries caused by these parasites, which then perish by a kind of suicidal death. 2nd, one among several maggots infesting the same silkworm may grow more actively and rapidly than the others, which will then die from the want of requisite food.

When the maggots once infest the ganglia, the silkworm becomes generally weakened, and its body presents an unusual aspect from severe irritation of the nervous system. The segments are swollen out like the caterpillars of some hawked moths, and the disease is usually known by the silkworm growers as Fushidaka or Fushiko (*swelled segment*) (*Pl. III, fig. 3, 4, 5*) This disease however is not always due to the parasite, but may be occasioned by several other causes, especially pébrine. Rarely, silkworms infested by the parasite show no marked indication of illness for a long time.

That the maggots chiefly infest these ganglia which lie close beneath the digestive canal of the silkworm, is easily seen by laying open several infested specimens. In most cases, those ganglia in which the maggots are lodged are anteriorly those of the second to the fifth or sixth segments, and posteriorly those of the eighth or ninth to the last. The intervening ganglia of the middle segments, being separated from the digestive canal by the large paired silk glands, are, as already stated,

almost always left intact.

Like the typical form of Annulose animals, the nervous system of the silkworm is composed of a series of thirteen ganglia connected by two nervous cords. Each of these ganglia is enveloped by a thin transparent membrane, and contains in its interior two yellowish masses of delicate ganglion cells, each cell being provided with a large nucleus (*Pl. III, fig. 6*).

It is still uncertain by what sense the maggot is guided to the ganglia. At any rate, if, after getting out of the digestive canal, the maggot gets close to the ganglia (*Pl. III, fig. 8*), it soon makes its way into them, and makes their envelopes its special covering (*Pl. III, fig. 9*). In this position it feeds on the ganglion-cells and grows larger and larger until its covering ruptures, through excessive dilatation (*Pl. III, fig. 1 g*).

At the time when the maggot first gets lodged in a ganglion, it is very inconspicuous in size, but we may easily recognize its presence with the naked eye by opening the diseased silkworms; for the ganglion which harbours the maggot assumes always a white color, while those which are uninjured or do not harbour the parasite is always tinged light yellow. The color is in the one case simply that of the parasite and in the other that of the ganglion-cells themselves. As the maggot grows in size, the envelope of the ganglion gradually dilating forms a sort of a membranous sac; and when this stage is reached, the diseased ganglion becomes still more easy to distinguish from the others. Previous to the rupture of the sac, it becomes oblong in shape like the form of the contained maggot, so that the nervous cord appears as if it bore a large white oblong sac in place of a ganglion (*Pl. III, fig. 7 a*). The largest maggot found in a sac of this sort measured 5 mm. in length.

Generally the maggot remains in a ganglion more than a week,

and when it has become 2 to 5 mm. long or even larger in size, it gets free and passes into the body cavity of the silkworm. After travelling through the mass of fat which occupies the greater portion of this cavity, it directly searches for the portions of the tracheal system of its host, where the stigmata open. On reaching one of these places, it forces its way into the chamber directly inside the stigma, and forms a sort of a cup (*Pl. III, fig. 10 b*) for the reception of its body by heaping up the fats and muscular fibres of its host round the opening made on entering, and sticking them together with its saliva. The mouth of this cup (*Pl. III, fig. 10 e; fig. 11.*) is directed toward the body cavity, while its bottom opens into the stigma (*Pl. III, fig. 10 c*) of its host.

The maggot (*Pl. III, fig. 10 a*) which rests in the newly-formed cup, projects its anterior end into the body-cavity from the mouth of the cup, while its posterior end (*Pl. III, fig. 10 d*) is directed towards the bottom of it. In this position, the maggot anteriorly consumes fat as its food, and posteriorly respires with the air which enters through the stigma.

The cup which the maggot thus inhabits has a dark brown color partly produced by the action of the saliva upon the fats and muscles which build up the cup, and partly by the faeces which the maggot voids.

When a cup thus colored is formed inside a stigma, there appears a dark brown or brownish-black patch around the stigma (*Pl. III, fig. 12 a; fig. 13 a; fig. 14 a*); so that the presence of the patch is a conclusive evidence of the fact that the silkworm is infested by this parasite. The similar marking which occurs on the body of a pupa enclosed in a cocoon is also due to the same cause (*Pl. III, fig. 15*).

As the maggot grows in size, the cup is able to enlarge in proportion, and the maggot remains in this abode until it attains its full maturity, no matter whether the silkworm meanwhile turns into a pupa or not

The manner in which the maggot becomes parasitic is so far as known, unique, and therefore of peculiar interest to us. The opinion of my father that the maggot hatches outside and becomes parasitic by entering through the stigma of the silkworm, must not be set aside altogether, but demands a fuller investigation.

IV.

Diseases and Symptoms either caused by or concomitant with the presence of the maggot in the silk-worm.

The maggot iufesting the silkworm causes not only the disease already referred to as "Fushidaka" but gives rise to many other diseases and symptoms. Of these, the following may be mentioned:—

1st. "Umiko" (*Luisettes*) (*Pl. IV, fig. 1*) in which the segments of the silkworm swell up, the fat and visceral organs are dissolved into a milky fluid, and the animal becomes quite inactive. If the silkworm once presents these symptoms, it dies sooner or later, before spinning a cocoon. On dissecting such diseased worms I have almost always found the parasite lodged in the ganglia.

2nd. "Tareko" (*Flacherie*). The silkworm affected by this disease (*Pl. IV, fig. 2, 3*) has its body much softened and tinged dark brown, and its internal organs all converted into a dark brown fluid. A slight touch on the surface of the body is sufficient to wear off the skin and cause the dark fluid to flow out. This disease occurs in the silkworms nearly at the end of the fourth moult or later at the time when it spins its cocoon, or after it has completed it, and is known to the silkworm growers to be very injurious for, if the infested silkworms perish within a cocoon, the latter is always injured by their dark brown

contents. This is mostly due to the presence of the injurious Uji. The parasites in such diseased silkworms die with their host, and never undergo their complete development.

3rd. Sometimes the silkworm contracts its body before spinning a cocoon, or if it begins one, gives it up and begins another afresh. The silkworm thus affected is called "Jijii," and harbours the Uji in its interior. It will die after a longer or shorter struggle, (*Pl. III, fig. 14*).

4th. Most of the silkworms which are able to spin only thin cocoons are infested by the parasite.

5th. Most of the silkworms which are dull and inactive before spinning a cocoon harbour this parasite.

6th. Those silkworms, one or more of whose segments are extraordinarily dilated either before or after spinning a cocoon, contain the maggot in the ganglia of the dilated segments.

7th. Those silkworms which die while changing into pupa, or those which die after they have changed into pupa, have also the parasite.

8th. Those larvæ or pupæ which exhibit dark brown patches around their stigma have the parasite just beneath that stigma (*Pl. III, figs. 12–15*).

An interesting fact which is worthy of mention is that, if rainy weather prevails during the fourth stage of the development of the silkworm, the number affected by the parasite is large, while if fair and dry weather prevails during the same period the number of the parasites is much less. It is evident then that moisture is favorable and warm dry air unfavorable to the development of the maggot.

V.

Habits and Anatomy of the Mature Maggot.

The maggot, when it reaches maturity, leaves its abode in the body of the silkworm or its pupa, and through a hole it makes in any part of the body of the host, passes into the cocoon and thence to the outside world (*Pl. IV, figs. 4, 6*). To effect the final release, the maggot first softens one pcle of the cocoon with saliva, and there makes an opening by separating the silk fibres by means of its hooked jaw. The action of the jaw upon the softened portion of it is so severe and constant that we can always hear a peculiar cracking sound inside the cocoon. When the maggot has succeeded in making a small hole in the cocoon by the repeated action of its jaw, it projects its anterior end out of the cocoon (*Pl. IV, fig. 4*), and by a combination of expansions, contractions, and screwings, it finally wriggles out of the cocoon, leaving a round hole where it passed through (*Pl. IV, fig. 5*). In rare cases the maggot changes into a puparium while leaving a cocoon, that is, when one half of its body has gained the outside while the other half still remains in the inside (*Pl. IV, fig. 6*).

It is very strange that the infested silkworm or its pupa enclosed in a cocoon produces almost always only one single maggot and no more, though many eggs are taken in by the silkworm. This circumstance has led our silkworm growers to entertain a very false idea in regard to the nature of the maggot. As only one maggot comes out of one silkworm or pupa, they believe that the silkworm directly changes into the maggot. I have already given the reasons why only one maggot out of several attains maturity, and need not go into them again.

The time of the day when the maggot leaves the cocoon, is generally in the morning, especially of bright and hot days ; the number of those that come out at other times is very small.

The mature maggot which thus becomes free is very active, and in order to change into the pupa, generally crawls into the ground, effecting its progress by means of alternate contractions and expansions of its body, aided by the setæ which cover it as well as by its hooked jaw.

The adult maggot (*Pl. IV. fig. 7, 7 A*) is yellowish white in color and cylindrical in form. Its anterior end is sharply pointed while its posterior end is broad and abruptly truncated. It is about 20 mm. in length and 6 mm. in breadth. The body (*Pl. IV, fig. 8*) is composed of twelve segments; each segment is provided in its front half with several transverse rows of minute black setæ (*Pl. IV, fig. 8 b*), and marked with a series of transverse wrinkles as well as with a few longitudinal wrinkles placed laterally.

On the front part of the first anterior segment, there are situated two pairs of processes, the top of each of which is formed into a circular space provided with a brown rim (*Pl. IV, fig. 8 c, fig. 9 b*). The anterior pair of these seems to correspond to the antennæ, and the hinder, to the maxillary palpi, which A. Weismann [(8)] has pointed out on the larvæ of the Muscidæ. On the posterior edge of the second segment lying close to the third segment, there is situated on each side a narrow brown ridge (*Pl. IV, fig. 8 d, fig. 10 a*), which is perforated by five small spiracles, each with a brown colored rim. Again, on the posterior edge of the fifth segment, there is on each side a fleshy protuberance (*Pl. IV, fig. 8 a*) the entire surface of which, excepting a small circular space in its centre, is thickly covered with fine setæ. This circular space becomes later a hole on the puparium-case which allows the respiratory tube of the contained pupa to project outward.

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(8) A. Weismann loc. cit. p. 205.

The digestive canal (*Pl. IV, fig. 11*) begins with a mouth which is followed by œsophagus, proventriculus, chylific stomach, ileum and rectum, and finally ends at the anus opening on the last segment of the maggot. The mouth is provided with a hooked jaw (*fig. 11 ja*) of a dark brown color, which may be either projected or retracted at will. The root of the jaw which lies within the anterior segments of the maggot is parted into two white lobes, showing that the hooked jaw is formed by an amalgamation of 2 similar jaws. Into the œsophagus opens a common duct of two salivary glands (*fig. 11 a*), each having the form of a long cylindrical sac. Its walls are composed of delicate glandular cells, provided with large nuclei (*Pl. IV, fig. 12*). In the mature maggot, these sacs are always filled up with saliva; this is the very material with which the maggot moistens and softens the cocoon of the silkworm while getting out of it, and with which the maggot sticks together fats snd muscles of its host in the trachea of the silkworm so as to form a sort of cup for its own lodgement. The proventriculus (*fig. 11 e*) is in the form of a globular sac provided close to its posterior end with a pair of blind sacs (*fig. 11 f*). The chylific stomach (*fig. 11 ch*) is a convoluted tube which is much longer in ength than the ileum and rectum (*fig. 11 il, r*). At the junction of the chylific stomach and the ileum, there are provided two malpighian vessels (*fig. 11 m*), each of which again bifurcates into two branches. These branches are long and slender, and have the aspect of a zigzag line. They are colored light yellow, and their free ends which are somewhat widened are white. One of these two branches is directed anteriorly, and the other posteriorly. These vessels are composed of a long canal, in the walls of which are imbedded large delicate cells (*Pl. IV, fig. 13*), each containing a large nucleus in the centre. The contents of the cells are granular and tinged light yellow, and to the peculiar arrangement of these cells is due the zigzag appearance

of the branches of the vessels.

Muscular system (*Pl. IV*, *fig. 14, 15*).—The inside of the walls of the body is provided with three sets of muscular bundles— longitudinal, transverse and oblique. The longitudinal bundles (*fig. 14 a*, *fig. 15 a*) are two in number and run, one on each side of, and in close proximity to, the ventral median line, thus leaving only a long narrow space between them. They are constricted transversely at regular intervals, i.e., at the junction of successive segments beginning with the 5th and ending with the last. The longitudinal bundles thus divide the body into two unequal longitudinal parts, a narrow ventral portion and a much larger dorsal portion. The narrow ventral portion is occupied with series of oblique bundles (*fig. 14 b*, *fig. 15 b*), which will be spoken of later. The larger dorsal portion is provided with three superposed layers of muscular bundles. The most external of them are the transverse bundles (*fig. 15 c*). Generally there are two pairs to a segment, each bundle starting ventrally from the longitudinal bundles and ending close to the dorsal median line. Internal to the transverse bundles is a set of oblique bundles (*fig. 14 d*, *fig. 15 d*). These arise symmetrically on each side of the dorsal median line, in close proximity to it, from the junction of the segments, and are directed backward and ventralward. They appear to be attached at the joints of the longitudinal bundles three segments behind the one from which it takes its origin, but in reality pass under the longitudinal muscles and dividing into several branches (*fig. 14 b*, *fig. 15 b*) run toward the ventral median line where the bundles of two sides interlock with their extremeties. The most internal set of muscular bundles (*fig. 14 e*, *fig. 15 e*) is likewise oblique, arising from exactly the same spot as the last, but taking their course forward and ventralward. The two sets of the oblique bundles together with the transverse will be seen to present an appearance something like a network.

Besides these bundles mentioned above, there are provided three pairs of short but broad bundles at the base of the jaw. Two (*Pl. IV, fig. 15 f*) of these have their distal end attached to the dorsal wall of the anterior segments, while the remaining one arises from their ventral wall. These make the jaw project outward or retract inward.

Dorsally just between the two hemispheres of the supera-œsophageal ganglion which will be described later on, there is placed an oval muscular ring (*Pl. V, fig. 3 j*) which is anteriorly supported by two air tubes (*Pl. V, fig. 3 k*) and posteriorly by a muscular string (*Pl. V, fig. 3 l*) arising from the posterior portion of the proventriculus (*Pl. V, fig. 3 pr*). Through the ring passes the delicate dorsal vessel (*Pl. V, fig. 3 he*).

The respiratory system consists of two long air-tubes or tracheæ (*Pl. V, fig. 1*) passing lengthwise on either side of the median line of the body. From the main trunk of the tracheæ (*Pl. V, fig. 1 a*) are given out smaller branches in each segment, and fine ramifications of these are distributed to all internal organs. The posterior broader end of the two main trunks opens separately into a spiracle (*Pl. V, fig. 1 b*) which is situated at the truncated end of the last segment of the maggot. The spiracle (*Pl. V, fig. 2 a*) is protected by an oval chitinous disc colored dark brown, and marked with several wavy ridges, as well as with a few strong radial lines. Just inside the spiracle, a single large branch (*Pl. V, fig. 1 c*) is given out, whose smaller branches are mostly distributed over the intestinal canal. The anterior end of the main tracheal trunks (*Pl. V, fig. 3 a'*) is connected with the five small spiracles (*Pl. IV, fig. 10 a*) on the second segment of the maggot On the third segment, these two trunks are connected with each other by a fine transverse air-tube (*Pl. V, fig. 3 a*). The four smaller branches given out at about the same portion of the trunks bear each an oval disc on its free end (*Pl. V, fig. 3 b, c, d, e*) What these discs

give rise to later in development has not been ascertained. On the 4th segment, each trunk bears on its branches an oval disc (so called "flugelscheiben[9]) (*Pl. V, fig. 3 f*) which afterward becomes a wing. Finally on the 5th segment, the same trunks on each side bear two oval discs—the one (untere metathoracalscheibe[10]) (*Pl. V, fig. 3 g*) is concerned later in forming the third leg while the other (obere metathoracalscheibe[11]) (*Pl. V, fig. 3 h*) is destined to form the balancer.

Nervous system—The oblong ventral mass (bauchmark) (*Pl. V, fig. 4 a, fig. 5 a, fig. 6 a*), which is formed out of the amalgamation of the ventral chain of ganglia, lies in the 5th segment of the maggot. On its anterior end which is in the fourth segment is attached a supra-œsophageal ganglion (obere schlundganglion) (*Pl. V, fig. 5, 6 b*) which bearing a constriction at the median line, assumes a form of two hemispheres arranged side by side. The infra-œsophageal ganglion could not be made out on the under side of the ventral mass. Weismann[12] says:—"Auch das untere schlund-ganglion is mit diesem Bauchmark so vollständig verschmolzen, dass es sich in keiner Weise hervorhebt, * * *." From the under side of the ventral mass, twelve pairs of nervous cords are given out, each of which extends respectively to each of the twelve segments of the maggot (*Pl. V, fig. 5* 1–12).

Each nervous cord of the 1st, 2nd and 3rd pairs (*Pl. V, fig. 5* 1, 2, 3) has along its side a spindle-shaped body bearing marks of concentric layers. The spindle-shaped body on the cord of the first pair (*Pl. V, fig. 5* 1) which lies on the "Augenscheibe," (*Pl. V, fig. 5 d*) seems later to form "Stirnscheibe" and "Stirnanhange" which has been pointed out by Weismann in the larva of Muscidæ. Of the

(9) A. Weismann. loc. cit. p. 238.
(10) " " " " p. 236.
(11) " " " " p. 241.
(12) " " " " p. 205.

spindle-shaped body on the cord of the 2nd and 3rd pair, the former (*Pl. V, fig. 5 f*) seems to correspond to the "untere prothoracalscheibe" and the latter (*Pl. V, fig. 5 g*) to the "untere mesothoracalscheibe" which Weismann has described in the same larva.

All the pairs following are directed posteriorly. The first (*Pl. V, fig. 5 4*) of these pairs which is really the 4th nervous cord, lying far apart from the rest which follow it, passes into the fourth segment, while each of the remaining pairs which are regularly arranged one after the other extends to the 5th, to 12th segments respectively.

VI.

The Pupa and its Development into the Mature Insect.

As the maggot makes its exit from the cocoon of a silkworm, it immediately searches for dark places, especially for moist ground by getting down through some cracks or fissures in the floor of the house where it comes out. When it has gained a suitable locality, it bores into it with the aid of its hooked jaw. Having reached the depth of two or three inches or even more beneath the surface, it begins to contract in length and then changes into a cylindrical puparium tinged light yellow (*Pl. V, fig. 7*). It is slightly smaller at the anterior than at the posterior end. The puparium, which is really formed out of the larval skin, gradually hardens, and loses the wrinkles which were previously visible. We can still, however, distinguish the segments of the maggot.

This newly formed puparium of light yellow, has its color soon changed into crimson red, which deepens gradually until at

the end of about thirty hours it becomes black. In this stage, the contents of the puparium are mostly dissolved into a milky white fluid containing large quantity of fat, without apparent structure of any sort.

At the end of the third day, this white fluid solidifies and becomes an oblong mass or pupa, enveloped with two thin membranes—the outer and the inner. The outer membrane, which is slightly thicker than the other and tinged light yellow, lies close inside the hardened larval skin or puparium, while the inner (pupa-skin) which is colorless and transparent covers directly the oblong mass which later becomes the pupa. The space between these two membranes is filled with a small amount of a colorless liquid which is necessary to protect the contained pupa from dessication. Generally, the anterior smaller end of the pupa can easily be distinguished from the broader posterior end (*Pl. V, fig. 8*). Dorsally the anterior end (*Pl. V, fig. 9 a*) is separated by a slight coustriction from the rest of the body, and near the front edge of the same end lies a pair of tubercles (*Pl. V, fig. 9 b*) from each of which is projected a sort of a short black horny spine (respiratory tube). Ventrally on the same end are developed three pairs of small incomplete legs (*Pl. V, fig. 8 a*), those of one side meeting with their fellows of the opposite side along the median ventral line. At the base of the legs are produced two lamellar sac-like paired bodies, the wings, (*Pl. V, fig. 8 b*), each of which covers the greater portion of the last two legs on its own side.

At the end of a week, the body of a pupa is plainly divided by a deep constriction (*Pl. V, fig. 10 c*) into two regions—the anterior being much smaller than the posterior. From the former, the head and thorax of the adult are developed; and from the latter, the abdomen. At this stage, the paired tubercles (*Pl. V, fig. 10 a*) which lay in the previous stage close with each other at the front end of the body

assume by further development a shape somewhat like that of a top, and, at the same time, they became separated from each other by a space between them. Also on each side near the posterior margin of the anterior region there arise two tiny tubular processes (*Pl. V, fig. 10 b*), which later form the balancer or halter. The three pairs of legs (*Pl. V, fig. 11 a*) are somewhat increased in length, but the bases of the last two pairs are still covered by the lamellar sac-like wings (*Pl. V, 10 b*).

On the eighth or ninth day, a large median process is developed from the space between the two top-shaped tubercles in front. This forms the head. Between it and the succeeding portion, a transverse constriction appears, marking out the regions of the head (*Pl. VI, fig. 1 a*) and thorax (*Pl. VI, fig. 1 b*). The head assumes a triangular form being composed of three large lobes — one median (*Pl. VI, fig. 1 e*) and two lateral (*Pl. VI, fig. 1 f*). On the dorsal median portion between these lateral processes, appear three tiny processes (*Pl. VI, fig. 1 g*) lying close together, which are the first appearance of the simple eyes. On each side of the base of the mouth-parts (*Pl. VI, fig. 2 a*) on the under side of the head arises a small process (*Pl. VI, fig. 2 b*), which subsequently forms the marginal ridges of the cavity in which the mouth-parts are inserted. The top-shaped bodies (*Pl. VI, fig. 1 h*), which were situated on the front edge of the anterior region in the last stage, alter their positions as the head is developed, and are at the junction of the head and thorax. On the ventral side of the thorax, the first pair of legs which were previously close to each other are now separated by a small space, which become occupied by the mouth-parts or proboscis (*Pl. VI, fig. 2 a*). The membranous flap or alula (*Pl. VI, fig. 1 c*) is also formed just below the balancer.

On the 12th day, the pupa (*Pl. VI, fig. 3* and *4*) is colored light yellowish brown, and the three regions of its body attain their

proper relative sizes. The two large lateral lobes of the head (*Pl. VI, fig. 3 a*), receiving a light copper red pigment, become the compound eyes. On the anterior edge of the head, a pair of antennæ (*Pl. VI, fig. 4 a*) appears, which are laid flat on the under side of the head, and lie in a depression excavated just above the insertion of the mouth-parts. The tiny processes (*Pl. VI, fig. 4 b*) on either side of the insertion of the mouth-parts grow larger; and the top-shaped bodies (*Pl. VI, fig. 3 b*) at the junction of the head and thorax recede backwards in proportion as the head grows in size, and thus become attached to the anterior corners of the thorax. The summit of each top-shaped body (*Pl. VI, fig. 3 b*) is covered with a round chitinous disc (*Pl. VI, fig. 3 e*), in the centre of which stands a long pointed chitinous tube (*Pl. VI, fig. 3 f*). The pointed end of the tube projects through a perforation (*Pl. V, fig. 7 a; Pl. VI, fig. 3 d*) at the side of the fifth segment of the puparium, and formed no doubt on the membranous disc on the top of the fleshy protuberance (*Pl. VI, fig. 8 a*) which we found on each side of the fifth segment of the maggot. From this circumstance, it will be seen that the head of the pupa is formed out of the anterior four segments of the maggot. Dorsally the space (*Pl. VI, fig. 3 g*) between the compound eyes is covered with long hairs which are arranged on each side in a single row along the inner side of the compound eye. The thorax (*Pl. VI, fig. 3 h*) is now separated into three segments by the presence of two slight transverse lines. The anterior two segments are covered with six longitudinal rows of long hairs; but the last has hairs only on its posterior rounded edge. The wings (*Pl. VI, fig. 4 c*) grow larger in size, and have each a fold on the outer edge. The abdomen (*Pl. VI, fig. 3 i*) is also separated into four segments by three transverse lines, but no hairs can as yet be seen on its surface.

On the 28th day (4 weeks), the pupa (*Pl. VI, figs. 5* and *6*) has

deepened much in color until it is now dark brown The top-shaped bodies (*Pl. VI, fig. 5 a, fig. 7*) are well developed; and on their summit rests a chitinous disc (*Pl. VI, fig. 7 a*). The smaller or proximal end of these bodies opens into a spiracle situated on the sides of the prothorax of the pupa. On the outside and very close to this small end lies a bright reddish yellow disc of an oval shape (*Pl. VI, fig. 7 b*), the surface of which is marked with reticulated ridges. The function of this oval disc seems to be to prevent the smaller end of the membranous tube from closing up and interrupting the passage for the entrance of the external air. The mouth-parts are developed further than on the last stage, and form a rudimentary proboscis which is now separable into basi-proboscis (*Pl. VI, fig. 6 bp*) and proboscis proper, (*Pl. VI, 6 pp*). The labrum which has a wedge-shape (*Pl. VI, fig. 6 b*) lies on the proboscis proper with its pointed end toward the labellæ (*Pl VI, fig. 6 d*). The maxillary palpi (*Pl. VI, fig. 6 a*) which arise on each side of the distal portion of the basi-proboscis are developed into short thread-like processes. The labium (*Pl. VI, fig. 6 c*) which forms a greater portion of the proboscis proper has its floor protected by a chitinous shield, and its free end divided into two expanded lobes (*Pl. VI, fig. 6 d*) by a slight constriction. The margin of these lobes is covered with a few bristles, and their inner surface is closely beset with many transverse ridges (pseudotracheæ). The segments of the abdomen become more distinct, and are covered with a few transverse rows of conspicuous hairs.

On the 35th day, the body of the pupa has become uniformly darker in color (*Pl. VI, fig. 8* and *9*). The simple eyes (*fig. 8 a*) become more distinct, and the insertion of the maxillary palpi (*fig. 9 a*) ascends a little above their previous position. In this state the pupa remains through the winter months to the following spring, generally to April. The only external changes which I have observed in the

space of time, is that of coloration, and the increase of hair: thus the color of the pupa changes gradually from light to grayish brown, and then to grayish black slightly tinged with yellow, the compound eyes become deep ochre-red, and the hairs on the body grow thickly all over its surface.

The pupa when lodged under ground is liable to die from two causes, (1) from the unfavorable condition of the earth in which it rests, and (2) from the ravages of a parasitic mite:—1st. When the earth keeps moist and unexposed to the sun-light, the pupa may remain alive and undergo its complete development; but if on the contrary the earth becomes dry or is exposed to the direct rays of the sun, the pupa will die in a longer or shorter period. 2nd. The mite which infects the pupa is probably a species of Tyroglyphus (*Pl. VI, fig. 10*). The body which is about 0.45 mm. in length, is oval in shape, faint yellow in color, and covered with a few long colorless bristles. There are four pairs of long jointed legs, of which each is composed of five segments, and ends with a single hooked claw. The male and female are similarly colored, but differ slightly in size. The eggs (*Pl. VI, fig. 11*) are oval in shape and very small, being only 0.13 mm. in the longest diameter. The egg-shells are hard and chitinous, and their surface is decorated with scattered masses of granules. When the young is in the process of hatching out, the one pole of the egg opens by a valve, through which it soon emeeges. Sometimes these parasites increase enormously in number, so that the interior of the puparium is filled up with hundreds of them instead of with the pupa of the fly.

About the middle part of April of the following year, the perfect fly begins to appear above ground, and continues to come out during the following three weeks. As I have mentioned before, the pupa remains at the depth of 2–3 inches or even more beneath the ground. The manner in which the perfect fly hatches out from a puparium is

a curious phenomenon to observe. On account of the tough yet brittle nature of the chitinous puparium, it breaks open easily when certain forces are applied. The puparium (*Pl. VI, fig. 12*) breaks invariably at its anterior end, and the lines of breakage are in two directions—one longitudinally in the direction of *c d.*, and the other transversely in the direction of *a b.*, so that the anterior portion above the fifth segment is divided into two pieces (*Pl. VI, fig. 13 a, b*). The transverse line of breakage (*Pl. VI, fig. 12 ab*) occurs at the junction of the fourth (4) and fifth (5) segments of the puparium, of which the latter is firmly attached to the body of the contained pupa (*Pl. VI, fig. 14 c*) by the chitinous tube or spine (*Pl. VI, fig. 14 e*) provided on the summit of the top-shaped bodies on the anterior corners of the thorax. The line of the longitudinal fracture (*Pl. VI, fig. 12 c d*) divides the part of the puparium in front of the transverse fracture into two bilaterally symmetrical halves (*Pl. VI, fig. 13 a, b*). Every specimen of the puparium always breaks open in this way. Now the questions arise: how do all the puparium break open in exactly the same mode, and how does the newly hatched fly, which is perfectly helpless, get above the ground? These are explained by a simple mechanical process which the young flies perform. In escaping from the puparium, the fly, in a most strange fashion, protrudes a curious air bladder (*Pl. VI, fig. 14 f*) from the oval space (*Pl. VI, fig. 15 a*) on the front edge of its head between the compound eyes. This oval space is membranous marked with a transverse line in the middle, and surrounded by a dark line. Both the transverse and the circumferential lines are due to the portion of the membranous space being folded into the interior of the head, so that when the air has fully expanded this space, no trace of these lines is seen. This becomes clearer by looking at a section of the head shown in (*Pl. VI. fig. 16*) in which *a a'* are the dark marginal, and *c*, the central transverse lines. The air-sac

is thickly covered with short strong setæ (*Pl. VI, fig. 16 b*), and it can be either folded in or projected out of the head at will, simply by driving the air from the air-sacs in the head into the sacs in the thorax and abdomen or by reversing this process.

As has been said before, the contained fly is firmly attached to the puparium or pupa case at the two points where the respiratory tubes stick out on the fifth segment (*Pl. VI, fig. 14 a a'*). These points act as the fulcrum when the air-bladder is expanded to force open the puparium. Again the air-bladder is not circular, but squarish in form, so that when it is expanded, it touches the puparium only at two points *b b'* (*fig. 14*). The greatest force is therefore applied at these two points, and the tendency will be to split the part of the puparium, in front of the 5th segment in the line *c d fig. 12.* The transverse fracture naturally occurs at the junction of the 5th and 4th segments as the body of the fly is firmly fixed to the former. These considerations will explain why the puparium invariably breaks open in exactly the same manner.

By carefully listening, we may easily recognize the hatching out of the fly (*Pl. VI, fig. 17 b*) under ground by a peculiar cracking sound produced at the time when the puparium is broken open by the action of the temporary air-bladder. When the perfect fly (*Pl. VI, fig. 17 c c'*) has made its exit out of the puparium (*Pl. VI, fig. 17 a a'*) by the process just described, its next step is to crawl up to the surface of the ground. As the wings of the newly-hatched flies are folded up, and their legs so soft that they are perfectly useless for locomotion, they cannot use the ordinary means of reaching the surface against the resistance of the earth which overlies them. This, however, they easily perform by the alternate expansion and contraction of the air-bladder of the head. They first project the air-bladder (*Pl. VI, fig. 17 e*) upward into the overlying layer of earth, and then, expand

it, so raising their body through a small distance. The space vacated is no doubt soon filled up with the earth so that they can not descend to their original position. They then contract the bladder and repeat the operation, effecting another slight ascent. By successive repetitions of the same they finally gain the surface of the ground (*Pl. VI, fig. 17 d*).

The days in which the flies hatch out beneath the ground are always fair and bright with a temperature above 50° F. They appear above the ground usually at from six to ten a. m. Those that appear later in the day are comparatively few in number. On the other hand, if the weather is rainy and the temperature below 50° F., they very rarely make their appearance.

Generally on appearing above ground, they rest quietly for 30 to 40 minutes, after first crawling into some shady place if the rays of the sun fall directly on the spot where they emerged. During this time, their body deepens much in color and their folded wings expand completely. When the wings attain their proper size, they begin to vibrate them at short intervals and finally fly off into the air.

After a few weeks, they attain maturity, and deposit their eggs on the under-surface of the leaves of the mulberry-trees to perpetuate their offspring. The life history which we have been following is thus completed.

Measures for Protecting the Silkworm from the Ravages of the Fly.

From the foregoing study of Ugimya sericaria, the following measures for protecting the silkworm from the ravages of the fly suggest themselves :—

1st. There is no danger of introducing the eggs of the fly into the silkworm, if every mulberry leaf given to the silkworm is examined, and the eggs are removed, if found attached. But this method has only a theoretical value ; it would be utterly impossible to carry it into effect, for it would require an enormous expenditure of time and labor. The only methods which I therefore believe to be practicable are by attending to the modes of planting mulberry trees by which the flies may be prevented from visiting the bushes and depositing their eggs.

2nd. As a general thing, the flies select for the deposition of their eggs mulberry trees planted on moist or shady grounds or near ponds, marshes, ditches &c. or in the vicinity of the spots where various much branched shrubs grow in numbers or in any other places where air currents do not freely circulate. On the other hand, those planted on dry and high grounds or in the neighbourhood of large streams or on spots where air passes freely, are mostly free from the eggs of the fly. On this account, those who desire to plant on new grounds should consider their features from this point of view.

3rd. In order to prevent the deposition of fly-eggs on the leaves of mulberry trees already planted very close to one another, one or two stumps should be rooted out at regular intervals so as to allow the air to pass freely between the remaining trees. If it is required on the other hand to plant the trees on new grounds, they should be distributed in the ratio of one stump to one Tsubo (6 feet square) or of 300 stumps to one Tan (300 Tsubo or 10,800 sq. ft.)

4th. If two varieties of mulberry trees which sprout at different times were planted in alternate rows, there would obviously be freer passage of air through the plantation than if all sprouted simultaneously. Such a precantion will certainly have a good effect.

5th. The old and much exhausted trees which bear their leaves crowded on shortened stems are selected most by the flies, so that, if

possible, such trees should be removed and new and vigorous trees planted in their place. In any case, the leaves of such old and exhausted trees should be given to the silkworm before its third moult, as the fly does not deposit its eggs till after that time.

6th. To avoid the visit of the fly, it seems better to plant mulberry trees in separate rows parallel to the direction of the prevalent wind in May.

7th. The raising of the summer and autumnal breeds of silkworms with the leaves which have sprung out later than the month of June would answer best the purpose of escaping the damages of the parasite. But stripping off the leaves at this season is very injurious to the growth of mulberry trees, and also the later breeds are much inferior in their growth to the spring breed, so that it is still most advantageous to raise the spring breed alone with great care and giving the leaves of the best sort.

8th. The silkworms afflicted by the diseases of Umiko, Tareko, Fushidaka &c. and which are not likely to spin the cocoon should be killed by putting them into salt or lime water, for most of them contain the Uji.

9th. In the month of May and June the flies wander in the bushes of mulberry trees for the deposition of their eggs. No pains should be spared in catching and killing them, for a single female may be able to deposit about six thousand eggs.

10th. Besides the silkworm, there are larvæ of other moths which are infested by the maggot of the same fly:—one is Kuwako (larva of a wild silkworm) which feed on mulberry trees, and another is the larva of Laurion atratus, Butler, which feeds on Simplocus cratezoides, Don. The specimen of the latter I have received from my friend Mr. C. Ishikawa. The larvæ of both should be caught and killed.

11th. The pupa of the silkworm and the maggot imprisoned in

a cocoon may generally be killed by thoroughly drying the cocoon either artificially or by means of the sun. The former answers best for the purpose, for the temperature of the sun changes from hour to hour and is not well adapted for killing both the contained pupa and maggot.

EXPLANATION OF PLATES.

PLATE I.

Fig. 1. Ugimya sericaria, Rond. (male). Dorsal view. *a a*. Chitinous rods. *al*, *b*, *b'*. Three alulæ.

Fig. 1A. Ditto. (female). Dorsal view $\frac{2}{1}$.

Fig. 2. Ditto. (side view). *a*. Spiracle. $\frac{2}{1}$.

Fig. 3. Wing of Ugimya. *a a*. Costal vein. *b*. Transverse shoulder vein. *c*. Auxilliary vein. *d*, *e*, *f*, *g*, *h*, *i*. 1st, 2nd, 3rd, 4th, 5th and 6th longitudinal veins. *j*. Small transverse vein. *k*. Hinder transverse vein. *l*. Anterior basal transverse vein. *m*. Posterior basal transverse vein. *n*, *o*, *p*. Three alulæ. *q*. Apex of wing. *r*. Outer oblique edge of wing. *s*. and *t*. Two chitinous rods. $\frac{4}{1}$.

Fig. 4. Antenna of Ugimya. *a*. Bristle. $\frac{20}{1}$.

Fig. 5. Mouth parts of Ugimya. Dorsal view. *A*. Basi proboscis. *B*. Proboscis proper. *a*. Pharynx. *b*. Muscles on upper processes of pharynx. *c*. Chitinous oval disc. *d*. Single jointed palp. *e*. Modified labrum. *f*. Long chitinous rod. *g*. Labellæ. *h*. Mouth opening. *l*. Processes on upper side of pharynx. *n*. Air-tube. *r*. Fine hairy chitinous rod. $\frac{10}{1}$.

Fig. 6. Ditto. Side view. *i*. Outer surface of labellæ. *j*. Labium. *m*. Processes on lower side of pharynx. *b'*. Muscles on lower processes of pharynx. Other letters same as *Fig. 5*. $\frac{10}{1}$.

Fig. 7. Ditto. Ventral view. *k*. T-shaped chitinous body. Other letters same as *Fig. 5*. $\frac{10}{1}$.

Fig. 8. Spiracle on the thorax magnified $\frac{60}{1}$.

Fig. 9. Spiracle on the abdomen magnified. $\frac{100}{1}$.

Fig. 10. Spiracles on the abdomen. *a*. Spiracle. 1, 2, 3 &c. show the number of segments. $\frac{5}{1}$.

Fig. 11. Showing the position of eggs on the leaves of mulberry trees. Highly magnified.

PLATE II.

Fig. 1. Sexual organ of Ugimya (male). *a.* Testes. *b.* vas deferens. *c.* Penis. *g.* Anus. *h.* Common tube. *k.* Seminal reservoir. $\frac{8}{1}$.

Fig. 2. Penis (*c*) with clasper (*b*). *a.* Anus. $\frac{20}{1}$.

Fig. 3. Sexual organ of Ugimya (female). *a.* Ovary. *b.* Oviduct. *c.* Vagina. *d.* Receptaculum seminis. *e.* Cement gland. *f.* Air-tubes. $\frac{8}{1}$.

Fig. 4. Ovipositor of ditto. *a.* Opening of vagina. *b.* Anus. *c.* Chitinous plate. $\frac{16}{1}$.

Fig. 5. Receptaculum seminis. *a.* Its broader end. *b.* Long chitinous tube. *c.* Delicate cellular layer. $\frac{40}{1}$.

Fig. 6. Surface view of an ovarian tube. *a.* Tunica propria with nucleii (*c*). *b.* Epithelial layer. *d.* Terminal thread. *e.* Germogen. *f.* Egg-tube proper. *g.* Egg-cell. *h, h.* Egg-cells of more advanced stage. *i.* Granulated yolk. *j.* Mature egg. *k.* Chorion. *l.* Micropyle. $\frac{200}{1}$.

Fig. 7. Egg taken out of the posterior portion of vagina. Seen from its flattened side. *m.* Micropyle. $\frac{200}{1}$.

Fig. 8. Maggot taken out of the egg. $\frac{200}{1}$.

Fig. 9. Newly laid egg of Ugimya. $\frac{200}{1}$.

Fig. 10. Germogen with a part of egg-tube proper *a.* Group of nucleated cells surrounded by the epithelial layer. $\frac{110}{1}$.

Fig. 11. Digestive canal of Ugimya. *a.* Œsophagus. *b.* Proventriculus. *c.* Sucking stomach. *d.* Chylific stomach. *e.* Malpighian vessels. *f.* Ileum. *g.* Rectalpapillen. *h.* Rectaltasche. *i.* Anus. *j.* Salivary glands. *re.* Rectum. $\frac{4}{1}$.

Fig. 12. Diagrammatic figure of the respiratory system of Ugimya. *a a'*, *b b'*, *c c'*. Air sacs in the head. *d d'*. Those in the proboscis. *e e'*, *f f'*. Those in the thorax. *g g'*. Those in the abdomen.

Fig. 13. Nervous system of Ugimya. *o g*. Optic ganglion. *s g*. Supra-œsophageal ganglion. *i g*. Infra-œsophageal ganglion. *t g*, *t g'*. Two thoracic ganglia. *cm*. Nerve commissures in the abdomen. *cd*. Optic disc.

PLATE III.

Fig. 1. Ideal section of silkworm showing its internal organs in which the maggot lodges. *a*. Digestive canal. *b*. Silkgland. *c*. Nervous system. *d*. Ganglion containing a maggot. *e*. Trachea. *f*. Tracheal chamber just beneath the stigma, into which maggot makes its way. *g*. Empty ganglion which maggot has left. *h*. Tracheal chamber with a maggot. *i*. Maggots hatched out in the digestive canal. *j*. Empty egg-shell. *k*. Chorion.

Fig. 2. Empty egg-shell of Ugimya with a fracture on its flattened side. $\frac{100}{1}$.

Fig. 2A. Maggot leaving chorion. $\frac{80}{1}$.

Fig. 3,4,5. Diseased silkworms having maggots in their ganglia. $\frac{1}{1}$.

Fig. 6. Ganglion-cells of silkworm. $\frac{300}{1}$.

Fig. 7. Nervous system bearing five ganglia (*a*, *b*, *c*, *d*, *e*.) infested by maggots. $\frac{1}{1}$.

Fig. 8. Maggots entering a ganglion. $\frac{80}{1}$.

Fig. 9. Ganglion containing a maggot. $\frac{80}{1}$.

Fig. 10. Cup-like chamber on the trachea of silkworm containing a maggot. $\frac{10}{1}$.

Fig. 11. Inside view of a cup-like chamber from which the maggot has been removed. $\frac{10}{1}$.

Fig. 12, 13, 14. Silkworms infested by maggots. *a.* Dark patches around stigma. $\frac{1}{1}$.

Fig. 15. Pupa of a silkworm infested by a maggot. *a.* Dark patches around stigma. $\frac{1}{1}$.

PLATE IV.

Fig. 1. Umiko. $\frac{1}{1}$.

Fig. 2. Tareko. $\frac{1}{1}$.

Fig. 3. " $\frac{1}{1}$.

Fig. 4. Maggot leaving the cocoon of silkworm. *a.* maggot. $\frac{1}{1}$.

Fig. 5. Cocoon with a round hole at one pole. $\frac{1}{1}$.

Fig. 6. Maggot changed into pupa while leaving cocoon. *a.* Hole in silkworm pupa through which maggot has got out. *b.* Puparium. $\frac{1}{1}$.

Fig. 7. Dorsal view of adult maggot. $\frac{1}{1}$.

Fig. 7A. Ventral view of adult maggot. $\frac{1}{1}$.

Fig. 8. Adult maggot. *a.* Fleshy protuberance. *b.* Minute black setæ. *c.* Circular space with a brown rim. *d.* Narrow brown ridge. *1, 2, 3–12.* Number of segments. $\frac{5}{1}$.

Fig. 9. Top view of the first segment of maggot. *a.* Hooked jaw. *b.* Circular space with a brown rim. $\frac{15}{1}$.

Fig. 10. Side view of the anterior three segments (*1, 2, 3.*) of adult maggot. *a.* Narrow brown ridge. *b.* Circular space with a brown rim. $\frac{15}{1}$.

Fig. 11. Digestive canal of maggot. *a.* Salivary gland. *c.* Œsophagus. *e.* Proventriculus. *f.* Blind sac. *ch.* Chylific stomach. *m.* Malpighian vessel. *il.* Ileum. *r.* Rectum. *an.* Anus. *ja.* Jaw. $\frac{3}{1}$.

Fig. 12. Salivary gland, magnified. *a.* Glandular cell with a large nucleus (*b*).

Fig. 13. Part of malpighian vessel, magnified. *a.* Large delicate cell with a nucleus (*b*).

Fig. 14 Muscular system of maggot seen from the ventral side. *a.* Longitudinal bundle. *b, d.* Oblique bundle. *c.* Transverse bundle. *e.* Most internal set of muscular bundle. $\frac{3}{1}$.

Fig. 15. Muscular system of maggot seen from the dorsal side. *f.* Muscular bundles at the base of jaw (*j*). Other letters indicate the same parts as the *Fig. 14.* $\frac{3}{1}$.

PLATE V.

Fig. 1. Respiratory system of maggot. *a.* Main trunk of tracheæ. *b.* Spiracle. *c.* Large branch of tracheæ given out at the base of a spiracle. $\frac{3}{1}$.

Fig. 2. Truncated end of the last segment of maggot. *a.* Spiracle. $\frac{10}{1}$.

Fig. 3. Anterior portion of the main trunk of tracheæ of maggot in relation with other organs. *a.* Branch of tracheæ connecting a trunk of tracheæ on both sides. *a'* Anterior end of the main trunk. *b, c, d, e.* Oval discs on the free end of the trunk. *f.* Flugelscheibe. *g.* Untere metathoracalscheibe. *h.* Obere metathoracalscheibe. *i.* Augenscheibe. *j.* Muscular ring. *k.* Air-tubes. *l.* Muscular string. *he.* Dorsal vessel. *pr.* Proventriculus. *ja.* Jaw. $\frac{10}{1}$.

Fig. 4. Nervous system of maggot (Dorsal view). *a.* Oblong ventral nervous mass. *b.* Muscular ring. *c.* Augenscheibe. *1, 2, 3–12.* Nervous cords passing from 1st to 12th segments respectively. $\frac{6}{1}$.

Fig. 5. Ditto (ventral view). *a.* Oblong ventral nervous mass. *b.* Supra-œsophageal ganglion. *d.* Spindle shaped body. *e.* Augenscheibe. *f.* Untere-prothoracalscheibe. *g.* Untere

mesothoracalscheibe. *1*, *2*, *3–12*. Twelve nervous cords. Highly magnified.

Fig. 6. Ditto. (side view). The letters indicate the same parts as in *Fig. 5.* Highly magnified.

Fig. 7. Puparium of Ugimya sericaria, R. *a.* Hole on the fifth segment. $\frac{3}{1}$.

Fig. 8. Ventral view of pupa, three days old. *a.* Legs. *b.* Wing. $\frac{3}{1}$.

Fig. 9. Dorsal view of ditto. *a.* Its anterior end. *b.* Tubercle. $\frac{3}{1}$.

Fig. 10. Dorsal veiw of pupa, a week old. *a.* Tubercle. *b.* Tiny tubular process. *c.* Deep constriction. $\frac{3}{1}$

Fig. 11. Ventral view of ditto. *a.* Legs. *b.* Wing. $\frac{3}{1}$.

PLATE VI.

Fig. 1. Dorsal view of pupa, eight or nine days old. *a.* Head. *b.* Thorax. *c.* Alula. *d.* Abdomen. *e.* Median lobe. *f.* Lateral lobe. *g.* Tiny processes. *h.* Top-shaped body. *i.* Balancer. $\frac{3}{1}$.

Fig. 2. Ventral view of ditto. *a.* Mouth-parts. *b.* Small process. $\frac{3}{1}$.

Fig. 3. Dorsal view of pupa, 12 days old. *a.* Large lateral lobe. *b.* Top-shaped body. *c.* Section of puparium. *d.* Perforation on the fifth segment of the puparium. *e.* Round chitinous disc. *f.* Pointed chitinous tube. *g.* Space between the compound eyes. *h.* Thorax. *i.* Abdomen. $\frac{3}{1}$.

Fig. 4. Ventral view of ditto. *a.* Antennæ *b.* Tiny process. *c.* Wing. $\frac{3}{1}$.

Fig. 5. Dorsal view of pupa, 28 days old. *a.* Top-shaped body. $\frac{3}{1}$.

Fig. 6. Ventral view of ditto. *a.* Maxillary palp. *b.* Labrum. *c* Labium. *d.* Expanded lobe. *bp.* Basi-proboscis. *pp.* Proboscis proper. $\frac{3}{1}$.

Fig. 7 Top-shaped body. *a.* Chitinous disc. *b.* Bright reddish yellow disc. *c.* Pointed chitinous tube. $\frac{30}{1}$.

Fig. 8. Dorsal view of pupa, 35 days old. *a.* Simple eyes. $\frac{3}{1}$.

Fig. 9. Ventral veiw of ditto. *a.* Maxillary palp. *b.* Labrum. $\frac{8}{1}$.

Fig. 10. Tyroglyphus sp. ♀ $\frac{100}{1}$.

Fig. 11. Eggs of Tyroglyphus. $\frac{100}{1}$.

Fig. 12. Breakages on puparium. *ab.* Transverse breakage. *cd.* Longitudinal breakage. *1, 2, 3–12.* Number of segments. $\frac{3}{1}$.

Fig. 13. Puparium with two broken pieces *a* and *b.* $\frac{8}{1}$.

Fig. 14. Section of puparium with contained pupa. *a, a′.* Two points where the respiratory tubes (*e*) stick out the fifth segment. *b, b′.* Two points where the temporary air bladder touches the puparium. *c.* Pupa. *d.* Puparium. $\frac{8}{1}$.

Fig. 15. Top view of the head of pupa. *a.* Oval spacs. *b.* Compound eye. $\frac{5}{1}$.

Fig. 16. Longitudinal section of the head of pupa through the space between compound eyes. *a, a′.* Dark marginal lines. *b.* Setæ. *c.* Central transverse line. *d.* Basal joint of antennæ. $\frac{80}{1}$.

Fig. 17. Manner of hatching out of the perfect fly. *a, a′.* Empty puparium. *b.* Fly getting out of it. *c, c′.* Newly-hatched fly fully expanding its air bladder on the head. *d.* Fly on the ground. *g.* Surface of the ground. $\frac{1}{1}$.

Notes on Distoma Endemicum, Baelz.

By

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With Plate VII.

Since 1883 it has been known that in certain districts of Okayama-Ken a species of Distoma often infests the liver of the natives, causing deplorable diseases or even deaths. The parasite has been described in no less than two papers, but not quite satisfactorily, a fact which induced me to apply to the Science Department of Tōkyō Daigaku for permission to visit infected districts, in order to obtain an exact knowledge of the parasite, preliminary to an attempt at the elucidation of its life-history. Experiments for attaining the latter purpose are being carried on. As I have however no expectation of bringing them soon to a close, I wish to fulfill my duty in submitting the following report to the authorities of the Imperial University. I venture to believe that my notes will put the characters of our Distoma in a clearer light than heretofore and hope that they may be of some service for future investigations by our naturalists and physicians.

As already referred to, there exist to my knowledge two published accounts of the Distoma parasitic in the liver of Japanese. The one is contained in the joint-work of Messrs. Kiyono, Nakahama, Suga and

Yamagata, acting-physicians in the hospital of Okayama. It appeared in January 1883 under the title of 肺臟及肝臟ジストマノ實驗 (*"Observations on Distomæ of the lung and liver"*). This pamphlet contains a series of observations on numerous cases of the Distoma-disease, which are no doubt of great importance to physicians, together with a short anatomical account of the liver-distome.—In "*Berliner Klinische Wochenschrift*," published on April 16th of the same year, Prof. Baelz gives some accounts of Japanese human-parasites (*"Ueber einige neue Parasiten des Menschen"*). Of liver-distomes he describes two new species, viz., Distoma hepatis endemicum s. perniciosum and Distoma hepatis innocuum. To me it is exceedingly doubtful whether these are to be regarded as really distinct species. At any rate it is certain that the species which I myself examined is identical with Dist. endemicum. The two species described by Baelz are very closely allied to Distoma spathulatum Leuckart (= Dist. Sinense Cobbold). The latter parasite, as described and figured by McConell*, is by no means satisfactorily known, and I should not be surprised if further investigations should show its identity with either or both of Baelz's species. Kiyono &c., as well as Baelz, mention the absence of seminal receptacle in the Japanese liver-distome as a chief point of difference between it and Dist. spathulatum; but I believe this is a mistake, for I have seen a distinct seminal receptacle in Dist. endemicum.—I remember that a few years ago Prof. Leuckart of Leipsic was engaged in the study of human Distomes from Japan. The publication of his results may bring the above mentioned identity or non-identity of the several species to a definite settlement.

I wish here to express my thanks to the medical staff of Okayama-hospital, especially to Messrs. Yamagata and Matsuwo, for giving me every facility in collecting information. The last named

* Lancet, 1875.

gentleman kindly supplied me with a number of Distomes from two sources. Those from human-liver could easily be identified as Dist. endemicum Baelz, while those from the liver of cat showed slight differences inasmuch as they possessed very fine spines in the skin (= cuticula) and were of smaller size than the former. Perhaps this cat-distome from Okayama is to be considered as a distinct species, but I firmly believe that in Tōkyō, Dist. endemicum does sometimes inhabit the liver of cats. During December of last year I have had occasion to dissect three cats. In one of them, I found the gall-bladder and hepatic ducts unusually enlarged. They were almost filled up with Distomes, which agreed in every respect with Distoma endemicum from Okayama. I counted over 600 of them. In the second cat, the gall-bladder and hepatic ducts were of normal appearance and only a single specimen of Distoma was found within. It is probable that some more were left undiscovered. The third cat did not seem to be at all infested.

It thus stands beyond doubt that Dist. endemicum infests not only the human-liver but also cats. This fact affords a great convenience in experimentally ascertaining its life-history, since the mode of infection must be the same in both cases. Again, it is clear that conditions requisite for the development of Distoma endemicum do also exist in Tōkyō, the cats examined having been reared up in this city or in its environs. But as yet no one has ascertained the occurrence of our Distoma in the inhabitants of Tōkyō. Notwithstanding, the possibility or even the probability of its occurrence is not to be denied. Perhaps it may be that the parasitic worm owing to some local circumstances, does not become introduced into the human-body in a number enough to cause any calamitous influences, and thus passes unnoticed. Baelz also assumes the occurrence of human liver-distome elsewhere than in Okayama-Ken, since he met with cases in Tōkyō,

analogous to the Distoma-disease of the above mentioned Ken. Recently some cases of Distoma-disease have been reported from a certain district in the neighborhood of Lake Biwa.

According to Kiyono and his colleagues, there are no less than 21 localities in Okayama-Ken and 1 small village in Hiroshima-Ken where the Distoma-disease has been observed. I could visit only a few of the infected villages in Kojima-Gōri, about 7 hours' voyage on steamer from Kōbe and about 8 miles distant from the town of Okayama. Amongst them a small village by the name of Nakaune was pointed out to me as the place suffering most from the disease. On the authority of Dr. Yamagata and of a local physician, there are about 10 *o/o* of inhabitants affected with the disease (Baelz gives as much as 20 *o/o*). The nature of the locality and its situation has been described by Kiyono &c. and also by Baelz. It is a strip of low land along the sea-shore, lying beneath the sea-level during high-tide. The sea-water is kept out by means of a dam, constructed some sixty years ago. The land is traversed by a broad ditch with sluggishly flowing water. The villages in which the Distoma-disease occurs are all situated along this ditch. There is no well in the whole strip of land and drinking water is brought from the adjoining hills. The ditch-water is drunk, according to the information I received, only in exceptional cases, if ever it is so taken. Its main use is for washing purposes. It is important to mention however that amongst other things kitchen-utensils, vessels of all sorts, and vegetables to be eaten half-raw, are always washed in the ditch. Moreover this ditch-water is usually resorted to in watering vegetables growing on farms and also in irrigating rice-fields which remain dry during uncultivated seasons.

It is a settled fact that the eggs of Distoma endemicum are discharged like those of other Distomes together with the fœces of patients In those villages as in other parts of Japan, human excrements are

used for manuring purposes. I frequently observed farmers transporting manure in boats on the ditch and unrestrainedly cleaning their manure-tubs in the water. Here are undoubtedly chances enough for millions of Distoma-eggs to reach the ditch-water, in which ciliated embryos would hatch out. Indeed it admits of hardly any doubt that the ditch-water stands in intimate relation with the development of our parasite; and one can not be going too far in asserting that the establishment of a new system of water-supply would ensure the annihilation of Distoma-disease, so far as the above mentioned villages are concerned.

The question into what animal those ciliated embryos next find their way must remain unanswered for the present. After the analogy of those Distomes whose life-history has been worked out, I naturally fixed my attention on molluscs. I found in abundance Limnæa japonica Jay, Melania libertina Gould, and a small species of Paludina. Less abundant were a large species of Paludina, and species of Planorbis, Cyclas, Corbicula and Anodonta. Land-snails were said to be exceedingly rare. Notwithstanding the special search made, I failed to discern any trace of Sporocyst, Redia or Cercaria in any one of these molluscs. Dr. Kiyono informed me that some years ago he found Sporocysts or Rediæ, whichever they were, in almost all Melania of the same locality that he examined. Unfortunately I was unable to verify this fact myself. It is less probable, although the possibility should not be excluded, that the ciliated embryo should first enter into some other invertebrates. At any rate it requires no comment to assume that somehow broods of Cercaria would finally be formed. As to the way in which these Cercariæ become introduced into the human body, one might think of four alternatives. Firstly, they may enter it together with water that is drunk. Baelz does not hesitate to assume this as *the* way of infection ; but I am inclined to put some doubt on this point,

for the ditch-water, as I have already said, does not form the usual drink of the natives. A number of patients whom I saw assured me of never having drunk the ditch-water. I do not mean however to exclude all chances of infection directly from the water. Secondly, they may be taken in together with the host in which the Cercariæ have developed. In this case the Distoma would have but one intermediate host. In this connection I should mention that Paludina and Corbicula are eaten but never in a raw condition. Oysters are abundantly cultivated in the neighborhood, but they are clearly above suspicion since most of them are sent to markets in Okayama where the Distoma-disease is unknown. Thirdly, they may be eaten together with vegetables in an encysted condition after the manner of Distoma hepaticum. I did not observe any edible plants in the ditch, but the fact that the ditch-water is often used for watering vegetable-farms must not be forgotten. Fourthly, they may enter the human body together with the second intermediate host. In this respect shrimps and small miscellaneous fishes as well as molluscs fall under suspicion. Eels and Carassius vulgaris are sent to Okayama markets and are probably harmless. If the words of villagers are to be trusted, symptoms of the Distoma-disease generally manifest themselves late in Summer or at the beginning of Autumn. This is suggestive of the fact that the immigration of young Distoma-broods takes place about that time. In the case of sheep-rot, it is known that symptoms appear in the interval from July to September in consequence of the immigration of young Dist. hepaticum, which attains maturity in 2—4 months.

With these remarks I proceed to give the description of the mature worm.

In the fresh state the worm is translucent, colorless or with a slight reddish shade. The dark-colored uterus is very conspicuous and other organs such as the testes, the seminal receptacle, the vas deferens, the

ovary and vitellarium can be discerned as whitish objects.

The total length of the body averages $11\frac{3}{4}$ mm. The largest measured 13 mm. and the smallest 8 mm. in length. At about the middle of the body, the breadth measures 2—$2\frac{3}{4}$ mm. (Kiyono &c. give 8—20 mm. × 4—5 mm. and Baelz 8—11 mm. × 3.5—4 mm. for the dimension). It has thus an elongated form resembling Dist. lanceolatum in shape. In *fig. 1* I have endeavored to reproduce the shape as exactly as possible. The anterior portion occupying about $\frac{1}{4}$ of the whole body-length tapers toward the blunt apex where the mouth is situated. This portion of the body is marked off from the hinder portion by a slight curving in of the sides. At the level of these notches is situated the ventral sucker (*v. s.*), at which point the breadth of the body measures $1\frac{1}{4}$—$1\frac{4}{5}$ mm. Behind the ventral sucker the body is broad and flat. The ventral side is more convex than the dorsal, while in front of the ventral sucker the cross-section of the body presents an oval form. The hind end does not taper away so gradually as in front, but forms a rounded angle of about 90° or less.

The skin ("*Cuticula*") is smooth, very thin and without spines. Its substance is finely granular. Much controversies were held as to the nature of "*cuticula.*" Biehringer* and Schwarze† have lately shown that it originally constitutes a layer of nucleated cells (ectoblast), an opinion that I can fully corroborate.

The peripheral system of muscles (*Hautmuskelschicht*) is weakly developed. It consists of a layer of exceedingly fine circular fibers lying in immediate contact with the skin, a middle layer of strong longitudinal fibers, and an innermost layer of less developed diagonal fibers. Dorso-ventral fibers are rather abundant.

The mesenchymatous connective tissue presents the usual aspect.

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* Beitr. z. Anat. ü. Entwickl.-gesch. d. Trematoden. Arbeiten aus dem Zool. Inst. zu

A number of large branched cells formerly taken for ganglionic cells and lately explained by Looss‡ as elements of the connective tissue, are distributed in the mesenchymatous mass, especially in the anterior portion. I have observed besides a number of large cells in the neighborhood of the pharynx. They are probably of a glandular nature.

The mouth opens on the ventral side at the anterior end. The oral sucker (*fig. 1, o.s.*) is slightly larger than the ventral sucker (*v. s.*). Baelz mentions numerous cuticular hooks as being present in the buccal-cavity, but I did not find any trace of them.

The cask-shaped muscular pharynx (*ph.*) is a well-marked organ closely following the oral sucker. The œsophagus (*œ.*) is very short. Only this portion of the alimentary tube is supplied with circular and longitudinal muscle-fibers on its wall.

The bifurcation of the intestine occurs a considerable distance in front of the ventral sucker. The blind ends of intestinal tubes lie near the posterior end of the body.

The unpaired terminal portion of the excretory vessel (*ex.*) extends from the minute excretory pore (*p. ex.*) anteriorly to the region of the seminal receptacle (*s. r.*), pursuing an irregular course. It runs in the middle of the body, on the dorsal side of the testes. Being a relatively wide canal, it can easily be seen in either fresh or hardened specimens as a transparent streak. I could not distinctly make out the point where this median vessel was continuous with the lateral vessels. The latter can be easily traced in the anterior two thirds of the body. On each side a clear vessel runs anteriorly just outside the intestine. Reaching the height of the point of intestinal bifurcation it turns on itself and traces back its former course posteriorly, its caliber gradually thinning out. I have observed a number of ciliated funnels

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Würzburg. VII.

† Die postembryonale Entwickl. der Trematoden Z. f. wiss. Zool. XLIII.

‡ Beitr. zur Kenntniss der Trematoden. Z. f. wiss. Zool. XLI.

(*Wimpertrichter*) but no ciliary patches in the lumen of the vessels.

The brain (*br.*) is situated, not above or in front of pharynx as is usual, but immediately behind it forming a bridge over the œsophagus.

Sexual organs. The ovarium (*ov.*) is a lobed organ of no definite configuration, situated at the beginning of the last $\frac{1}{3}$ of the body. It lies nearer the ventral side than the dorsal. A very fine oviduct arises from its dorsal side (*vide fig. 2*) and soon turns forward in order to become continuous with the uterus. At the turning-point it is joined by the Laurer's canal and soon afterwards by the median duct of the vitellarium. The oviduct does not show any special widening that might be designated as ootyp. Unicellular shell-glands (*sh. g*, *fig. 2*) are grouped together in front of the ovarium. They do not color very strongly and seem to have been hitherto entirely overlooked.

The Laurer's canal (*l. c.*) opens on the dorsal surface by means of a minute pore, after it has pursued an irregular course posteriorly and dorsally.

Shortly before its junction with the oviduct the Laurer's canal stands in connection with a large oval or pear-shaped seminal receptacle (*s. r.*), usually lying to the right of the median line. I always found it filled with spermatozoa. In preserved specimens the mass of spermatozoa has contracted, thus leaving a space between it and the receptacular wall. It is this organ that has hitherto been taken for the ovary.

The vitellarium (*vit.*) on each side of the body consists of numerous small groups of yolk-cells, presenting a clustery appearance. It extends through all that part included between the ovary and the ventral sucker, along the body-margin outside of the intestinal tube. Sections show that it is confined to the dorsal side of the body. I did not happen to notice it arranged in separate coils, nor did I perceive any

segmentation of the body such as Baelz mentions.

Among the vitellarium, branched efferent ducts filled with yolk-cells may be seen here and there. From the hind portion of the organ a main efferent duct (*vit. d.*), one from each side, proceeds posteriorly and towards the median line over the intestine. Above the ovary the two ducts meet, and from this point a short unpaired duct descends ventrally and opens into the oviduct at the place already indicated.

The Uterus (*ut.*) makes manifold convolutions in the region bounded laterally by intestinal tubes and situated between the ovary and the ventral sucker. The approximate number of uterine loops along the lateral border varies from 16 to 24. On account of innumerable eggs contained within, the uterus forms, as is generally the case, a most conspicuous organ. The eggs have in the anterior portion a dark-brown color, which gradually passes into brown in the middle and then into white in the posterior portion of the uterus. The anterior and much narrowed end of the uterus passes over the ventral sucker and opens externally just in front of the latter on the ventral median line. This opening, which is minute and not recognizable except in sections, is the genital pore common to both male and female products. Two distinct sexual openings have been described, but this must be an error.

There are always two testes (*t.*) situated in the hind quarter of the body. They lie one after the other, ventrally to intestinal tubes. Their shape is varying, being irregularly lobed or branched. One might estimate the number of main lobes at 6 or 7. Each testis sends out a very fine vas deferens (*v. d. fig. 2*) from its central portion. The vasa deferentia run anteriorly above the seminal receptacle and the uterus but beneath the transversal efferent ducts of the vitellarium. At about the middle of the region occupied by the uterus and above it the vasa deferentia unite and form a single duct (*v. d. fig. 1*) of a

considerable calibre. This portion is always filled with spermatozoa and might be termed the seminal reservoir. After an irregular winding course, it reaches the ventral sucker, assumes a position to the right of the median line and finally joins the front end of the uterus a short distance within the sexual opening.

It remains yet to say a few words about the eggs. They are unusually small, measuring 0.028—0.03 mm. in length and 0.016—0.017 in breadth. In those eggs contained in the hind end of the uterus the shell is colorless and transparent. Each encloses an egg-cell and a number of yolk-cells, the nuclei of which can easily be demonstrated by coloring. In the anterior portion of the uterus, where the egg-shells have assumed a dark-brown or dark-olive color, embryos are already formed. Such an egg is represented in *fig. 3*, magnified 640 times. In the interior 3 distinct bodies beside some yolk-granules are seen. One of these bodies is a granular mass of triangular or irregular shape. Behind this body, away from the operculum, there is a second mass of larger size and clearer appearance. The third body lies mainly on the side of the second and has the form of a rod, slightly curved and often showing constrictions. This elongated body does not form a part of the embryo ; it is probably the remnant of yolk-matter.

Embryos can be forced out of shells by a sharp tap on the cover-glass. In *fig. 4* I have drawn an embryo as examined in weak acetic acid. It has an elongated oval shape, measuring 0.025 m.m. in length. The body slightly tapers toward the hind end. I believe I have seen an indication of head-papilla. The delicate skin is covered all over by cilia, which are turned posteriorly. The anterior portion of the body is made up of a few large cells with granular contents. The hind portion contains small cells of clear appearance, probably germinal cells. These two groups of cells apparently correspond with the two granular bodies that we have seen within the egg-shell. There are no eye-spots.

Embryos artificially pressed out remain perfectly quiet. Only once I saw an embryo moving slowly by means of its cilia, after a preliminary pause.

Eggs from the gall-bladder and intestine of a cat have been kept from December last until this day for over five months, but I observe no changes in them. Nor did the artificial warmth of an incubator enhance the hatching out of embryos.

Tōkyō, June 5th, 1886.

EXPLANATION OF FIGURES.

br., brain.
ex., median excretory vessel.
int., intestine.
l. c., Laurer's canal.
o., dorsal opening of Laurer's canal.
œ., œsophagus.
o. s., oral sucker.
ov., ovary.
ph., pharynx.
p. ex., excretory pore.
s. r., seminal receptacle.
sh. g., shell-gland.
t., testes.
ut., uterus.
v. d., vasa deferens.
vit., vitellarium.
vit. d., efferent duct of vitellarium.
v. s., ventral sucker.

Fig. 1. Distomum endemicum Baelz, magnified 12 times. Seen from the dorsal side.

Fig. 2. Female sexual organ, half-diagramatically represented. Seen from the dorsal side.

Fig. 3. An egg, containing an embryo. Magnified 640 times.

Fig. 4. En embryo, examined in weak acetic acid. Magnified 640 times.

Fig. 5. Distomum endemicum, natural size.

Comparison of Earthquake Diagrams simultaneously obtained at the Same Station by two Instruments involving the Same Principle, and thereby proving the Trust-worthiness of these Instruments.

By

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With Plates VIII.—XI.

In 1883, Prof. J. A. Ewing contributed a valuable, and now well-known, work on Earthquake Measurement to the memoirs published by this University, in which he described the theory and construction of seismographs of his own invention together with earthquake diagrams that were obtained by the use of these instruments. These diagrams have given many new and important results, and in several cases settled, in a decisive manner, questions relating to the character of earthquake motions.

This short paper is intended to treat of the further examination of similar diagrams recently obtained, and to prove the trust-worthiness of Horizontal Pendulum Seismographs. This is done by comparing the two diagrams of the same earthquake simultaneously registered by two instruments placed side by side in the Observatory. With this object in view two pairs of earthquake diagrams are given, which were originally photographed from the smoked glass plates on which they were recorded.

It is the writer's intention to give, in the next number of this Journal, the general results of observations especially relating to Vertical Motion and therefore the record of that motion is here studied, as a preliminary, in conjunction with horizontal movement. It is only by compounding the vertical and horizontal motions that we can obtain the complete tracing of the motion of the earth particles during an earthquake.

PLATE VIII. AND PLATE IX.

The Earthquake of May 18th, 1886.—This earthquake disturbed the plain of Musashi and its neighbouring provinces including a radius of 85 miles around; the origin of disturbance was inland 32 miles N.N.W. from the Observatory. It did scarcely any damage to buildings.

Instruments.—The record of the earthquake is shown by Plate VIII, which was taken by Prof. J. A. Ewing's Horizontal Pendulum Seismograph; the waves on the inner circle were traced by a pointer which registered north to south components of motion, and those on the outer circle by another which registered east to west motion. The pointers tracing these components are prolongations of horizontal pendulums, which magnify the motion of the ground four times, and produce their records on a revolving glass plate which is started by means of an electric contact maker. The plate is driven by a clock-work train, which, after starting, quickly reaches a steady rate under the control of a friction governor. The speed of rotation was one revolution in 80 seconds; the short radial lines mark seconds, so that the successive movement of the earth may be studied in conjunction with time.

Plate IX. which is the record of the same earthquake as that of Plate VIII. has the record of both horizontal and vertical motions;

the former was given by Horizontal Pendulum Seismograph of somewhat modified form, but similar in principle to the first. The horizontal motion of the ground was magnified nearly six times in this instrument, and a larger smoked glass plate of slower rate was employed. It is to be observed that the driving clock did not move at a perfectly uniform rate, but was slighly retarded towards the end. The circles are divided up to the eightieth second so as to facilitate the comparison of the two diagrams.

The vertical motion, which is given on the outermost of the three circles, was registered by Ewing's Vertical-Motion Seismograph; it has, for its astatic mass, a heavy bob suspended from a horizontal axis with an automatically changeable leverage. The magnifying ratio of this instrument is 1 to 8, or the motion of ground is magnified eight times on the record ; the outside of the circle corresponds to up-motion and inside to down-motion of the ground.

Horizontal Motion.—In order to compare the two records of the horizontal motion, Plates VIII and IX. may be examined conjointly. It will be observed that the plates revolved in opposite directions as indicated by the directions of the arrows.

The records begin at *O*, but they are quite feeble till the end of the fifth second ; then both the North-South and the East-West components suddenly exhibited vigorous movements ; allowing for multiplication introduced by the recording levers (1 to 4 in Plate VIII. and 1 to 6 in Plate IX.) the displacement on the earth surface at the sixth second was 1.4 mm. from South to North and 0.9 mm. from East to West. When these two components are compounded together, they give a resultant of 1.8 mm. in the direction of N. 33° W. and S. 33° E. The complete period at this point is approximately 0.8 seconds.

Later on there are constant changes of phase-relations in the

waves of the both components, *i. e.*, waves in one circle are sometimes in advance of those in the other circle or *vice versâ.* This means that the particles of the earth moved more or less in a curvilinear path. At the twenty-third second there is a large East-West movement which has nothing to correspond with the North-South components. On the contrary, from the thirty-third second a series of vigorous undulations appear on the North-South components, while that of the East-West is still at rest; however at the thirty-sixth second, the latter suddenly takes up its motion along with the former. The measurements made at this point give a motion of 2.1 mm. in the direction of N. 47° E. and S. 47° W. with a complete period of 0.8 seconds, which is the maximum horizontal motion registered in this shock.

On these points readers are reminded to compare Plates VIII. and IX. and they will not fail to discover a close agreement between them.

During the interval of 80 seconds 91 complete waves may be counted, which makes the average period of one complete wave equal to 0.9 seconds. The undulations may be traced over one complete revolution of Plate IX. making the duration 2 minutes 24 seconds.

Vertical Motion.—The vertical motion begins at *O*, but up to the end of the sixth second it is nothing more than a series of very minute tremours recurring from four to six times in a second. At the sixth second when there is a large horizontal displacement, a vertical motion of only 0.2 mm. is registered; at the seventh second it is 0.4 mm. with a complete period of nearly 0.5 seconds, and is the maximum vertical displacement in this shock; then the motion continues with comparatively large amplitude till the end of the fourteenth second. The vertical motion may be traced up to the forty-fourth second, but the subsequent disturbance is entirely in the horizontal plane.

The horizontal and vertical motions thus measured may be compounded together so as to make the complete tracing of the motionpath of the earth-particles, which is then by referred to three rectangular co-ordinate axes. By introducing the vertical component to the already complex character of horizontal motion, we make the result still more complex. Without going into details it will be observed that at the seventh second while the ground was moving nearly 1 mm. in ES., it oscillated first 0.15 mm. downward and then 0.4 mm. upward. In this and other parts of the principal disturbance the period of vertical motion is less than one-half of that of horizontal movement, or in other words for one to-and-fro oscillation of the soil there are more than two simultaneous up-and-down strokes.

The ratio of the maximum vertical to the maximum horizontal motion is 1 to 5.3 for amplitude, and 1 to 3.3 for duration of disturbance.

PLATE X. AND PLATE XI.

Earthquake of December 19th, 1885.—This was quite a severe and extensive shock, shaking the country within a radius of 160 miles from its centre of disturbance, which was in the flattest part of Musashi Plain, near the sea and 37 miles E. 35° N. from the Observatory. The latter was in the midst of the affected portion of the country. No damage was done by this shock.

Instruments.—Plates X., and XI. which are the records of the above earthquake, are the exact repetition of Plates VIII. and IX. respectively. The same instruments were used in both cases under the same circumstances, except that the smaller plate was turned in the opposite directions at the rate of 84 seconds to complete one revolution, and the outer side of the East-West circle was made to correspond to West and the inner side to East.

The later and feebler parts of the record are cut out from Plate XI. in order to reduce the side of the plate.

Horizontal Motion.—The records begin at *O* and from the fifth to sixth second there suddenly appears one large undulation with the period of 1.7 seconds in the East-West circle with comparatively short-period waves on the other, and hence when these two are compounded together it appears that the ground moved approximately 1.1 mm. 55° W. and then 1.7 mm. N. 30° W. within the interval of 1.7 seconds. This is the largest horizontal motion in this shock. The record on the smaller plate may be traced over one and one-quarter revolution of the plate, corresponding to 106 seconds of time.

Vertical-Motion.—This begins at *O*, and up to the end of the seventh second there only but minute tremours of from .03 mm. to .09 mm. with a period of nearly 0.25 seconds. At the sixth second when the largest horizontal displacement appears the vertical motion is almost nothing, or at this point the soil still moved purely in a horizontal plane. From the tenth to the fourteenth second, a series of 8 distinct shocks of nearly equal amplitude is recorded, giving an average period 0.5 seconds. The greatest motion is 0.22 mm. at the eleventh second. The vertical motion may be traced up to the forty-third second with occasional rests between.

If we combine the three components exhibited on the diagram at the tenth second we will discover that, at this point, the ground was describing a loop or an elliptical figure whose major axis (pointing East and West) is 1 mm. and whose minor axis 0.4 mm. if we look from above downward, but at the same time the particles of the earth were actually performing two double oscillations of 0.2 mm. in the vertical plane. In the next second the motion was more nearly East and West with the usual rapid vertical oscillation. The composition of the three components may be carried out further in similar ways, but always

leading to the same result which shows that the particles of the earth move in space in a most irregular and complex manner, and to try to picture the path pursued at the time of an earthquake is almost a hopeless task.

The ratio of the principal vertical to the horizontal movement ranges from 1 to 7 to 1 to 3 for amplitude, 1 to 2 for average period, and 1 to 2.4 for duration of disturbance.

Conclusions regarding the agreement of two diagrams of Horizontal Motion given by the same earthquake.

The object of the present paper is not so much to discuss the character of earthquake motion as to examine the agreement of two diagrams of the same earthquake taken by two Horizontal Pendulum Seismographs. If we go right through the records, wave by wave, or by means of the successive short radial lines, we will notice (taking into account the multiplying ratio of Plates VIII. and X. to be four and that of IX. and XI. to be nearly six) that the corresponding pointers in the two seismographs produced waves of exactly the same amplitude and period, both marking even irregular minor ripples between the principal undulations. These coincidences are faithfully repeated through all changes of amplitudes and periods to the end of the disturbance.

Such coincidence means either of two things,—(1) the steady mass in the instrument which serves as a datum line for registering earthquake motion remained stationary; or (2) the said mass in each instrument was moved exactly in the same manner during the prolonged shaking, and produced exactly the same error throughout. But what is called the steady mass is a hanging bob suspended by a horizontal lever, and therefore is free to oscillate round the axis of support with its own period of five to six seconds, or in other words, the bob

is sensibly stable. If it had been at all affected by the continuous shaking of the earth it must have ultimately swung with its own period of oscillation; however it did *not* swing, as is clearly proved on the diagram by the shortness of the periods of undulation.

To find out experimentally the steadiness of horizontal pendulums, Prof. Ewing placed two of them on a shaky table, and by rigidly fixing the bob of one instrument to a neighbouring wall by means of a bracket, and letting the other bob free, he moved the table in such a manner as would resemble the earthquake motion. The free and fixed instruments produced waves which were almost alike to each other. This shows that the steady mass of the free pendulum remained stationary during the whole of the varied motion.

Allowing then for certain instrumental errors which cannot in all cases be eliminated, we have, in the agreement of records produced by similar instruments during the same earthquake, or during artificial shaking, evidence which conclusively proves the accuracy and trustworthiness of the Horizontal Pendulum Seismograph.

Ueber die Deformation der Metallplatten durch Schleifen.

von

H. Muraoka.

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§ 1. *Zweck der Untersuchung.*

In meiner früheren Arbeit über japanischen Spiegel * theilte ich die merkwürdige Eigenschaft der festen Körper mit, dass sie nämlich als dünne Platte durch Risse oder Schleifen nach der geritzten oder geschliffenen Seite convex werden. Die Ursache dieser Deformation suchte ich in der molekularen Spannung. Da diese Eigenschaft der festen Körper bis jetzt vollständig unbekannt war und ein etwas eingehenderes Studium derselben wohl zur Kenntniss der molekularen Kräfte beitragen könnte, so unternahm ich einige messende Bestimmungen für Metalle auszuführen. Damals bemerkte ich, dass die Convexität von der Dicke der Platte abhängig sei. *Im Folgenden soll der Versuch gemacht werden, den Krümmungsradius zunächst als Function der Dicke darzustellen und dann Beziehungen zwischen der biegenden Wirkung des Schleifens und der elastischen Constante zu finden.*

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* Muraoka, Wied. Ann. Bd. XXII. p. 249. 1884.

§ 2. *Vorbereitung zum Schleifen.*

Zur Bestimmung der Krümmung darf man nicht etwa die zu untersuchende Platte ohne weiteres schleifen und den Krümmungsradius messen, sondern man muss die Deformation während des Schleifens verhindern; denn sonst kann die Platte unmöglich gleichmässig geschliffen werden, was für die messenden Zwecke nothwendig ist. Hiezu ist die Anwendung des Wood'schen Metalles vorzüglich geeignet, wie ich dies in der Notiz * „über japanischen Spiegel" beschrieben habe. Es wurde nämlich vom Holz ein Kästehen verfertigt, dessen Boden die Form der zu untersuchenden Metallplatte hatte und etwas grösser war als diese. In dasselbe wurde die zu schleifende Platte hineingelegt, so dass diese auf der Mitte des Bodens zu liegen kam. Das Kästchen wurde dann in warmes Wasser hineingesenkt und das Wood'sche Metall auf der Metallplatte geschmolzen und langsam abgekühlt. So erhielt man die Platte mit dicker Belegung von Wood'schem Metalle, welche man schleifen konnte ohne dass die Krümmung während des Schleifens stattfand. Hat man das Schleifen und Poliren in geeigneter Weise vollgeführt, so braucht man die Platte in warmes Wasser hineinzulegen und vom Wood'schen Metalle zu befreien, um die Deformation erfolgen zu lassen. Es zeigte sich aber, dass das Wood'sche Metall stellenweise nicht gut an die Metallplatte haftete. Um das gleichmässige Haften zu erzielen, wurde die Platte zunächst unter Anwendung des gewöhnlichen Löthwassers mit Wood'schen Metalle überzogen und darauf in obiger Weise behandelt. Im warmen Wasser wurde die Platte von dem Wood'schen Metalle so benetzt, als ob sie amalgamirt werde.

§ 3. *Schleifen und Poliren.*

Zum Schleifen wurde aufänglich harte ebene Thonschiefer benutzt. Da aber für die angewandte Messungsmethode günstiger war die Platte

* Muraoka, Wied. Ann. Bd XXV. p. 138. 1885.

von vornherein etwas rund zu schleifen, so wurde von dickem Glas concave Schleifschale verfertigt und die in beschrichener Weise preparirte Metallplatte mit Granatpulver von verschiedener Feinheit geschliffen, wie man bei der Herstellung der Glaslinsen verfährt. Das letzte Feinschleifen geschah mit japanischer Kirschkohle, wie diese gewöhnlich zur Bearbeitung der Schmucksachen gebraucht wird. Schliesslich wurde die Platte mit einem Gemisch von Schleifsel und Alaun unter Anwendung von Pflaumensäure polirt. Die Fläche war dann blank genug, um für die angewandte Spiegelungsmethode benutzt zu werden. Für weiche Metalle wie Zinn war zum Poliren die Anwendung von Leder und Ochsenhornpulver günstig.

§ 4. *Messungsmethode.*

Zur Bestimmung des Krümmungsradius bediente ich mich der Methode der Spiegelung. Die geschliffene Metallplatte wurde vertical aufgestellt, ihr gegenüber eine Lichtquelle in gleiche Höhe angebracht, welche man mittelst einer Schlittenvorrichtung parallel zur Metallplatte verschieben konnte. Als Maassstab an der Metallpatte dienten zwei an einem Rahmen gespannte feine Seitenfaden, zu welchen noch ein dritter senkrecht stand. Die mit diesem Maassstab versehene Platte wurde auf ein beliebig verstellbares Gestell angebracht und so aufgestellt, dass die parallelen Faden vertical zu liegen kamen und das Lichthild auf dem horizontalen Faden sich hewegte, wenn die Lichtquelle auf dem Schlitten verschoben wurde. Die Verschiebung des Lichtes geschah immer so weit bis sein Bild mit einem der verticalen Faden zusammenfiel. Die Lichtquelle wurde mit einem Zeiger und einem feinen Spalt verschen, deren Breite man beliebig ändern konnte. Die Einstellung des Lichtes konnte man an dem Maassstab des Schlittens ablesen. Das Spiegelbild wurde natürlich mit Fernröhre beobachtet. Bezeichnet:

L die Verscbiebung des Lichtes,

l diejenige des Bildes,

A die Distanz zwischen der Metallplatte und dem Schlitten.

r den Krümmungsradius,

so findet bekanntlich bei convexem Spiegel die Beziehung statt:

$$r = \frac{2Al}{L - 2l} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Sollte dieses r der Krümmungsradius sein, welchen man erhalten würde, wenn die Krümmung nur durch die Wirkung des Schleifens erfolgte, so müsste man zu diesem Zwecke die Metallplatte anfänglich vollständig eben geschliffen haben. Es ist aber eine schwere Arbeit, eine Fläche vollkommen eben zu machen und eine solche zu bestätigen. Deshalb schliff ich die Platte von vornherein convex und bestimmte den Krümmungsradius vor und nach der Ablösung des Wood'schen Metalles. Aus diesen beiden Werthen der Krümmungsradien kann man denjenigen Krümmungsradius berechnen, welchen man erhalten würde, wenn die Platte anfänglich eben geschliffen wäre und ich nenne ihn kurz den „*reducirten Radius.*"

§ 5. *Berechnung des reducirten Radius und die Curvengleichung.*

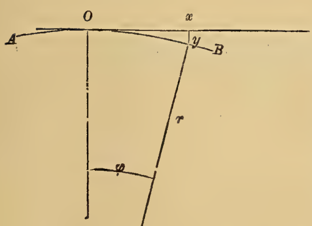
Um den reducirten Radius r aus den Radien r_1 und r_2 vor und nach der Deformation zu berechnen, muss man sich zunächst eine Vorstellung verschaffen, von welchen Art eigentlich die Deformation ist. Nehmen wir der Einfachheit wegen einen paralleloepipedischen dünnen Stab und machen auf einer Seitenfläche senkrecht zur Längsrichtung parallele Risse, so findet an jedem Risse die Hebung statt*

* Muraoka, Wied. Ann. Bd. XXII. p. 246. 1884.

und wenn alle Risse und ihre Entfernungen gleich sind, so muss der Stab nach der Deformation die Form eines regelmässigen Poligons annehmen, dessen Grenzwerth ein Kreis ist. Wenn wir also anstatt Risse zu machen, senkrecht zur Längsrichtung gleichmässig schleifen ——was dasselbe bedeutet, wie unzählig viele feine Risse machen*——, so müssen wir aus dem paralleloepipedischen Stab einen kreisförmig gebogenen Stab erhalten. Nun wollen wir aber, weil ein paralleloepipedischer Stab von vollkommen ebenen Seitenflächen schwer herzustellen ist, statt dessen einen von vorherein kreisförmigen Stab herstellen und diesen der deformirenden Wirkung des Schleifens aussetzen. Es fragt sich dann, welche Form bekommt der Stab nach der Deformation? Die Antwort ist leicht. Bezeichnet man das Bogenelement zwischen den beiden Rissen des Schleifens mit ds und den zugehörigen Contingenzwinkel mit $d\varphi$, so ist, wenn das Schleifen gleichförmig geschieht, ds und $d\varphi$ constant, somit auch $\frac{ds}{d\varphi}$. Die Form des Stabes nach der Deformation ist also ein Kreis, denn $\frac{ds}{d\varphi}$ giebt den Krümmungsradius an.

Es handelt sich nun um die Berechung des reducirten Radius r aus den beobachteten Werthen r_1 und r_2.

Fig. 1.



Sei AOB (Fig. 1.) ein Kreisbogen, dessen Radius r ist. Wird eine Tangente als Abscissenaxe und die Normale dazu als Ordinatenaxe angenommen, so gilt für den Kreis die Gleichung:

* l. c. p. 249.

$$x^2 + (r - y)^2 - r^2 = O.$$

Ist der Winkel φ klein genug, so kann man das Quadrat von y vernachlässigen und dadurch wird aus der Gleichung:

$$y = \frac{x^2}{2r}.$$

Darnach haben wir für den Metallstab vor und nach der Deformation die Gleichungen:

$$y_1 = \frac{x^2}{2r_1}$$

$$y_2 = \frac{x^2}{2r_2}$$

woraus folgt:

$$y_2 - y_1 = \frac{x^2}{2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Dies ist der Ausdurck für die Senkung der Stelle x durch Deformation. Da wir überhaupt mit kleiner Krümmung zu thun haben, so nehme ich hier an, dass im Falle der Stab anfängleich gerade, auch die nämliche Senkung $y_2 - y_1$ stattfindet. *Es stellt dann die Gleichung:*

$$y = \frac{x^2}{2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \dots \quad \dots \quad \dots \quad \dots \ (2)$$

die Curve nach der Deformation des parallelоepipedischen Stabes dar. Weil die Krümmung sehr klein ist, so kann man dieselbe einfach dem zweiten Differentialquotienten gleich setzen und erhält als Werth des reducirten Radius den Ausdruck

$$r = \frac{1}{\frac{1}{r_2} - \frac{1}{r_1}} \dots \quad \dots \quad \dots \quad \dots \quad \dots \ (3)$$

Man sicht, dass $r = r_2$ wird für $r_1 = \infty$.

Hiernach kann man die Curvengleichung (2) auch schreiben:

$$y = \frac{x^2}{2r} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (2a)$$

§ 6. *Beziehung zwischen Krümmungsradius und Dicke.*

Die Formel (2) und (3) leitete ich *ab* unter der Annahme eines paralleloepipedischen Stabes, dessen eine Seitenfläche senkrecht zur Längsrichtung geschliffen wird. Die Behandelung eines solchen Stabes war äusserst schwierig. In erster Linie war das Schleifen senkrecht zur Längsrichtung fast unmöglich. Wenn nämlich das Schleifen so weit war, dass man die Platte poliren konnte, so blieben immer gröbere Risse zurück und wenn man sie mit grosser Mühe wegschaffte, so kamen wieder neue hinzu. Noch schlimmer war die Unregelmässigkeit der Deformation, besonders bei grösserer Länge des Stabes. Diese Schwierigkeiten waren bei kreisförmiger Seheibe, wenn sie ohne Richtungsvorzug geschliffen, viel geringer und ich nahm daher für alle Versuche die letztere und machte die Annahme, dass die Formel (3) annäherungsweise giltig sei. Die Versuche zeigten, dass, wenn das Schleifen und die Politur gleichmässig geschieht, die Grösse der Krümmung unabhängig ist, von der Richtung des Schleifens oder des Polirens.

Anfänglich verfertigte ich vom Kupfer verschiedener Dicke kreisförmige Scheibe von 4^{ccm} Durchmesser und machte bei jeder Scheibe die Bestimmung des Krümmungsradius an verschiedenen Stellen derselben und nahm das Mittel. Die erhaltenen Werthe von r für verschiedene Dicke waren aber so unregelmässig, dass man aus den Zahlen keinen Zusammenhang zwischen Dicke und Krümmungsradius finden konnte. Die Ursache dieser Unregelmässigkeit ist sieher das, dass die Elasticität sehr leicht von der Behandelung abhängig ist. Es

ist bekannt, dass eine Metallplatte, die man käuflich bekommt, nicht an allen Theilen gleiche Elasticität besitzt und diese ist ja auch abhängig von der Richtung bei gewalzten Platten. Zur grösseren Sicherheit nahm ich daher eine einzige Kreisscheibe, glühte sie in starkem Kohlenfeuer zwischen zwei ebenen Ziegelsteinen aus und liess sie an. Nach der Abkühlung wurde sie mit Säure gewaschen, um sie von Oxydschicht zu befreien. Darauf wurde sie an einer Seite mit Wood'-schem Metalle in beschriebener Weise (§ 2) belegt und an der anderen Fläche geschliffen und polirt. War die Messung des Krümmungs-radius vor und nach der Ablösung des Wood'schen Metalles fertig, so wurde dieselbe Scheibe wieder ausglüht und in genau gleicher Weise behandelt und gemessen. Diese Operation wurde wiederholt vor-genommen, so dass die Metallscheibe allmälig dünner wurde. Selbst bei dieser Sorgfalt kamen Unregelmässigkeiten noch vor. Es wurde daher bei wenig differirenden Dicken Mittel genommen.

Die für vier Metalle Kupfer, Messing, Stahl und Gold erhaltenen Zahlen zeigten, dass *die Abhängigkeit des Krümmungsdradius von der Dicke am besten durch die Formel:*

$$r = A\,a^3 \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

sich ausdrücken lässt, worin r den reducirten Krümmungsradius, a die Dicke und A eine von der Substanz abhängige Contante bedeutet. Geometrisch ist A der Krümmungsradius für die Einheit der Dicke. Der Werth von A wurde aus den beobachteten Zahlen berechnet und aus demselben wieder der Krümmungsradius r zurückberechnet. Die Werthe sind in der folgenden Fabelle enthalten :

KUPFER

$A = 84\,100$

Dicke a in Centimeter	Krümmungsgradius r in Centimeter	
	beobachtet	berechnet
0.158	324	332
0.145	262	256
0.130	193	185
0.117	135	135
0.108	112	106

MESSING

$A = 49\,100$

0.157	357	364
0.149	317	312
0.142	278	269
0.131	206	216

STAHL

$A = 169\,000$

0.158	594	667
0.137	444	435
0.132	345	389
0.121	313	300
0.110	264	225

GOLD

$A = 268\,000$

0.130	688	589
0.121	428	475
0.115	310	408
0.0931	250	216
0.0855	175	168

An diesen Zahlen sieht man, dass die Uebereinstimmung zwischen beobachteten und berechneten Grössen allerdings eine sehr rohe ist, aber gegenüber den Unregelmässigkeiten, die sich für andere Metalle ergaben, muss ich sie eine befriedigende nennen. Uebrigens zeigten die Zahlen für Silber und Zink, wenn auch sie unregelmässig, für die Beziehung zwischen r und a weder eine lineare noch quadratische, sondern eine cubisehe Gleichung am geeignetesten ist. Ein aufallendes von den andern gänzlich abweichendes Verhalten zeigte das Antimon. Für dieses war mehr eine lineare Gleichung giltig.

§ 7. *Beziehunj der Deformation zur elastischen Constante.*

Suchen wir jetzt nach der Beziehung zwischen der Deformation und der elastischen Constante. Die Grösse der Deformation ist offenbar abhängig erstens davon, in welcher Weise die Wirkung des Risses oder des Schleifens rings um die gestörte Stelle ausbreitet und zweitens von der elastischen Constante, solange die Elasticitätsgrenze nicht überschreiten wird. Da ich noch nicht im Stande bin, die allgemeine Theorie darüber aufzustellen, so knüpfe ich hier eine provisorische Betrachtung an, welche wenigstens annähernd der Wahrheit entsprechen möge.

Zu dem Zwecke mache ich die Annahme, dass wenn auf ein Flächenelement df der Oberfläche eines elastischen Stabes eine Störung vor sich geht, also wenn df geschliffen wird, so breitet sie sich nach beiden Seiten in gleicher Weise aus, so aber, dass sie in der Entfernung λ verschwindet. λ ist also voraussichtlich eine sehr kleine von der Art der molckularen Beschaffenheit und Umlagerung abhängige Grösse und mag die „*Wirkungsweite*“ genannt werden.

Sei b die Breite eines paralleloepipedischen Stabes, dessen Längsrichtung als Abscissenaxe gewählt werden kann, so verursacht das Schleifen des Flächenelementes $b. d\xi$ eine Biegung des Stabes nach

beiden Seiten desselben. Die Biegung ist offenbar an der Stelle $b.\,d\xi$ am stärksten und nimmt allmälig *ab*, bis sie bei λ null wird. Dieser Bedingung entspricht die bekannte Curvengleichung für die Biegungselasticität

$$y = \frac{12P}{Ea^3b}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) \dots \quad \dots \quad \dots \quad \dots \quad \dots \ (5)$$

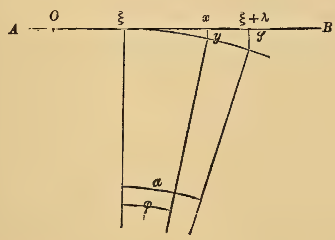
worin l die ganze Länge des an einem Ende befestigten Stabes bedeutet, an welches andere Ende die Kraft P senkrecht zur Stabrichtung wirkt. Denn, wenn man die Krümmung, da sie überhaupt sehr klein, durch den zweiten Differentialquotienten darstellt, so ist sie:

$$\frac{d^2y}{dx^2} = \frac{12P}{Ea^3b}\,(l - x),$$

welcher Ausdruck am grössten ist, für $x = o$ und gleich null für $x = l$. Nimmt man also die Gleichung (1) für den vorliegenden Fall als giltig an und bezeichnet mit P die biegende Kraft, welche durch das Schleifen der Flächeneinheit verursacht wird, so ist $P.b\,d\xi$ diejenige für das Flächenelement $b\,d\xi$.

Ist AB (Fig. 2) die Längsrichtung des Stabes, welcher in der Mitte O befestigt wird, so wird eine an der Stelle ξ wirkunde Elementarkraft $P.b\,d\xi$ eine Senkung der Stelle x innerhalb der Wirkungsweite λ hervorbringen, welche gleich ist

Fig. 2.



$$yd\xi = \frac{12P}{Ea^2}\left[\frac{\lambda(x-\xi)^2}{2} - \frac{(x-\xi)^3}{6}\right]d\xi$$

oder

$$\left.\begin{aligned} yd\xi &= C\left[\frac{\lambda(x-\xi)^2}{2} - \frac{(x-\xi)^3}{6}\right]d\xi \\ C &= \frac{12P}{E\,a^3} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (6)$$

Die Senkung der Stelle $\xi + \lambda$ ist daher

$$sd\xi = \frac{C\lambda^3}{3}\,d\xi \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6a)$$

φ und α sind die Richtungsänderungen der Normale an der Stelle x resp. $\xi + \lambda$. Von der Wirkung der Störung $P.b\ d\xi$ nach der anderen Seite $\xi\ O$ wollen wir varläufig absehen.

Die Senkung irgend eines Punktes x' ist das Integral aller Elentarsenkungen, welche durch die Elementarkraft $P.b\ d\xi$ von O bis x' hervorgebracht wird. Dieses Integral zerfällt in zwei Theile, da die Senkung theils durch Störungen ausserhalb der Wirkungsweite, theils aber durch solche innerhalb derselben hervorgebracht wird. Um die Integration auszuführen, setze ich

$$\left.\begin{aligned} x' &= n\lambda + x \\ O &\lesseqgtr x \lesseqgtr \lambda \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

so dass der Stab in lauter gleichen Intervallen (Fig. 3) nach λ getheilt wird

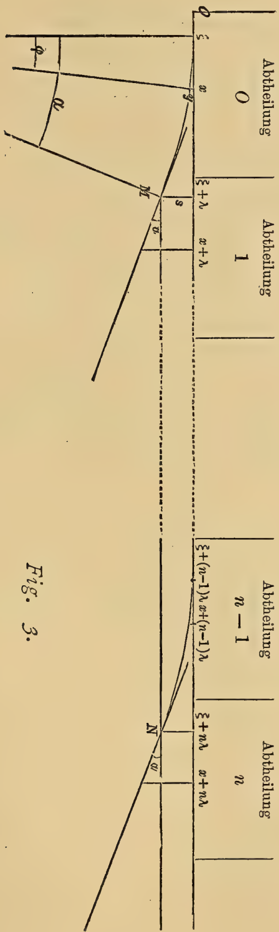

Abtheilung
0
Abtheilung
1
Abtheilung
n − 1
Abtheilung
n
ξ
x
ξ+λ
x+λ
ξ+(n−1)λ
x+(n−1)λ
ξ+nλ
x+nλ
M
N
s
φ
α
y

Fig. 3.

Ich numerire die Abtheilungen mit 0, 1, 2, 3............, so dass x' in der n-ten Abtheilungen zu liegen kommt. Dem Punkte x' in der n-ten Abtheilung entsprechen die Punkte $x, x + \lambda, x + 2\lambda$ ....in den Abtheilungen beziehungsweise 0, 1, 2 ...., wie dies in der Figur ersichtlich.

Die Senkung einer Stelle durch die Störung $P.b\ d\xi$ an der stelle ξ ausserhalb der Wirkungsweite λ besteht augenscheinlich darin, dass zunächst die Stelle $\xi + \lambda$ um $s.\,d\xi$ gesenkt wird und von hier an der Stab als die geometrische Tangente an dem Punkte M sich fortsetzt. Sei $O \gtreqless \xi \gtreqless x$, so verursacht die Störung $P.\,b\ d\xi$.

im Punkte	die Senkung
x	$y\,d\xi$
$x + \lambda$	$s.\,d\xi + (x - \xi)\alpha\,d\xi$
$x + 2\lambda$	$s.\,d\xi + (x + \lambda - \xi)\alpha\,d\xi$
$x + 3\lambda$	$s.\,d\xi + (x + 2\lambda - \xi)\alpha\,d\xi$
$\vdots$	$\vdots$
$x + (n - 1)\lambda$	$s.\,d\xi + [x + (n - 2)\lambda - \xi]\ \alpha\,d\xi$
$x + n\lambda$	$s.\,d\xi + [x + (n - 1)\lambda - \xi]\ \alpha\,d\xi$

Da x höchtens gleich λ werden kann, welches letztere selbst eine sehr kleine Grösse ist, so vernachlässige ich x gegen x' und schreibe

$$n = \frac{x'}{\lambda}$$

Dies in das Integral eingeführt, wird:

$$y' = C\int_0^\lambda (\tfrac{1}{2}x'^2\lambda + \tfrac{1}{6}x'\lambda^2 - \tfrac{2}{3}\lambda^3 + \lambda^2 x - \lambda^2\xi)d\xi + C\int_0^x (\tfrac{2}{3}\lambda^3 + x\lambda^3 - \xi\lambda^3)d\xi$$
$$+ C\int_0^\lambda [\lambda(x-\xi)^2 - \tfrac{1}{3}(x-\xi)^3]d\xi.$$

Wir sehen, dass alle Glieder unter dem Integralzeichen sehr kleine Fatcoren x, ξ, λ enthalten. Das erste Glied aber enthält nur einen derselben, während die anderen zwei oder drei enthalten. Vernach-

lässigen wir demnach die letzteren gegen das erste, so wird aus dem Integral:

$$y' = C \int_0^\lambda \tfrac{1}{2} x'^2 \lambda \, d\xi = \frac{C}{2} \cdot x'^2 \lambda^2$$

oder, wenn wir nun mehr den Strich weglassen, *so ergiebt sich als Curvengleichung:*

$$y = \frac{C\lambda^2}{2} \cdot x^2 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (11)$$

Dies ist eine Parabel, deren Krümmungsradius

$$r = \frac{[1 + C^2 \lambda^4 \lambda^4]^{\frac{3}{2}}}{C\lambda^2}$$

und welcher, solange $C^2 \lambda^4 x^4$ gegen 1 klein genug, gesetzt werden kann:

$$r = \frac{1}{C\lambda^2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (12)$$

Vermöge dieses Werthes r *nimmt die obige Curvengleichung die Form an:*

$$y = \frac{1}{2r} \cdot x^2 \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (11a)$$

Die eben erhaltene Curvengleichung (11a) und diejienige (2a) welche letztere wir durch rein geometrische Betrachtung erhalten haben, sind identisch. Indem wir jetzt die Gleichung für die Biegungselaticität zu Hülfe nahmen, haben wir eine Beziehung des Krümmungsradius zu der elastischen Constante bekommen. Führen wir nämlich den Werth C *aus (6) ein, so wird*

$$r = \frac{E}{12} \cdot \frac{1}{P\lambda^2} \cdot a^3 \quad \ldots \ldots \ldots \ldots \ldots \quad (13)$$

und es ist nach (4)

$$A = \frac{E}{12} \cdot \frac{1}{P\lambda^2},$$

eine Constante, welche für verschiedene Substanzen characteristisch ist. Wäre P *d. i.* die Kraft auf der Flächeneinheit, welche beim Schleifen auftritt und λ *d. i.* die Wirkungsweite der Elementarkraft $P.b.\ d\xi$ auf dem Flächenelement $b\ d\xi$, constant für alle Substanzen, so müsste

man für A dem Elasticitätsmodul E proportionale Zahlen bekommen. P und λ sind aber wahrscheinlich abhängig von der Art der molekularen Lagerung des Körpers, wie dies sich beim Antimon, dem Körper mit deutlichen krystallinischem Gefüge, durch sein merwürdiges Verhalten sich Kund giebt (§ 6). Ferner sind P und λ gewiss bedingt durch die Feinheit des Schleifens und der Politur. Der Grad derselben war, wie man durch die Schärfe des Spiegelbildes urtheilen konnte, für verschiedene Metalle ungleich und selbst bei demselben Metalle und sorgfälltiger gleichmässiger Behandlung war es schwierig immer die gleiche Feinheit zu erlangen. Dies gehört wohl mit zu dem Grund, warum die beobachteten Zahlen nicht so regelmässig waren, wie man es nach der angewendeten Methode erwarten konnte.

Der Werth des Produktes $P\lambda^2$ kann man für die 4 Metalle aus dem Werthe A und dem Elasticitätsmodul berechnen. Es ist

$$P\lambda^2 = \frac{E}{12A} \qquad \ldots \ldots \ldots \ldots \ldots \ldots \quad (14)$$

Nimmt man für Kupfer, Messing, Stahl und Gold annähernd $E =$ bezüglich 12 400, 9 000, 21 000, und 8 100 an und wenn man ferner bedenkt, dass diese Zahlen für E auf Kilogramm und Quadratmillimeter bezogen sind, dass man also A auf Millimeter beziehen muss, *so ergiebt sich als Werth des Produktes* $P\lambda^2$ *folgende Zahlen:*

Kupfer	**Messing**	**Stahl**	**Gold**
1.23	0.795	1.04	0.253

Zur Kenntniss der molekularen Kräfte wird es nicht von wenigem Interesse sein, wenn es gelingen sollte, die Werthe von P *und* λ *oder wenigstens des Produktes* $P\lambda^2$ *für verschiedene Substanzen zu bestimmen.*

A Note on Glaucophane.

By

B. Kotō, Ph. D.

Professor of Geology, Science College, Imperial University.

With Plate XII.

It is proposed in this paper to give a brief description of that interesting mineral—glaucophane, which the writer has collected during his excursions in two summer-vacations; hence exclusively confined to its occurrence in Japan.

In the island of Shikoku, we see the crystalline schist-system extensively developed, especially in the central mountain-chains which trend from S.W.-N.E. Among the schists a very characteric rock is found, which calls for our special attention, on account of its beautiful colour varying in tinge from a greyish-blue to a purplish-blue. It was formerly known among us as a cyanite-schist; but this is, however, not the case as will become perfectly clear from the discussion to which we shall refer hereafter. The typical specimens were brought from Mount Ōtakisan near the city of Tokushima, and from the copper mine at Besshi; the rock itself being not confined to the above-mentioned localities, but being widely distributed in the region of the crystalline schists. The glaucophane-bearing schist contains, besides other ingredients, a long rectangular, olive-green epidote, yellowish-green crystals of garnet, quartz, some felspars, rutile, also specular iron, and the last, but not the least important one is piemontite or manganese epidote. The glaucophane, when observed macroscopically,

is of an indigo-blue colour, and shows flat prismatic forms ; all arranged parallel to each other, and also parallel to one prevailing horizontal direction in regard to the rock itself. Thus a thin slide of the rock in any one direction shows only the same side of glaucophane contained in it.

I. Basal Section.

Any one of the vicinal basal sections in a thin slide presents usually an approximate hexagonal outline, being bounded by four long, and two short sides. (*fig. II.*) These sides are the traces of ∞P (110), and $\infty P \grave{\infty}$ (010) ; those of the orthopinacoid never happened to be observed in any slide. The prismatic angle is about 124°, similar to that of a hornblende. The cleavage-traces upon ∞P (110) are distinct and well developed, but all are widely separated. (*fig. II. & fig. V.*) Only in a few cases the cleavage-direction parallel to the clinopinacoid ($\infty P \grave{\infty}$) is seen, although less clear. (*fig. V.*) On rotating the section cut nearly at ⊥ to the vertical axis, over the polarizer, the crystals are seen to change from a light brown to a light bluish purple; and extinguish the light in the direction of both diagonals. We often observed the crystals attaching by the right prismatic face of the one with that of the left of another individual, or even one individual penetrates the other. (*fig. II.*) In the third case then, three or more crystals, are packed together by the prismatic faces so as to form a definite aggregate. Again, we have still another instance in which the glaucophane shows an intergrowth of two individuals, placed in such a manner that the orthopinacoid ($\infty P \bar{\infty}$) is the contact plane. On account of their minute size, here the character of interference ring could not be made out. Whether all these cases stated above are the outcomes from a mere intergrowth of crystal-individuals, or in some way connected with the twining formation, I am at present not able to say, as these are not capable of a rigid proof.

II. Clinopinacoidal Section.

As the glaucophane-individuals are all more or less arranged parallel with the orthopinacoid, to the plane of a schistose structure of the rock, slides taken in the longitudinal direction of the schist present quite a different aspect; here the crystals show all a light green with a slight tinge of blue. The sections are slender, and terminate in sharp points at both ends with unsymmetrical sides, thus indicating the probable presence of one of orthodomes. The surface of the section taken parallel to the clinopinacoid shows usually a striated appearance, while only a few stripes, if ever present, have been observed on the orthopinacoid. This is, indeed, a very characteristic feature. The direction of extinction :— $c : C = 11° - 12°$; these figures* seem to be somewhat large in comparison with those already well known to us.

III. Orthopinacoidal Section.

The section in this direction presents a broad tabular form without any distinct termination at both ends, being often resolved more or less into radiating fibres, so this particular feature gives a clue as to the direction in which a section is taken.

Various tinges of colour are discernible even in the same individual; the central part is lighter in tinge and more purple in its hue, while the periphery is intensely coloured, and a shade of green predominates. This zonal arrangement has also been observed by Stelzner † in the Swiss glaucophane, and G. H. Williams ‡ assigned the cause to the new formation of arfvedsonite from glaucophane.

* $c : C = 3° - 4°$ as is observed by Luedecke in the specimen from Syra in Greece: Zeitschrift d. deutch. geol. Ges. 1876, XXVIII, p. 249. Vide also Tschermak's Mineralogische u. petrographische Mittheilungen, 1879, II, p. 71.

† Neues Jahrbuch für Mineralogie etc., 1883, I Band, p. 209.

‡ ibid. 1882, II Band, p. 202.

Such an assumption is, as it seems to me, precarious ; for, the writer has observed the same phenomenon in the pargasite from Pargas, Finnland, especially on the periphery, where the Finnish mineral shows a slight inclination to transformation. Even in the central intact part such green spots are by no means rare.

The orthopinacoid of our glaucophane extinguishes the light, of course, parallel to $\acute{C}$, and also at $\perp$ to it; but the shade sweeps over the section in an undulating manner as we rotate the table. Owing to this fact, the direction of extinction might easily be mistaken for an oblique one with a small angle.

Taken as a whole, the crystal-individuals of our glaucophane possess a broad columnar form whose basal section presents an elongated rhombic outline, truncated at the acute corners by traces of the clinopinacoid; (*fig. II.*) and are terminated at both ends by an orthodome or hemi-pyramid. The axial colour: $C =$ greenish blue; $B =$ lavender blue ; $A =$ bluish brown. $C > B > A$.

This mineral has been isolated from other constituents of the schist by the Thoulet solution, and the analytical result obtained by Mr. Yoshida,* is as follows :—

$Si\,O_2$	56, 71
$Al_2\,O_3$	15, 14
$Fe_2\,O_3$	9, 78
$Fe\,O$	4, 31
$Ca\,O$	4, 80
$Mg\,O$	4, 33
$Na_2\,O$	4, 83
$K_2\,O$	0, 25
	100, 15

$Sp.\,Gr. = 2,9912$

* I gladly take this opportunity of returning my hearty thanks to him for the chemical analysis of this mineral.

The mineral, glaucophane is of comparatively rare occurrence. The typical one is found in Syra, in Greece, and in a few localities in the Western Alps. Quite recently, one more locality is found in the island of Groix in France, the descriptions of the latter occurrence were given by Barrois * and v. Lasaulx, but their works are unfortunately not accessible to me.

But this mineral is found in abundance in the *Japanese Islands*, and indeed, the glaucophane-bearing schist makes up a normal member amongst the crystalline schist-system, just occupying the upper horizon of chlorite-schist. Being of a beautiful colour, the rock can at once be recognized in hand specimens, and on this very account, it may be advantageously used in a classification of the crystalline schist-system of our country.

Secondary Glaucophane.

There yet remains to be described, a purple-green, more or less fibrous, mineral whose definite chemical relations, and also the crystallographic forms of which are of a somewhat doubtful nature. Before entering into the details, it is here to be remarked that the writer has been obliged to speak at some length of the rocks, and the augite contained in them, in order to give a clearer view of the formation of the secondary (paramorphosed) glaucophane.

According to the kind of rocks in which the glaucophane makes its appearance, descriptions are brought under the three headings.

I. Schalstein.†

(Slaty diabase-tuff.)

This rock plays an important rôle in the paleozoic group in Japan. There are two kinds of it, one being of a dark-red, the other of a green

* Cited by Rosenbusch in his 'Physiographie,' I Band, 2te Auflage, p. 471.

† Prof. Bonny is of opinion that some "schalsteins" are compact basalts that have undergone mechanical crushing. 'Quart. Journ. Geol. Soc.' Vol. XLII., 1886, Anniversary address, p. 60.

colour, and they are called respectively "brown or green slate." Both varieties come together in the same geological horizon, insensibly merging into one another. They decidedly belong to the paleozoic age, and are overlaid by the *sub*-carboniferous (?) limestone* which is extensively developed in Japan† and China.‡ The writer has often observed in fields, even, interstratification of the diabase-tuff with the subcarboniferous limestone.

It is slaty in its outer appearance, but less fissil than common roofing slates, and is so named "slate" by geologists of the geological survey of Japan. This diabase-tuff becomes not unfrequently massive and at the same time porous, being filled with calcite; thus it has then, all the appearances of melaphyre.

The lamellar-granular ('flaserige') rock-mass encloses the round, bent augite, and produces a structure similar to that seen on a large scale in the "augen-Gneiss." The typical specimen comes from Ananai, in Tosa province, near the Kunimi-yama.

The *augite* which the writer supposes to be diallage, but without being able to give a decisive proof, presents quite a fresh vitreous aspect, although traversed by irregular transversal fissures in various imaginable directions. The augite shows neither the tendency of a parting in the orthopinacoidal direction, nor that characteristic interposition very common in the gabbro-diallage. In short, it is just like augite in younger eruptive rocks ; the mineral is olive-green or brown in colour.

The augite is weakly pleochroic on the clinopinacoid, while on the orthopinacoid pleochroism is scarcely discernible. The extinction-direction varies from 23° — 31° with the trace of cleavage parallel to $\acute{C}$. The writer does not wish to lay much stress on the angular

* Cf. C. Gottsche, 'Science,' Vol. I, p. 166.

† E. Naumann, 'Ueber den Bau und die Entstehung der japanischen Inseln.'

‡ v. Richthofen, 'China.'

measurement, as it is impossible to find a good orientated section.

Paramorphosis of the diallage is abrupt, and usually begins from peripheries or from transversal fissures. The glaucophane is a small columnar or very fine fibrous; in the latter case it may, perhaps, be proper to call it *crocidolite*, although it is not capable of a rigid proof, on account of the minuteness of its size. The direction of fibres coincides, as a rule, with that of the general mass of the rock without regard to the position of diallage. The compact glaucophane again resolves at its terminations into a fibrous crocidolitic mass which, then, becomes grass-green in colour, and insensibly merges in the general rock-mass. The colour of glaucophane and its derivative—crocidolite, is deep green when the longest side is parallel to the short diagonal of the lower nicol; at right angles to it, it is lavender-blue.

The glaucophanized portion and the intact diallage are orientated in the same direction, and extinguish the light in the direction of C', when viewed from the orthopinacoid. On the clinopinacoid, however, the extinctions do not occur simultaneously, but show great deviations; and as the glaucophane-prisms are usually minute in size and the extinction-angles are so small, it appears as if were in the direction of the axis C'.

II. Amphibolite.

Amphibolites form a member in the archæan complex; their external appearances are various, on one side they approach to an ash-grey clay slate; on the other to a compact chlorite-schist; thirdly they have a close resemblance to an ordinary serpentine. So multifarious are their outward aspects that the writer has been struck with the simplicity of the mineralogical composition, when viewed under the microscope.

The microscope reveals to us no other ingredients than diallage and its derivatives–glaucaphane, and a hardly definable chloritic

matter. Only a few quantities of felspar, and a yellowish-green, highly pleochroic epidote, are sporadically discernible.

(a.) A typical rock of the appearance of a compact chlorite-schist is found in Nakakubo, in Kanra Gōri,* Kōzuke.

Here the pyroxene presents many characteristic features quite distinct from those already mentioned. The most striking aspect of the pyroxene is its light yellowish-brown colour, very common in augites of Basalts, but less so in andesites and trachytes. It is, as usual, traversed by various transversal cracks, from which the process of 'glaucophanization' has been commenced as well as from peripheries. The paramorphosed glaucophane is compact so as to lead observers to think that there is an intergrowth with a pyroxene. In other respects it presents nothing very special, except its intensely blue colour. The glaucophane, in turn, passes into a somewhat finely striated, deep green substance, and its transformation is very gradual; the only difference between them is in the colour of both minerals. *(Fig. III.)* The above-mentioned, deep yellowish-green mineral might be classed near arfvedsonite, judging by its external habitus and colour after comparing it with a specimen from a locality unknown to me, in Greenland. This, as well as the glaucophane, resolves into a confused aggregate of a yellowish, fibrous mass and the latter may fitly be called crocidolite.

(b.) On the southern side of the Tsuyetate pass, in the village of Kitagawa, Tosa province, there occurs an ash-grey, earthy slate which under a simple macroscopical consideration, gives scarcely any indication as to nature of the mineral contained in it.

Under the microscope thin slides show nothing more than glaucophane, and its derivatives of an extremely fine fibrous structure, together with some remnants of the original diallage. We may follow,

* Kōri or Gōri is nearly synonymous with a township or 'Kreis' in Germany.

step by step, every stage of transformation from diallage to glaucophane, and furthermore to a fibrous asbestiform substance. (*Fig. III.*) Compact glaucophane is comparatively rare, and if it is present then, shows a columnar form terminated at both ends by radiating tufts of the glaucophane-asbestus.

Both the glaucophane, and glaucophane-asbestus extinguish the light at the same time, and the ray vibrating parallel to C' is of a light blue; at $\perp$ to it light lavender-blue. Sometimes the asbestus-fibres assume a light yellow colour or become even colourless; this mass alone makes up nearly the whole rock.

Sometime ago, a stone-ax (10 *cm.* × 2 by $1/2$ *cm.* in size) was brought from the village Nomiya near Sapporo, Yesso (Hokkaidō); this is supposed to have been used by Ainos, the aborigines of that island. A slide made from this possesses exactly the same feature as that of Kitagawa, and therein we find a few diallage-remnants, one portion of each of them has been already changed into glaucophane, showing, if proved on the extinction-direction, a considerable deviation in their angular measurement.

It elucidates most typically the various stages of fibrillation from the intact diallage to an infinitely small crocidolitic fibres. A resolved part displays a corymbose ramification, proceeding from both ends of diallage-individual, and afterwards it makes undulating curves, indicating the effect of pressure upon the rock. (*fig. I.*) Farther researches of glaucophane-rocks from other localities might bring forth many not uninteresting facts, considered especially from the ethnological point of view.

(*c.*) The amphibolite of a serpentinous aspect is most typically developed near Izushi in Kodama Gōri, Musashi province. By a macroscopical consideration it looks just like an ordinary blue serpentine, for which, indeed, it has often been taken; and in which here

and there a brown diallage (2-4 *cm.*) is sporadically distributed.

The rock has precisely the same structure possessed by the "schillerspath" or "bastite" of the Harz Mountains, and following the suggestion given by Geo. H. Williams,* the writer here uses the term "poicilitic" for that structure. The diallage is provided with that peculiar, finely striated appearance, which is well-known to be the result of pressure.† In a large number of cases crystals are bent, thereby presenting an appearance as in the case of mica-lamellæ.

The outlines of the diallage are very irregular, being traversed by various cracks in all possible directions; consequently the original crystallographic form is nowhere visible.

The process of 'glaucophanization' seems to have commenced from the above-mentioned fissures and peripheries of individuals, precisely in the same manner as that of olivine in becoming a serpentine. One portion of crystals is, however, resolved into somewhat larger columnar individuals of glaucophane which have the same optical orientation as the mother-mineral. Transformation proceeds molecule after molecule in so gradual a manner, that when the section of diallage is at ⊥ with the shorter diagonal of the polarizer, both the glaucophane, and the original mineral are of a light brown, and we can scarcely discern the boundary of the two. If the section is at right angles to the former position, then the difference of colour becomes at once quite distinct, one being a lavender-blue, the other a brown. (*Fig. I.*)

The glaucophane thus produced resolves itself again into the glaucophane-asbestus which in turn, passes into a confused aggregate of minute fibres, assuming at the same time a grass-green colour.

Prof. Bonny‡ has ably described a gabbro from Pegli near

* Ameri. Journ. of Science, vol. xxxi, January, 1886, p. 30.

† Vide. O. Mügge, Neues Jahrbuch für Min. etc., 1883, I., p. 84.

‡ Geological Magazine, vol. VI., 1879, p. 363. 'Lingurian and Tuscanian Serpentine.'

Genoa, and found in the groundmass "rather fibrous or filmy patches, many of which show a peculiar blue colour." "On testing for dichroism," he came to the conclusion that "there can be no doubt it is glaucophane" and of a secondary origin. Prof. Bonny was, perhaps, the first who had spoken of a secondary glaucophane, and his very specimen from Italy, has a close resemblance to that of Musashi, as may be judged from his descriptions, as far as the groundmass is concerned.

A fresh portion of our diallage has been mechanically separated by the Thoulet solution, and a chemical analysis prosecuted by the writer in the laboratory of Prof. W. Knop in Leipzig, gave the following result:—

$Si\,O_2$	46, 40	
$Al_2\,O_3$	15, 59	
$Fe\,O$ } $Fe_2\,O_3$	12, 62	$FeO > Fe_2O_3$
$Mg\,O$	7, 15	
$Ca\,O$	13, 52	
Na_2O	2, 23	
$K_2\,O$	0, 93	
$H_2\,O$	1, 60	
	100, 04	

As it contains over 2 °/₀ of $Na_2\,O$, it approaches in its chemical composition to an alkaline augite, and to this very fact the formation of the secondary glaucophane may be attributable.

III. Melaphyre.

We have still another occurrence of glaucophane to be considered, and this is in the melaphyre-enclosure in a slate at Akaya, east of Ōmiya, Musashi province.

It is generally supposed that the occurrence of glaucophane in eruptive rocks is exceptionally rare. The mineral glaucophane, says Rosenbusch,* appears exclusively confined within the formation of crystalline schists, and phyllite. "Als Ausnahme† ist das Auftreten in einer Minette der Gegend von Wackenbach in Breuschthal zu erwähnen." Therefore, it may be of some interest to give a short description of it. The rock itself appears dark grey, and contains small amygdaloidal cavities filled with the carbonate of lime.

Under the microscope thin slides show a typical eruptive character, but the decomposition has so far advanced that the fresh porphyritic pyroxene is nowhere visible. The whole groundmass is made up of a trichitic substance exhibiting a skelet-like arrangement. *(Fig. IV.)* The glaucophane in this rock is surely of a secondary origin, as it contains still remnants of a brown pyroxene in the centre. The colour of this glaucophane is a light indigo-blue with a slight tinge of purple. Sometimes amygdaloidal spaces are filled with radiating needles of the same mineral. In all other respects it differs in no wise from those already described.

Before leaving the subject it may be well here to make some remarks on the difference between the primary, and secondary glaucophane; and furthermore, on a probable mode of procedure, by which the latter has been derived from the former.

As to the distinctions of both, the crystallographic forms are most apparent; the primary one is bounded by the prism together with the clinopinacoid, while in the other, the above pinacoid is lacking. Secondly, the primary mineral is considerably large in size of individuals. One more difference still exists, and this is in colour; the

* Mikroskopische Physiographie, I Band, 2te Auflage, p. 471.

† The writer may add one more exception of its occurrence in Europe. Lossen mentioned its presence in an acidic eruptive rock in the Devonian, near the basin of Elbingerede, in the Harz Mountain. Here unfortunately, the veteran-geologist does not give the exact description of it Zeitschrift d. deutch. geol. Ges. XXXIII, 1881, p. 175.

original glaucophane being bluish or purple, while the other is rather greenish.

The very first step of transformation is the production of stripes in the original, compact diallage, of which mineral alone the primitive rock seems to have been composed, and may then be termed pyroxenite. The cause of stripes or rather a lamellar structure may be sought in the action of pressure, under which the rock has been subjected in the the course of geological time. The action of pressure was so great as in many cases to have caused an actual crushing of massive diallage, and pulled it asunder in the direction of schistosity. *(Fig. I.)* This act of mechanical deformation of rocks, accompanied by the molecular rearrangement in mineral contained in them, has been so ably discussed by J. Lehmann,* Teall,† G. H. Williams,‡ and others that it is quite unnecessary to enter into details on the present occasion, and the writer expresses but his agreement with the view advocated by the above-named geologists.

In the weak, unprotected part, that is peripheries, and fissures of the mineral, paramorphosis readily begins to set forward in becoming glaucophane; the latter mineral seems, however, to be a chemical nature of unstable equilibrium, and represents only an ephemeral stage in the process of transformation; for, the secondary glaucophane soon assumes a fibrous structure, and becomes crocidolite. The writer observed a fact that might corroborate the above statement, and it is a matter of no small importance. For microscopic examination, a number of slides had been prepared of specimens from different localities, and glaucophane in these slides in the course of some weeks, totally lost its original blue colour, and turned into a light green substance.

* Ueber die Entstehung der altkrystallinischen Schiefergesteine, p. 190 et seq;

† Q. J. G. S., Vol. XLI, 1885, p. 139.

‡ Ameri. Journ. of science, Vol. XXVIII, 1884, p. 267.

Concerning the geographical distribution of the glaucophane-bearing rocks in Japan, a few words may still yet be said. As it is already stated, the rock containing the primary one is of a tolerably wide distribution; but the secondary glaucophane-rock is by far the most extensive.* E. Naumann † has already pointed out the importance of chlorite-schist among the crystalline schists, and it is a startling fact that nearly the half of what he called chlorite schist, turns out, on microscopic examination, to be the glaucophane-amphibolite. It is no wonder that such a rare rock as this, has been prodigiously developed in this part of the world.

Imperial University, Tōkyō, October, 1886.

* We have still to mention the other localities:—Yoshinobu village, Tosa-gōri, Tosa province; Yamakami and Hashikura, both in the Kanra-gōri in the Kōzuke province.

† loc. cit. p. 9.

EXPLANATION OF THE PLATE XII.

Fig. I. A striped diallage-fragment, showing the gradual transition of diallage into glaucophane, and then into crocidolite. Slide is taken from a stone-ax found in the village Nomiya, near Sapporo, Yesso.

Fig. II. Intergrowth and the penetration of one individual of glaucophane into the other upon the prismatic face.

Fig. III. Microscopic appearance of a slide of a slaty amphibolite from the Tsuyetate pass, Tosa, showing some remnants of diallage, secondary glaucophane and the derivative mineral of the latter.

Fig. IV. Secondary glaucophane with a diallage remnant, found in the trichitic groundmass of Melaphyre, found in Akaya, Musashi province.

Fig. V. Intergrowth of two individuals of glaucophane upon the clinopinacoid.

Fig. VI. A diallage-remnant in a slide of amphibolite from Izushi, Musashi province, showing the same orientation of diallage, and the paramorphosed glaucophane; but with different angles of extinction.

Mercury Sulphites, and the Constitution of Oxygenous Salts.

By

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and

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Much less can be done in the direction of determining the constitution of inorganic compounds than of carbon compounds. One consequence of this is the one-sided development of descriptive chemistry, in which our knowledge of carbo-hydrogen compounds far exceeds that of all the others. Modern views about the constitution of inorganic salts is based almost entirely upon observations of the nature of organic compounds, since it is usually considered that very little can be learned on this point by a study of the salts themselves. In the course of our own studies, the main results of which have been already published, it has seemed, however, to us that in the metals mercury, silver, and to a less extent, copper, we have elements differing sufficiently in their chemical affinity from other well-known metals, to enable us by their means to test at least one important point in the constitution of salts. The metals we have named do not decompose water or set free hydrogen from an acid; their affinities are for nitrogen and for sulphur, rather than for oxygen; and they

form two series of salts in which they are strongly basic. Thus differentiated from most metals, their behaviour ought to throw light upon the relations of oxygen salts to 'haloid' salts; and in order to show that they may thus be used, we shall take sulphates to represent oxygen salts, and sulphides as haloid salts.

A sulphate may be constituted as a sulphide with its sulphur oxidised, or it may be oxidised metal combined with oxidised sulphur, or, again, have other constitutions which we here pass over. An examination of the mercury and, say, the magnesium salts seems at once to settle this point, because mercury sulphide is one of the most stable compounds known, and mercury sulphate is instantly decomposed by water, while magnesium sulphide is decomposed by water, and magnesium sulphate is a most stable salt, quite unaffected. For magnesium here preserves its oxylic union with sulphur, ($Mg{=}O_2{=}SO_2$), and drops that with sulphur direct, ($Mg{=}S$), while mercury behaves in the reverse way, as the result, we may well conclude, of the well-known affinity of mercury for sulphur rather than oxygen, and that of magnesium for oxygen rather than sulphur.

Many elements, and sulphur is one of them, form more than one oxygen acid and corresponding series of salts, and the question suggests itself, whether in the less oxidised salts there will not be a constitution intermediate to those of haloid and fully oxidised salts. In organic chemistry this question has long been answered, and the less oxidised salts, distinguished by the termination, —*onates*, recognised as constituted partly as oxylic salts, partly as haloid salts, as the following names and formulæ will serve to make clear :—

Ethyl potassium sulphate	$Et\,K\,SO_4$	$= \left.\begin{matrix} Et\,O \\ KO \end{matrix}\right\} SO_2$
Ethyl potassium sulphonate	$Et\,K\,SO_3$	$= \left.\begin{matrix} Et \\ KO \end{matrix}\right\} SO_2$
Ethyl potassium sulphide	$Et\,K\,S$	$= \left.\begin{matrix} Et \\ K \end{matrix}\right\} S.$

Here it will be seen that in the sulphonate, the ethyl is directly united to the sulphur, and the potassium only by the intervention of an atom of oxygen. Whether these sulphonates are true organic sulphites, and whether, therefore, inorganic sulphites are similarly constituted, or have their base wholly in oxylic union, has long been uncertain, although now the opinion most generally held is that sulphites have half their metal directly united to sulphur, thus, $Mg\overset{O}{\sqsubset\!\sqsupset}SO_2$, and not, $Mg{=}O_2{=}SO$. This opinion has been based, however, wholly upon evidence afforded by organic bodies, and it seemed to us desirable that evidence from the inorganic sulphites themselves should be sought for, by examining the mercury, silver, and copper sulphites. Hence, the reason of our working upon mercury sulphites. We have also done a little upon silver sulphite, already pretty well known, but have not yet examined copper sulphites.

Mercury sulphites, we found, had been but very imperfectly studied. Several had been described, but we were soon convinced that of these only one really existed. To this, however, we succeeded in adding two new ones in the separate state, besides one which we only got in dilute aqueous solution. Yet after all, most remarkable to state, we could get neither the normal mercuric sulphite, $Hg\ SO_3$, nor the mercurous sulphite, $Hg_2\ SO_3$, but only a basic salt; an acid salt; a mixed salt, partly mercurous, partly mercuric; and another mixed salt, partly hypomercurous, partly mercuric. Besides these, there are double sulphites, one of which, sodium mercuric sulphite, we have examined more fully than had been done before.

The evidence afforded by the mercury sulphites, and by silver sulphite, as to the constitution of sulphites, proved even greater than we had expected. They are not decomposed by dilute nitric or sulphuric acid, and in this they resemble the chlorides and cyanides of these metals, instead of their oxygen, (or rather oxylic), salts, while in

the same point they differ from most sulphites, which, as every chemist knows, are exceedingly easily decomposed by acids. They are not decomposed by water, except when hot, and then they are resolved into metal, whether mercury or silver, and SO_3, which with the water forms sulphuric acid, and this again, where very little water is present, may react with any unchanged sulphite, and give mercurous or silver sulphate and sulphurous acid. This property of decomposing into metal and a non-metallic element, or group of elements, at once recalls the decomposition of mercury or silver cyanide, or gold chloride, by heat.

One more point of relationship with haloid salts is the particular readiness with which mercury and silver sulphites form double sulphites with very oxidisable metals, such as potassium and sodium, and the comparative stability of these salts. Thus, while a mercury or silver sulphite is decomposed by hot water, and sodium sulphite even by cold water, (if we may judge from the strongly alkaline character of its solution), yet the mercuric (or silver) sodium sulphite is neutral to litmus, and may be boiled with water, or with alkali, without decomposition. Having now discussed some of the points in chemical doctrine, which these bodies serve to throw light upon, we proceed to notice the several mercury sulphites, which in other respects are not without interest.

Mercuric oxysulphite, $(OHg_2\ SO_3)_2$, H_2O, first prepared by Péan de St. Gilles, is best prepared by adding a solution of sodium sulphite to an excess of somewhat concentrated solution of mercuric nitrate, as free as possible from nitric acid. Some mercuric nitrate remaining undecomposed, the oxysulphite almost at once makes its appearance as a flocculent precipitate, rapidly becoming granular and dense. It is of a pale yellow colour, and very unstable. When dry, it is explosive; sudden and complete, though very gentle, detonation being caused by

the touch of a hot wire at any point of a mass of it, or a rise of temperature to about 73° C., or percussion. Another of the properties of the sulphite, calling for notice, is its indifference to sufficiently dilute nitric acid, although it is a basic or oxysalt. Another is that, when dissolved by stronger acid, it rapidly changes into the metameric substance, mercurous sulphate. The same change takes place in the oxysulphite, under any circumstances, upon keeping it for a day. The metamerism shown by mercurous sulphate and mercuric oxysulphite, is of very rare occurrence in inorganic chemistry, though so common in organic chemistry. Treated with potassim hydroxide, three-fourths only of its mercury are left as oxide, the rest dissolving as double sulphite. Sodium chloride leaves half the mercury as oxide, and dissolves the other as double salts.

Mercury hydrogen sulphite, $Hg\,(SO_3H)_2$. This salt we can get only in dilute solution, by cautiously adding precipitated mercuric oxide suspended in water to excess of sulphurous acid. Mercuric oxide rashly added precipitates the sulphite next described. So, too, strange to state, does a little sulphuric or nitric acid added to the sulphurous solution of the mercuric oxide. The solution is unstable, slowly decomposing into metallic mercury and sulphuric acid. Potassium hydroxide does not precipitate the mercury.

Mercurosic sulphite, $Hg\,(SO_3)_2\,Hg'_2$, $4H_2O$, can be prepared in two sets of ways, either by hydrolysis of mercuric hydrogen sulphite, or by some form of double decomposition. When sulphurous acid is treated with precipitated mercuric oxide, mercuric hydrogen sulphite is first formed, as already described, but the attempt to make much of it in the same solution is at once followed by the precipitation of mercurosic sulphite, and the generation of sulphuric acid, thus:—

$$3\,Hg\,(SO_3\,H)_2 + OH_2 = Hg_3\,(SO_3)_2 + SO_4\,H_2 + 3\,SO_3\,H_2$$

Mercuric oxide, in a thin paste with water, is treated with a stream of

sulphur-dioxide gas, at first slowly, as else some black hypomercurosic sulphite may be formed. Or, to the paste of mercuric oxide strong sulphurous-acid solution containing a little sulphuric acid is added until in excess. The presence of the sulphuric acid prevents any formation of hypomercurosic sulphite. Mercurosic sulphite, thus prepared, is a splendent-white, distinctly crystalline, voluminous precipitate, insoluble in water and in dilute sulphuric or nitric acid, and when dry comparatively stable, except in being efflorescent. It can also be prepared from hypomercurosic sulphite and mercuric nitrate, but it is then of a pale buff colour, and amorphous :—

$$Hg_4\,(SO_3)_2 + Hg\,(NO_3)_2 = Hg_3(SO_3)_2 + (Hg\,NO_3)_2.$$

In thus existing in two forms, white and buff coloured, this sulphite resembles mercurous chloride. Like the white, the buff-coloured variety is insoluble in dilute sulphuric or nitric acid. By hydrochloric acid mercurosic sulphite is decomposed giving both chlorides of mercury. In other respects, too, it behaves as a compound half-mercurous, half-mercuric ; thus potassium hydroxide yields with it mercuric potassium sulphite and mercurous oxide :—

$$Hg_3\,(SO_3)_2 + 2\,KOH = Hg\,(SO_3\,K)_2 + Hg_2\,O + H_2\,O.$$

Heated with water, it suddenly decomposes into metallic mercury and sulphuric acid ; while when little or no water is present, mercury, sulphur dioxide, and mercurous sulphate are the products.

Hypomercurosic sulphite, $Hg_4(SO_3)_2$, H_2O, is best prepared by treating precipitated mercurous sulphate with sulphurous acid, in presence of a little sulphuric acid, this acid having the property of preventing decomposition of the hypomercurosic sulphite by the sulphurous acid, just as it prevents that of mercurosic sulphite. Hypomercurosic sulphite can also be prepared by the addition of sodium sulphite, not in excess, to mercurous nitrate, and in other ways. It is a flocculent, voluminous, greyish-black precipitate, insoluble in dilute nitric or sul-

phuric acid, but slowly decomposed by hydrochloric acid into the two chlorides of mercury, metallic mercury, and sulphurous acid. It is similarly decomposed by sodium chloride, and by potassium hydroxide. Sulphurous acid, or sodium sulphite, rapidly converts it into mercury, sulphuric acid, and mercuric hydrogen sulphite, or mercuric sodium sulphite. Heated alone, or with very little water, it yields the same products as mercurosic sulphite; and heated with much water it is wholly and suddenly converted into mercury and sulphuric acid.

On the Reduction of Nitrites to Hydroxyamine by Hydrogen Sulphide.

By

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and

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The solution of an *alkali nitrite*, saturated with hydrogen sulphide, and then acidified with hydrochloric or sulphuric acid, yields sulphur, nitric oxide, and ammonia, but no hydroxyamine, the presence of this being incompatible with the presence of what is known as *free* nitrous acid. When the escaping gases are collected out of contact with air, they slowly deposit sulphur, and do so more quickly still, when bubbled through water into the air, in consequence of reactions between hydrogen sulphide, nitric oxide, and oxygen,—but still no hydroxyamine appears.

But when *silver nitrite* in water is treated with hydrogen sulphide, then besides the sulphur, nitric oxide, and ammonia, still abundantly formed, (and also silver sulphide), there is found a considerable quantity of hydroxyamine. On filtering off the silver sulphide, adding hydrochloric acid to convert the hydrosulphides into hydrochlorides, and evaporating to dryness, there is obtained a mixture of ammonium and hydroxyammonium chlorides. (Heating the filtrate before adding an acid causes destruction of the hydroxyamine by the hydrogen sulphide).

Two estimations were made of the quantity of hydroxyamine in the solution, after this had been heated with hydrochloric acid. Silver nitrite, 0.6926 gram, was found, by titration with iodine, to have yielded about one-sixth of its nitrogen as hydroxyamine; while, in another case, 0.0644 gram yielded as much as three-elevenths of its nitrogen in this form.

Mercurous nitrite, supposed not to exist, but which will be described in a future communication, has been prepared by the authors, and found also to yield hydroxyamine when treated with hydrogen sulphide. But this salt forms dense, hard crystals, and is exceedingly insoluble in water, and thus becomes difficult to decompose fully, even by soluble chlorides, including hydrochloric acid. It, accordingly, resists for a long time complete decomposition by hydrogen sulphide, so that when that which appears to be only the mercury sulphide and sulphur precipitate is boiled with water, a nitrous smell is observed, due, no doubt, to decomposition of some residual nitrite.

The green solution prepared by mixing alkali nitrite with copper sulphate also yields hydroxyamine when treated with hydrogen sulphide.

It will hence be seen that those nitrites of which the metals,—mercury, silver, copper,—have especially marked affinities for nitrogen, and which nitrites, therefore, have a certain stability in presence of acids, are capable of being reduced to hydroxyamine. In large part, indeed, even these nitrites are decomposed by the hydrogen sulphide, so as to yield only the products of the decomposition of this acid by water and additional hydrogen sulphide; but the rest appears to act as follows :—

$$Ag\,NO_2 + 2H_2S = Ag\,N\left\{\begin{matrix}OH\\H\end{matrix}\right. + H_2O + 2S$$

and

$$2Ag\,N\left\{\begin{matrix}OH\\H\end{matrix}\right. + H_2S = Ag_2S + 2HN\left\{\begin{matrix}OH\\H\end{matrix}\right.$$

As a means of the preparation of hydroxyamine, the action of hydrogen sulphide upon a nitrite has no apparent value. The interest of this reaction lies in the light it may help to cast upon the constitution of the oxygen compounds of nitrogen. Experiments made in our laboratory, and already published in the *Journal of the Chemical Society of London*, serve to show that the metals of the zinc-tin class convert *nitric acid* into *ammonia*, and not hydroxyamine, when another acid is absent; and into *hydroxyamine*, without ammonia, when acting in conjunction with hydrochloric or sulphuric acid (J.C. S., 47, 615); and, further, that in the conversion of nitric acid to hydroxyamine, no *nitrous acid*, *nitric peroxide*, or *nitric oxide* shows itself as an intermediate product. It therefore, remained uncertain whether an (inorganic) *nitrite* could be converted into hydroxyamine. By the reaction of the nitrites of the silver class of metals with hydrogen sulphide this uncertainty is now removed.

In connection with the subject of this paper, the well-known behaviour of the nitro-hydrocarbons is of interest to consider. The application of reducing agents, if appropriate, converts them to *amines*, but never to *hydroxyamines*. Yet the paraffin members of the series yield simple or substituted hydroxyamines by certain treatment. Plainly, it must be impossible by simple reduction to convert the *nitro*-compound into the corresponding *oximido-* compound, because of the difference in valency between the nitroxy- and oximido- radicals and therefore the reaction proceeds probably as follows :—

$$Et\,N \triangleleft O_2 + 2H = Et\,N\,(OH)_2$$

and

$$Et\,N\,(OH)_2 + 4H = Et\,NH_2 + 2OH_2$$

But when, under special conditions, the hydrocarbon radical is made to suffer change, it becomes possible to form an oximido compound (Kissler), or, at least, hydroxyamine itself (V. Meyer). In these cases, the hydrocarbon acts as the reducing agent upon the nitroxyl. Thus,

nitroethane, at 140°, and in presence of water and hydrochloric acid, yields hydroxyamine, passing probably through the following change:—

$$MeC\begin{cases}H_2\\NO_2\end{cases} + \begin{matrix}O_2N\\H_2\end{matrix}\Big\}CMe = MeC\begin{cases}O\\NH.OH\end{cases} + \begin{matrix}HO.HN\\O\end{matrix}\Big\}CMe$$

in which acet-hydroxamic acid results from the interaction of two molecules of the nitroethane, and then, as usual under such circumstances, suffers hydrolysis into hydroxyamine and acetic acid. Again, by the interaction of sodium-nitroethane and benzoic or acetic chloride, diacet-hydroxamic acid is actually secured as the end-product:—

$$2\,Me\,CNa\,HNO_2 + 2\,\overline{Bz}\,Cl = (Me\,CO)_2\,NOH + \overline{Bz}_2\,NOH + 2Na\,Cl.$$

In this case, each oximido-radical finds ready for it two univalent radicals, in place of the one united to the nitroxyl, in consequence of the separation of the sodium and chlorine as salt.

It is thus evident, that, under suitable conditions, both metal nitrites (or nitrous acid itself) and organic nitrites (nitronites) reduce to hydroxyamines. It is, further, seen that the conversion of inorganic nitrites to hydroxyamine lends no support to the view that they have an *oxylic* constitution; because organic nitrites, beyond doubt non-oxylic, also reduce to oximido-compounds.

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Beitraege zur Theorie der Bewegung der Erdatmosphaere und der Wirbelstuerme

von

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EINLEITUNG.

Die Bewegung der Erdatmosphäre, welche als Wind in der Meteorologie eine so ausserordentlich wichtige Rolle spielt, bietet unstreitig eins der schwierigsten analytischen Probleme dar. *

Die Schweierigkeit der Lösung liegt nicht so wohl in dem beschränkten Mittel der reinen Analysis, als in der völligen Unkenntniss der exakten mathematischen Ausdrücke für gewisse Ursachen, welche auf die Bewegung der Erdatmosphäre maassgebend einwirken.

Suchen wir nus die Verhältnisse klar zu vergegenwärtigen, welche bei der Aufstellung der Differentialgleichungen für die allgemeine Bewegung der Erdatmosphäre zu berücksichtigen sind! Die Erdoberfläche, auf welcher die Atmosphäre ruht, zeigt zwischen der

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* Eine mathematische Theorie der Bewegung der Erdatmosphäre ist schon mehrfach mit Erfolg versucht worden. Von den darauf bezüglichen neueren Arbeiten erwähne ich nur.

Etudes sur les mouvements de l'atmosphère par C. M. Guldberg et H. Mohn. Christiana 1876.

Über die Bewegung der Luft an der Erdaberfläche. Oberbeck. Wiedemann Annalen Band XVII 1382.

Zur Mechanik des Windes—Dr Sprung. Archiv der deutschen Seewarte. Jahrgang II 1879.

Leider war mir die Arbeit der beiden norwegischen Forscher unzugänglich, und konnte desshalb in der gegenwärtigen Abhandlung keine Berücksichtigung finden.

Aequatorial—und Polarzone beträchtliche Unterschiede der Temperatur; es ist dadurch in der unteren Schichte der Erdatmosphäre eine fortwährende Strömung von den kälteren Polargegenden zum wärmeren Aequator veranlasst. Indem nun diese Luftströmungen von den beiden Polen her gegen den Aequator fliessen, und dort sich gegen einander stauend, einen Gürtel der verticalen Strömung bilden, so muss gleichzeitig auch in der oberen Schichte der Atmosphäre eine entgegengesetzte polwärts herabfliessende Strömung vorhanden sein. Wenn nun diese Aequatrialströmungen nach der böheren Breite fortschreitet, sinken sie, ihre höhere Temperatur mit der Umgebung in Gleichgewicht setzend, immer mehr vertical abwärts, und gehen schliesslich, eine reine verticalabwärts gerichtete Strömung bildend, zur Polarstromung über, um auf's nene nach dem Aequator zu fliessen.

Wäre die Erde eine rotationslose reine Kugel, welche sich in einer Flüssigkeit ohne Reibung bewegte, so würden sowohl die Aequatrial-als die Polarströmung in der Richtung fortschreiten, in welche sie zu fliessen begann,—die Bewegung der atmosphärischen Luft wäre in so fern eine einfache, als sie einen gewaltigen Wirbel bildend entweder theilweiese, oder ganz die Erdhemisphäre umfasst. Die analytische Aufgabe, die zu lösen wäre, bestünde darin, die Wirbelbewegung einer Flüssigkeit zu finden, welche einerseits durch eine feste Kugeloberfläche-und anderseits durch die Grenzfläche der Atmosphäre begrenzt ist, vorausgesetzt, dass die Gestalt des wirbelerfüllten Raumes bekannt ist. Die Erde dreht sich indessen täglich von West nach Ost um ihre Axe; die Bewegung der atmosphärischen Luft ist desswegen eine aus dem nord-südlichen, thermisch bedingten Antrieb und aus dieser west-östlichen Rotationsbewegung zusammengesetzte. Die Aequatorialströmung sowohl als Polarströmung erleidet daher eine Ablenkung von ihrer Nord Süd; respectiv Nord-Süd-Richtung im eutgegengesetzten Sinne, die erstere nämlich

von ihrer nach Norden gerichteten Bahn nach Osten, und die letztere hingegen von ihrer Südrichtung nach Westen.

Dieser Einfluss der Erdrotation kann unter allen Umständen in Rechnung gezogen werden, weil wir von der rotatorischen Bewegung, welche ein Lufttheilchen in Folge der Erdrotation gegen ein im Raum festes Coordinatensystem hat, ganz absehn dürfen, sobald wir statt eines festen Coordinatensystems ; alle bewegten Theilchen auf ein mit der Erde rotirende Coordinatensystem beziehen. * Die Berücksichtigung der Erdrotation führt also immer nur auf verwickeltere Bewegungungsgleichungen und verursacht demnach weiter keine principielle Schwierigkeit. Was aber das vorgelegte Problem wesentlich erschwert, ja fast unmöglich macht, ist die Berücksichtigung eines anderen Einflusses der rotirenden Erdoberfläche; nämlich der Reibung und des Widerstandes, welchen einen Luftströmung entlang der Erdoberfläche erleidet. Offenbar ist so wohl der Widerstand als die Reibung vom ebenso tief eingreifendem Einfluss auf die Bewegung der Erdatmosphäre, wie die Umdrehung der Erde selbst, und die Vernachlässigung dieser Factoren müsste so wohl in der Richtung der Luftströmung als in ihrem Verlauf auffallendeste Abweichungen von der Wirklichkeit veranlassen, wie die Nichtberücksichtigung der Erdrotation; denn die Erde hat im Vergleiche mit der Höhe ihrer Lufthülle eine verhältnissmässig rauhe unebene Oberfläche. Hohe Gebirgeskette, und tiefe Thaleinsenkungen erregen lokale Stauungen der Luftströme oder Gegenströmungen, und bieten der ungestörten Fortschreitung der Luftströmung Hemmnisse dar, welche als Widerstand die nur thermisch, und durch die Erdrotation bedingte Bewegung auf's mannichfaltigste modificiren müssen. Und wenn selbst der Widerstand, den die Luftströmung entlang einer ausgedehnten Ehene, wie weite Steppen, oder Meere, erleidet, wo also bedentende lokale Störung, wie

* Kirchhoff Vorlesungen über die Mechanik. pag. 90 (Zweite Auflage.)

Stauung oder Gegenströmung von vorn herein ausgeschlossen ist, wo also der Widerstand demnach als nichtbedentend angesehn werden dürfte, kann die Reibung, welche die lewegten Lufttheilchen entlang der Erdoberfläche erleiden, nicht vernachlässigt werden, wenn nicht die gröbste Abweichung der theoretischen Resultate von der Wirklichkeit hervorgerufen werden soll. Den von der Unebenheit der Erdoberfläche herrührenden Widerstand zu berücksichtigen, ist natürlich unmöglich.

Es kann sich daher bei einer analytischen Untersuchung über die Bewegung der Erdatmosphäre nur darum handeln, die Erdoberfläche als eine reine ellipsoidische oder kugelförmige Fläche anzusehen Unter dieser einschränkenden Voraussetzung hat man dann nur die Reibung zu berücksichtigen, welche die Luftströmung entlang der ellipsoidischen, oder kugelförmigen Erdoberfläche erleidet.

Die strenge Berüchsichtigung dieses Einflusses der Erdoberfläche ist indessen schwierig; schon desshalb weil wir uns noch keine Vorstellung bilden können, welche Bewegungsformen eine Flüssigkeit annimmt, wenn sie an der reibenden Oberfläche eines starren Körpers entlangströmt, und weil wir daher kein Gesetz kennen, welches uns gestattet, den Einfluss der Reibung der starren Körperfläche auf ein bereits näher bekanntes Gesetz zurückzuführen und eine einigermassen strenge mathematische Formulirung desselben herzustellen. Um diese principielle Schwierigkeit des vorliegenden Problems, wenn auch nicht zu überwinden, doch zu umgehen, ist daher eine Hypothese nöthig und zwar eine solche, welche geeignet zu sein scheint, den etwaigen aus Bewegungsgleichungen gezogenen Schlüssen die möglichste Allgemeinheit zu sichern.

Man betrachte ein Stücke der Erdoberfläche, längs welcher die Luft strömt. Da die als starr zu denkende Erdoberfläche nicht die Bewegung theilen kann, welche eine unmittelbar darauf gelagerte

Luftschichte besitzt, so muss die Resultante der Geschwindigkeit der Lufttheilchen entlang der Erdoberfläche jedenfalls zur Normale derselben senkrecht stehen.

Wäre die Oberfläche der Erde reibungslos, so würden die Geschwindigkeit der Lufttheilchen parallel zur Erdoberfläche unverändert bleiben, wenn die Geschwindigkeit der Luftschichten parallel zur Oberfläche unverändert bleibt, und die Lufttheilchen, welche ursprünglich in einer Normallinie der Erdoberfläche enthalten waren, werden auch immer in einer solchen sein, alle Lufttheilchen, welche in verschiedenen Luftschichten dieselbe horizontale Geschwindigkeit besassen, sie auch immer besitzen. Allein; eine solche Bewegung macht die auf der Erdoberflache stattfindende Reibung unmöglich. Denn die untere unmittelbar auf der Erdoberfläche hinstreichende Luftschichte erleidet eine gewisse Verzögerung, und zwar in der Richtung ihrer resultirenden Geschwindigkeit. Die Bewegung aller übrigen Schichten, werden auch zugleich mitverzögert; allein; da der Einfluss der Reibung für die unterste Luftschichte am grössten sein muss, so müssen die Lufttheilchen in der oberen Schichte während der Bewegung nach und nach den bewegten Theilchen der unteren Schichte vorauszueilen streben, als ob eine gewisse Kraft, deren Intensität mit der verticalen Erhebung über der Erdoberfläche abnimmt, in allen Luftschichten auf die bewegten Theilchen in der Richtung der resultirenden Geschwindigkeit entgegengewirkt und so jene Verzögerung bewirkt hätte. Wenn wir demnach für die oben geschilderte Art der Luftströmung den Einfluss der an der Erdoberfläche stattfindenden Reibung zu berücksichtigen haben, so ist klar, dass wir den verlangten Zweck erreichen, wenn wir uns auf die bewegten Lufttheilchen gewisse Kräfte als einwirkend denken, welche ihren Sitz in der Erdoberfläche haben und dem Einfluss der Reibung an derselben gleichwerthige Effecte hervorbringen. Durch die Ein-

führung solcher Kraftcomponenten in die Bewegungsgleichungen wird man dann offenbar die ganze Luftströmung betrachten, können als bewegten die Lufttheilchen längs der Erdoberfläche ohne Reibung. Hat eine Luftströmung gegen die Normale der Erdoberfläche eine andere Richtung, als die senkrechte, so kann man die Bewegung eines Lufttheilchens jederzeit zerlegen, einerseits in die der Normale der Erdoberfläche parallele-und andererseits in die zur Normale senkrechte Bewegung. Die zur Erdoberfläche senkrechte Componenten werden nun offenbar durch den Einfluss der auf der Erdoberfläche stattfindenden Reibung nicht modificirt; wohl aber die Componenten der Geschwindigkeit parallel der Erdoberfläche. Die Reibung, welche die Geschwindigkeit der unmittelbar auf der Erdoberfläche gelagerten Luftschichte verlangsamt, wird nämlich die Bewegung der Lufttheilchen in der oberen Schichte auch zu verzögern streben, und zwar wiederum in der Richtung parallel zur Erdoberfläche. Der Einfluss der Reibung auf der Erdoberfläche auf alle übrigen Schichten wird demnach auch hier wieder ersetzbar sein, durch gewisse Kräftecomponenten, welche ihres Sitz in der Erdoberfläche haben, und auf jedes Lufttheilchen in der Richtung wirkt, welche derjenigen der horizontalen d. h. der Erdoberfläche parallelen componenten Geschwindigkeit gerade entgegengesetzt ist.

Ich nenne diese Kräfte, deren Wirkung den Einfluss der auf der Erdoberfläche stattfindenden Reibung ersetzen, einfach Reibungswiderstand, und denke mir sie so gegeben, dass sie eine Resultante haben, deren Richtung in der Atmosphäre überall derjenigen der resultirenden Horizontalgeschwindigkeit des bewegten Lufttheilchens gerade entgegengesetzt ist, und deren Intensität aber mit der verticalen Erhebung über der Erdoberfläche mehr oder weniger rasch abnimmt.

Indem ich durch die Einführung dieser Kräfte in die Bewegungsgleichungen die oben erwähnte principielle Schwierigkeit des vorlie-

genden Problems zu umgehen suche, verhehle ich mir keineswegs, dass es damit im Grund genommen so gut, wie nichts gewonnen ist; da ja der analytische Ausdruck für solche Kräfte, welche, wie es verlangt wird, in einer Flüssigkeit dieselbe Wirkung ausübt wie der Einfluss der reibenden Oberfläche eines starren Körpers, nicht bekannt ist. Es kann ferner nicht in Abrede gestellt werden, dass es viel einfacher und der Sache gemäss wäre, wenn die Reibung der Erdoberfläche erst in den Grenzbedingungen berücksichtigt werden würde. Allein; die unmittelbare Einführung der Reibungskräfte in die Bewegungsgleichungen hat den Vortheil, Folgerungen zu gestatten, welche sonst nur auf Umwegen und obenein nicht so allgemein zu gewinnen wären.

§ *I.* *Hydrodynamische Differentialgleichungen mit Rücksickt auf die Rotation der Erde.*

Es seien Ξ'' H'' Z'' die rechtwinkligen Componenten der Reibungskräfte bezogen auf ein rechtwinkeliges Coordinatensystem x'' y'' z'', dessen z''—Achse mit der Erdachse zusammenfallen möge. Es sei ferner G' das Potential der Erdanziehung auf ein Flüssigkeitstheilchen von der Masse 1. und p der Druck und μ die Dichtigkeit der Flüssigkeit. Die hydrodynamischen Differentialgleichungen sind dann

$$\left.\begin{aligned}
\frac{d^2x''}{dt^2} &= \frac{\partial\,(G'-\Pi)}{\partial x''} - \Xi'' \\
\frac{d^2y''}{dt^2} &= \frac{\partial\,(G'-\Pi)}{\partial y''} - H'' \\
\frac{d^2z''}{dt^2} &= \frac{\partial\,(G'-\Pi)}{\partial z''} - Z''
\end{aligned}\right\} \quad (1)$$

wo

$$\Pi = \int \frac{dp}{\mu}$$

aus der gegebenen Relation zwischen μ und p zubilden ist. Nun

führen wir ein zweites dem ersten congruentes Coordinatensystem (x' y' z') ein, welches sich jedoch mit der rotirenden Erde sich dreht, mit der constanten Winkelgeschwindigkeit λ, die, wenn man die Secunde zur Zeiteinheit und das Meter zur Längeneinheit nimmt

$$= \frac{2\pi}{24^h.60'.60''} = 0{,}00007643 \; *$$

ist.

Wir setzen

$$x'' = x' \cos \lambda t - y' \sin \lambda t \qquad z'' = z'$$
$$y'' = x' \sin \lambda t + y' \cos \lambda t$$

woraus folgt.

$$x' = x'' \cos \lambda t + y'' \sin \lambda t \qquad z' = z''$$
$$y' = y'' \cos \lambda t - x'' \sin \lambda t$$

Weil der Stundenwinkel λt in *dem* Sinne wachsend angenommen werden muss, wie die Erde sich dreht, so ist durch diese Gleichungen ausgesprochen, dass das neue Coordinatensystem, seine positive x Achse stets gegen Süden und seine positive y Achse aber gegen Osten kehrt, wie die Fig. I. (Tafel) es veranschaulicht.

Wir differentiren x'' y'' z'' nach t, um die neuen Coordinaten in die Gleichungen (1) einzuführen. Es ist.

$$\frac{dx''}{dt} = \cos \lambda t\left(\frac{dx'}{dt} - \lambda y'\right) - \sin \lambda t\left(\frac{dy'}{dt} + \lambda x'\right)$$
$$\frac{dy''}{dt} = \cos \lambda t\left(\frac{dy'}{dt} + \lambda x'\right) + \sin \lambda t\left(\frac{dx'}{dt} - \lambda y'\right)$$
$$\frac{dz''}{dt} = \frac{dz'}{dt}$$

Eine nochmalige Differentiation ergiebt weiter

$$\frac{dx^{2\prime\prime}}{dt^2} = \cos \lambda t\left(\frac{d^2x'}{dt^2} - 2\lambda\frac{dy'}{dt} - \lambda^2 x'\right) - \sin \lambda t\left(\frac{d^2y'}{dt^2} + 2\lambda\frac{dx'}{dt^2} + 2\lambda\frac{dx'}{dt} - \lambda^2 y'\right)$$
$$\frac{d^2y''}{dt^2} = \sin \lambda t\left(\frac{d^2x'}{dt^2} - 2\lambda\frac{dy'}{dt} - \lambda^2 x'\right) + \cos \lambda t\left(\frac{d^2y'}{dt^2} + 2\lambda\frac{dx'}{dt} - \lambda^2 y'\right)$$
$$\frac{d^2z''}{dt^2} = \frac{d^2z'}{dt^2}$$

* Genauer 0,00007292 (Meter) da ein Sterntag $23^h\,56'\,4'',09 = 86164''$ beträgs.

Andererseits haben wir

$$\frac{\partial G'}{\partial x''}=\frac{\partial G'}{\partial x'}\cos\lambda t-\frac{\partial G'}{\partial y'}\sin\lambda t$$

$$\frac{\partial G'}{\partial y''}=\frac{\partial G'}{\partial x'}\sin\lambda t+\frac{\partial G'}{\partial y'}\cos\lambda t$$

$$\frac{\partial G}{\partial z''}=\frac{\partial G}{\partial z'}$$

$$\frac{\partial \Pi}{\partial x''}=\frac{\partial \Pi}{\partial x'}\cos\lambda t-\frac{\partial \Pi}{\partial y'}\sin\lambda t$$

$$\frac{\partial \Pi}{\partial y''}=\frac{\partial \Pi}{\partial x'}\sin\lambda t+\frac{\partial \Pi}{\partial y'}\cos\lambda t$$

$$\frac{\partial \Pi}{\partial z''}=\frac{\partial \Pi}{\partial z'}$$

Die Geschwindigkeiten des Flüssigkeitstheilchen sind unabhängig von der Lage der Coordinatenaxen, und die Componenten der Reibungskräfte sind es auch, weil sie überall in entgegengesetzter Richtung, wie die Geschwindigkeitscomponenten wirken. Bezeichnet man daher die Componenten der Reibungskräfte bezogen auf die Axen des neuen beweglichen Coordinatensystems mit $\Xi'\ H'\ Z'$, so ist

$$\Xi''=\Xi'\cos\lambda t-H'\sin\lambda t$$

$$H''=\Xi'\sin\lambda t+H'\cos\lambda t$$

$$Z''=Z'$$

Die Gleichungen (1) werden dann

$$\cos\lambda t\left(\frac{d^2x'}{dt^2}-2\lambda\frac{dy'}{dt}-\lambda^2x'\right)-\sin\lambda t\left(\frac{d^2y'}{dt^2}+2\lambda\frac{dx'}{dt}-\lambda^2y'\right)$$

$$=\cos\lambda t\left(\frac{\partial(G'-\Pi)}{\partial x'}-\Xi'\right)-\sin\lambda t\left(\frac{\partial(G'-\Pi)}{\partial y'}-H'\right)$$

$$\sin\lambda t\left(\frac{d^2x'}{dt^2}-2\lambda\frac{dy'}{dt}-\lambda^2x'\right)+\cos\lambda t\left(\frac{a^2y'}{dt^2}+2\lambda\frac{dx'}{dt}-\lambda^2y'\right)$$

$$=\sin\lambda t\left(\frac{\partial(G'-\Pi)}{\partial x'}-\Xi'\right)+\cos\lambda t\left(\frac{\partial(G'-\Pi)}{\partial y'}-H\right)$$

Inden wir die erste von diesen Gleichungen mit $cos\ \lambda t$, und die Zweite mit $sin\ \lambda t$, multipliziren, so finden wir durch Addition

$$\frac{d^2x'}{dt^2} = \frac{\partial(G'-\Pi)}{\partial x'} - \Xi' + 2\lambda\frac{dy'}{dt} + \lambda^2 x' \qquad (2\,a)$$

Multipliziren wir hingegen die erste mit $sin\ \lambda t$ und die Zweite mit $cos\ \lambda t$, so finden wir durch Subtraction

$$\frac{d^2y'}{dt^2} = \frac{\partial(G'-\Pi)}{\partial y'} - H' - 2\lambda\frac{dx'}{dt} + \lambda^2 x' \qquad (2\,b)$$

Diese beiden Gleichungen (2 *a*) und (2 *b*) und

$$\frac{d^2z'}{dt^2} = \frac{\partial(G'-\Pi)}{\partial z'} - Z' \qquad (3\,c)$$

sind die Differentialgleichungen für die Bewegung der Flüssigkeit unter Berücksichtigung der Erdrotation, und jedes bewegte Flüssigkeitstheilchen, dessen Ort zur Zeit t durch $x'\ y'\ z'$ bestimmt ist, kann nun mehr betrachtet werden als bewegte es sich auf der rotationslosen Erdoberfläche ohne Reibung, da der Einfluss der letzteren durch Einführung der Componenten $\Xi'\ H'\ Z'$ berücksichtigt worden ist.

Wir wollen jetzt wieder ein nenes Coordinatensystem ($x\ y\ z$) einführen, indem wir die $+\ z'$ – Achse so verlegt denken, dass dieselbe gerade den Zenith eines Ortes auf der Erdoherfläche trifft, dessen geographische Breite θ heissen möge, und zwar so, dass die y' – Achse mit sich parallel bleibt und dieselbe Richtung behält. Wir setzen daher.

$$x = x'\ sin\ \theta - z'\ cos\ \theta$$
$$y = y'$$
$$z = z'\ sin\ \theta + z'\ cos\ \theta$$

Es folgt hieraus umgekehrt.

$$x' = x\ sin\ \theta + z\ cos\ \theta$$
$$y' = y$$
$$z' = z\ sin\ \theta - x\ cos\ \theta$$

Es ist dann wieder

$$\frac{\partial(G'-\Pi)}{\partial x'}=\frac{\partial(G'-\Pi)}{\partial x}\sin\theta+\frac{\partial(G-\Pi)}{\partial z}\cos\theta$$

$$\frac{\partial(G'-\Pi)}{\partial y'}=\frac{\partial(G'-\Pi)}{\partial y}$$

$$\frac{\partial(G'-\Pi)}{\partial z'}=\frac{\partial(G'-\Pi)}{\partial z}\sin\theta-\frac{\partial(G-\Pi)}{\partial x}\cos\theta$$

$$\Xi'=\Xi\sin\theta+Z\cos\theta$$

$$H'=H$$

$$Z'=Z\sin\theta-\Xi\cos\theta$$

wo Ξ H Z die Componenten der Reibungskrüfte in Bezug auf das neuen Coordinatensystem bedeuten.

Indem wir diese Grössen in die Gleichungen (2 a) (2 b) (2 c) einführen, erhalten wir

$$\frac{d^2x}{dt^2}\sin\theta+\frac{d^2z}{dt^2}\cos\theta=\left(\frac{\partial(G'-\Pi)}{\partial x}-\Xi+\lambda^2x\right)\sin\theta$$
$$+\left(\frac{\partial(G'-\Pi)}{\partial z}-Z+\lambda^2z\right)\cos\theta+2\lambda\frac{dy}{dt}.$$

$$\frac{d^2y}{dt^2}=\frac{\partial(G'-\Pi)}{\partial y}-H-2\lambda\left(\frac{dx}{dt}\sin\theta+\frac{dz}{dt}\cos\theta\right)+\lambda^2y.$$

$$\frac{d^2x}{dt^2}\cos\theta=-\left(\frac{\partial(G'-\Pi)}{\partial x}-\Xi\right)\cos\theta$$
$$+\left(\frac{\partial(G'-\Pi)}{\partial x}-Z\right)\sin\theta$$

Hieraus leiten wir auf ganz dieselbe Weise ab, wie die Gleichungen $2a$, $2b$, $2c$ abgeleitet wurden,

$$\frac{d^2x}{dt^2}=\frac{\partial(G'-\Pi)}{\partial x}-\Xi+\lambda^2(x\sin\theta+z\cos\theta)\sin\theta+2\lambda\sin\theta\frac{dy}{dt}\qquad(3)$$

$$\frac{d^2y}{dt^2}=\frac{\partial(G'-\Pi)}{\partial y}-H-2\lambda\left(\frac{dx}{dt}\sin\theta+\frac{dz}{dt}\cos\theta\right)+\lambda^2y.\qquad(4)$$

$$\frac{d^2z}{dt^2}=\frac{\partial(G'-\Pi)}{\partial z}-Z-\lambda^2(x\sin\theta+z\cos\theta)\cos\theta+2\lambda\cos\theta\frac{dy}{dt}.\qquad(5)$$

Wir hönnen diese Gleichungen wesentlich vereinfachen, durch die Bemerkung, dass die Grössen

$$\lambda^2(x \sin\theta + z \cos\theta) \sin\theta, \qquad \lambda^2 y, \qquad \lambda^2(x \sin\theta + z \cos\theta) \cos\theta$$

nichts auderes bedeuten, als die Componenten der Centrifugalkraft nach den Axcu des neuen Coordinatensystems geschätzt. Da ferner die Componenten der Erdanziehung

$$\frac{\partial G}{\partial x}, \quad \frac{\partial G}{\partial y}, \quad \frac{\partial G}{\partial z},$$

wesentlich negatives Vorzeichen hesitzen müssen, so bedeuten die Grössen

$$-\left[\frac{\partial G'}{\partial x} - \lambda^2(x \sin\theta + z \cos\theta) \sin\theta\right], \quad -\left[\frac{\partial G'}{\partial y} - \lambda^2 y\right]$$

$$-\left[\frac{\partial G'}{\partial z} - \lambda^2(x \sin\theta + z \cos\theta) \cos\theta\right]$$

die Componenten der aus der Centrifugalkraft und der Erdanziehung resultirenden Kraft. Bezeichnet man das Potential dieser Kraft mit G, und setzt

$$G = G' + \frac{\lambda^2}{2}\left[(z \cos\theta + x \sin\theta)^2 + y^2\right]$$

und macht zugleich

$$\psi = G - \Pi$$

sodass man durch Differentiation nach x, y, und z erhält

$$\frac{\partial \psi}{\partial x} = \frac{\partial(G-\Pi)}{\partial x} = \frac{\partial(G'-\Pi)}{\partial x} + \lambda^2(z \cos\theta + x \sin\theta) \sin\theta$$

$$\frac{\partial \psi}{\partial y} = \frac{\partial(G-\Pi)}{\partial y} = \frac{\partial(G'-\Pi)}{\partial y} + \lambda^2 y$$

$$\frac{\partial \psi}{\partial z} = \frac{\partial(G-\Pi)}{\partial z} = \frac{\partial(G'-\Pi)}{\partial z} + \lambda^2(z \cos\theta + x \sin\theta) \cos\theta$$

Die Gleichungen (3) (4) (5) lassen sich dann auch so schreiben

$$\frac{d^2x}{dt^2} = \frac{\partial \psi}{\partial x} - \Xi + 2\lambda \sin\theta \frac{dy}{dt}$$

$$\frac{d^2y}{dt^2} = \frac{\partial \psi}{\partial y} - H - 2\lambda\left(\frac{dx}{dt} \sin\theta + \frac{dz}{dt} \cos\theta\right)$$

$$\frac{d^2z}{dt^2} = \frac{\partial \psi}{\partial z} - Z + 2\lambda \cos\theta \frac{dy}{dt}.$$

Nun setzen wir, wie es in der Hydrodynamik gebräuchlich ist, die Componenten der Geschwindigkeit

$$\frac{dx}{dt} = u \qquad \frac{dy}{dt} = v \qquad \frac{dz}{dt} = w$$

und entwickeln wir

$$\frac{d^2x}{dt^2} \quad \frac{d^2y}{dt^2} \quad \frac{d^2z}{dt^2},$$

indem wir uns sowohl u, v, w als x, y, z; als Functionen von der Zeit t vorstellen, dann erhalten wir

$$\left.\begin{aligned}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2\lambda \sin \theta\, v &= \frac{\partial \psi}{\partial x} - \Xi \\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + 2\lambda(u \sin \theta + w \cos \theta) &= \frac{\partial \psi}{\partial y} - H \\
\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} - 2\lambda \cos \theta\, v &= \frac{\partial \psi}{\partial z} - Z.
\end{aligned}\right\} \quad (6)$$

als verlangte Differentialgleichungen für die Bewegung der Erdatmosphäre, in denen so wohl der Einfluss des Reibungswiderstandes, als die Rotation der Erde berücksichtigt sind.

Zu den Gleichungen (6) tritt noch eine vierte hinzu, nämlich die Bedingung der Massencontinuität

$$\frac{\partial \mu}{\partial t} + \frac{\partial \mu u}{\partial x} + \frac{\partial \mu v}{\partial y} + \frac{\partial \mu w}{\partial z} = 0 \qquad (6a)$$

eine Gleichung, welche für jedes rechtwinkelige Coordinatensystem in unveränderter Gestalt giltig bleibt. Es dürfte aber nicht ganz überflüssig sein, nachzuweisen, dass diese Gleichung ($6a$) auch dann dieselbe Gestalt behält, wenn das Coordinatensystem, worauf u v w bezogen sind, in beliebiger Bewegung begriffen ist.

Man führe ein rechtwinkeliges Coordinatensystem (ξ η ζ) ein, welches im Raum fest sein möge, und setze

$$\begin{aligned}
\xi &= \alpha + \alpha_1 x + \alpha_2 y + \alpha_3 z \\
\eta &= \beta + \beta_1 x + \beta_2 y + \beta_3 z \\
\zeta &= \gamma + \gamma_1 x + \gamma_2 y + \gamma_3 z
\end{aligned}$$

wobei die 12. Grössen $\alpha\ \beta\ \gamma$ beliebige Functionen von t sind, von denen die jenigen mit Indices, den Bedingungen genügen müssen

$$\alpha_1^2+\alpha_2^2+\alpha_3^2=\beta_1^2+\beta_2^2+\beta_3^2=\gamma_1^2+\gamma_2^2+\gamma_3^2=1 \qquad (6b)$$

$$\alpha_1\beta_1+\alpha_2\beta_2+\alpha_3\beta_3=\alpha_1\gamma_1+\alpha_2\gamma_2+\alpha_3\gamma_3=\gamma_1\beta_1+\gamma_2\beta_2+\gamma_3\beta_3=0 \ (6c)$$

weil das System ($x\ y\ z$) ein rechtwinkeliges bleiben soll.

Es ist nun zunächst.

$$\frac{\partial\mu u}{\partial x}=\frac{\partial\mu u}{\partial\xi}\frac{\partial\xi}{\partial x}+\frac{\partial\mu u}{\partial\eta}\frac{\partial\eta}{\partial x}+\frac{\partial\mu u}{\partial\zeta}\frac{\partial\zeta}{\partial x}$$
$$=\alpha_1\frac{\partial\mu u}{\partial\xi}+\beta_1\frac{\partial\mu u}{\partial\eta}+\gamma_1\frac{\partial\mu u}{\partial\zeta}.$$

Es ist ferner umgekehrt

$$x=\alpha_1(\xi-\alpha)+\beta_1(\eta-\beta)+\gamma_1(\zeta-\gamma)$$
$$\eta=\alpha_2(\xi-\alpha)+\beta_2(\eta-\beta)+\gamma_2(\zeta-\gamma)$$
$$\zeta=\alpha_3(\xi-\alpha)+\beta_3(u-\beta)+\gamma_3(\zeta-\gamma)$$

Wenn wir setzen

$$\frac{d(\xi-\alpha)}{dt}=u^1 \qquad \frac{d(\eta-\beta)}{dt}=v^1 \qquad \frac{d(\zeta-\gamma)}{dt}=w^1$$

so folgt durch Differentiation nach t

$$u=\alpha_1 u^1+\beta_1 v^1+\gamma_1 w^1+(\xi-\alpha)\frac{d\alpha_1}{dt}+(\eta-\beta)\frac{d\beta_1}{dt}+(\zeta-\gamma)\frac{d\gamma_1}{dt}$$

$$v=\alpha_2 u^1+\beta_2 v^1+\gamma_2 w^1+(\xi-\alpha)\frac{d\alpha_2}{dt}+(\eta-\beta)\frac{d\beta_2}{dt}+(\zeta-\gamma)\frac{d\gamma_2}{dt}$$

$$w=\alpha_3 u^1+\beta_3 v^1+\gamma_3 w^1+(\xi-\alpha)\frac{d\alpha_3}{dt}+(\eta-\beta)\frac{d\beta_3}{dt}+(\zeta-\gamma)\frac{d\gamma_3}{dt}.$$

Wir erhalten somit

$$\frac{\partial\mu u}{\partial x}=\alpha_1\frac{\partial\mu(\alpha_1u^1+\beta_1v^1+\gamma_1w^1)}{\partial\xi}+\beta_1\frac{\partial\mu(\alpha_1u^1+\beta_1v^1+\gamma_1w^1)}{\partial\eta}$$
$$+\gamma_1\frac{\partial\mu(\alpha_1u^1+\beta_1v^1+\gamma_1w^1)}{\partial\zeta}+(a.f.S.)$$

$$\left.\begin{aligned}
&+ \alpha_1 \frac{d\alpha_1}{dt}\left[\frac{\partial \mu(\xi-\alpha)}{\partial \xi} + \frac{\partial \mu(\xi-\alpha)}{\partial \eta} + \frac{\partial \mu(\xi-\alpha)}{\partial \zeta}\right] \\
&+ \beta_1 \frac{d\beta_1}{dt}\left[\frac{\partial \mu(\eta-\beta)}{\partial \xi} + \frac{\partial \mu(\eta-\beta)}{\partial \eta} + \frac{\partial \mu(\eta-\beta)}{\partial \zeta}\right] \\
&+ \gamma_1 \frac{d\gamma_1}{dt}\left[\frac{\partial \mu(\zeta-\gamma)}{\partial \xi} + \frac{\partial \mu(\zeta-\gamma)}{\partial \eta} + \frac{\partial \mu(\zeta-\gamma)}{\partial \zeta}\right]
\end{aligned}\right\} \quad (6d)$$

Auf ganz dieselbe Weise erhalten wir

$$\begin{aligned}
\frac{\partial \mu v}{\partial y} = \alpha_2 &\frac{\partial \mu(\alpha_2 u^1 + \beta_2 v^1 + \gamma_2 w^1)}{\partial \xi} + \beta_2 \frac{\partial \mu(\alpha_2 u^1 + \beta_2 v^1 + \gamma_2 w^1)}{\partial \eta} \\
&+ \gamma_2 \frac{\partial \mu(\alpha_2 u^1 + \beta_2 v^1 + \gamma_2 w^1)}{\partial \zeta}
\end{aligned}$$

$$\left.\begin{aligned}
&+ \alpha_2 \frac{d\alpha_2}{dt}\left[\frac{\partial \mu(\xi-\alpha)}{\partial \xi} + \frac{\partial \mu(\xi-\alpha)}{\partial \eta} + \frac{\partial \mu(\xi-\alpha)}{\partial \zeta}\right] \\
&+ \beta_2 \frac{d\beta_2}{dt}\left[\frac{\partial \mu(\eta-\beta)}{\partial \xi} + \frac{\partial \mu(\eta-\beta)}{\partial \eta} + \frac{\partial \mu(\eta-\beta)}{\partial \zeta}\right] \\
&+ \gamma_2 \frac{d\gamma_2}{dt}\left[\frac{\partial \mu(\zeta-\gamma)}{\partial \xi} + \frac{\partial \mu(\zeta-\gamma)}{\partial \eta} + \frac{\partial \mu(\zeta-\gamma)}{\partial \zeta}\right]
\end{aligned}\right\} \quad (6e)$$

$$\begin{aligned}
\frac{\partial \mu w}{\partial z} = \alpha_3 &\frac{\partial \mu(\alpha_3 u^1 + \beta_3 v^1 + \gamma_3 w^1)}{\partial \xi} + \beta_3 \frac{\partial \mu(\alpha_3 u^1 + \beta_3 v^1 + \gamma_3 w^1)}{\partial \eta} \\
&+ \gamma_3 \frac{\partial \mu(\alpha_3 u^1 + \beta_3 v^1 + \gamma_3 w^1)}{\partial \zeta}
\end{aligned}$$

$$\left.\begin{aligned}
&+ \alpha^3 \frac{d\alpha_3}{dt}\left[\frac{\partial \mu(\xi-\alpha)}{\partial \xi} + \frac{\partial \mu(\xi-\alpha)}{\partial \eta} + \frac{\partial \mu(\xi-\alpha)}{\partial \zeta}\right] \\
&+ \beta_3 \frac{d\beta_3}{dt}\left[\frac{\partial \mu(\eta-\beta)}{\partial \xi} + \frac{\partial \mu(\eta-\beta)}{\partial \eta} + \frac{\partial \mu(\eta-\beta)}{\partial \zeta}\right] \\
&+ \gamma_3 \frac{d\gamma_3}{dt}\left[\frac{\partial \mu(\zeta-\gamma)}{\partial \xi} + \frac{\partial \mu(\zeta-\gamma)}{\partial \eta} + \frac{\partial \mu(\zeta-\gamma)}{\partial \zeta}\right]
\end{aligned}\right\} \quad (6f)$$

Mit Rücksicht auf die Gleichungen (6 *b*) (6 *c*) und auf die aus (6 *b*) fliessenden Gleichungen

$$\begin{aligned}
\alpha_1 \frac{d\alpha_1}{dt} + \alpha_2 \frac{d\alpha_2}{dt} + \alpha_3 \frac{d\alpha_3}{dt} &= \beta_1 \frac{d\beta_1}{dt} + \beta_2 \frac{d\beta_2}{dt} + \beta_3 \frac{d\beta_3}{dt} \\
&= \gamma_1 \frac{d\gamma_1}{dt} + \gamma_2 \frac{d\gamma_2}{dt} + \gamma_3 \frac{d\gamma_3}{dt} = 0
\end{aligned}$$

findet man durch Addition der Gleichungen (6*d*) (6*e*) (6*f*) in der That

$$\frac{\partial \mu u}{\partial x} + \frac{\partial \mu v}{\partial y} + \frac{\partial \mu w}{\partial z} = \frac{\partial \mu u'}{\partial \xi} + \frac{\partial \mu v'}{\partial \eta} + \frac{\partial \mu w'}{\partial \zeta}$$

Das ist, was wir beweisen wollten.

Es ist hier der geeignete Ort, die Bedingungen festzustellen, denen die Functionen u, v, w an der Begrenzungsfläche der Erdatmosphäre zu genügen haben.

Wenn irgend ein Stück der Erdoberfläche in Folge der stärkeren Erwärmung eine vertical aufsteigende Strömung veranlasst, werden die Lufttheilchen, welche das Gebiet der Verticalströmung ringsum umgeben, von allen Seiten her längs der Erdoberfläche hereinstürzen, und das gestörte Gleichgewicht wieder herzustellen streben. Allein; ehe sie dazu gelangen, werden sie die Folge des verticalen Auftriebs emporsteigen, und dadurch, dass die Theilchen in allen Horizontalschiehten einen Zug nach dem Innern des Raumes der verticalen Strömung erhalten, wird eine Horizontalströmung ausserhalb dieses gedachten Gebietes verlasst, *d. h.* eine Strömung, deren Richtung zur Normale der Erdoberflöche senkrecht steht, während die Richtung der Strömung innerhalb des gedachten Raumes, die Normale der Erdoberfläche im Allgemeinen schief schneiden wird.

Die Richtung der Strömung in derjemgen Luftschichte, welche unmittelbar auf der Erdoberfläche liegt, kann aber auch innerhalb des Raumes der verticalen Strömung nur eine zur Normale der Erdoberfläche senkrechte sein; da die vertical aufwärts gerichtete Geschwindigkeit der Lufttheilchen unmittelbar auf der Erdoberfläche verschwinden muss, weil der Autrieb vertical aufwärts zu strömen, nicht unmittelbar auf der Erdoberfläche eine endliche Grösse besitzen kann, sondern erst in endlicher Höhe.

Steigt hingegen irgendwo in der Atmosphäre eine Luftströmung

vertical niederwärts in Folge der stärkern Erkältung der Erdoberfläche, ader eines Theils der Luftschichte, so stürzt die Luftmasse vertical auf die starre Erdoberfläche und da sie dann keinen auderen Ausweg findet, so wird sie aus dem Raum der verticalen Strömung nach allen Seiten herausfahren, und so ausserhalb des Raumes des verticalen Strömung eine Strömung senkrecht zur Normale der Erdoberfläche veranlassen. Auch in diesem Fall werden also unmittelbar auf der Erdoberfläche alle vertical gerichteten Componenten verschwinden.

Man kann hiernach die bewegte Atmosphäre in zwei Arten Raumgebiete getheilt denken; einerseits in Raumgebiete der verticalen Strömung, welche durch thermische Ungleichheit auf der Erdoberfläche bedingt und bestimmt ist, und andererseits in Raumgebiete der Horizontalbewegung, welche erst durch jene verticale Strömung veranlasst wurde. Wir denken uns nun; es sei die Gestalt des Raumgebietes gegeben, und auch die Componenten der verticalen Strömung. Wir bezeichnen dieselben mit A B Γ, und die Componenten der horizontalen Geschwindigkeit so wohl im Raumgebiete der verticalen Strömung, als in demjenigen der horizontalen Strömung mit u', v', w'. Offenbar sind die Componenten der Geschwindigkeit, im inneren Raumgebiete, wenn wir darunter jedesmal das Raumgebiete der verticalen Strömung verstehen

$$u = v' + A \qquad v = v' + B \qquad w = w' + B \qquad (7)$$

während die Componenten der Geschwindigkeit im äusseren Raumgebiete, mit welchem Ausdruck wir fortan das Raumgebiet der Horizontalströmung bezeichnen wollen;

$$u = v' \qquad v = v' \qquad w = w'$$

Die Werthe der Componenten u' v' w' sind im allgemeinen für beide Raumgebiete verschieden; sie werden aber gewisse Bedingung erfüllen müssen, wenn die Bewegung der Lufttheilchen durch die Trennungs-

fläche der beiden Raumgebiete continuirlich vor sich gehen soll. Wenn wir unter dem Raumgebiete der verticalen Strömung den Raum verstehen, welche einerseits durch das Gebiet der verticalen Strömung auf der Erdoberfläche und andererseits durch eine zur Horizontalbewegung aller Luftschichten senkrechte Oberfläche begrenzt ist, dann ist die Bedingung, welcher u' v' w' genügen müssen

$$u' = u' \qquad v' = v' \qquad w' = w' \qquad (8)$$

wo die linker Hand stehenden Grössen die Componenten im inneren Raumgebiete bedeuten mögen. Die Bedingung dass die horizontale Bewegung continuirlich durch die Trennungsfläche der beiden Raumgebiete geschieht, ist sonach,

$$(u' + A) \cos(nx) + (v' + B) \cos(ny) + (w' + \Gamma) \cos(nz)$$
$$= u' \cos(nx) + v' \cos(ny) + w' \cos(nz) \,*$$

woraus mit Rücksicht auf (8) folgt

$$A \cos(nx) + B \cos(ny) + \Gamma \cos(nz) = 0 \qquad (8a)$$

d. h. die Oberfläche des inneren Raumgebietes sind entweder durch Strömungslinien der verticalen Strömung gebildet, oder durch die Curven

$$A = B = \Gamma = 0$$

In dem ersten Fall werden die horizontal bewegten Lufttheilchen plötzlich in die Höhe gerissen, sobald sie in das innere Raumgebiet eintreten. In dem letzteren Fall geht die Horizontalbewegung allmählig in die verticale Bewegung über.

Die Componenten der Geschwindigkeit im inneren Gebiete u' v' w' und A B Γ sind dabei von einander nicht unabhängig; denn da die Resultanten auf einander senkrecht stehen; so folgt

$$u'A + v'B + w'\Gamma = 0 \qquad (8b)$$

Was die Bedingung anbelangt, welche auf der Erdoberfläche zu erfüllen ist, so ergiebt sich dieselbe durch die Bemerkung, dass die Erdoberfläche nach der Einführung der Reibungskräfte und eines mit

* n bedeutet die Normale an der Begrenzungsfläche.

der Erde rotirenden Coordinatensystems, als eine reibungslose, reihenden Kugelfläche betrachtet werden darf, und dass daher jede vertical gerichtete Componente der Geschwindigkeit an der Erdoberfläche verschwinden muss; *d. h.*

$$u \cos(nx) + v \cos(ny) + w \cos(nz) = 0 \qquad (8f)$$

Für das äussere Gebiet folgt weiter

$$u' \cos(nx) + v' \cos(ny) + w' \cos(nz) = 0 \qquad (8c)$$

und für das innere Raumgebiet hingegen

$$(u' + A) \cos(nx) + (v' + B) \cos(ny) + (w' + \Gamma) \cos(nz) = 0$$

Da aber auf der Erdoberfläche $A \cos(nx) + B \cos(ny) + \Gamma \cos(nz) = \sqrt{A^2 + B^2 + \Gamma^2}$ gleich der Resultante der verticalen Geschwindigkeit ist, so folgt, weil $u' \cos(nx) + v' \cos(ny) + w' \cos(nz)$ verschwindet, dass auf der Erdoberfläche

$$\sqrt{A^2 + B^2 + \Gamma^2} = 0$$

d. h.

$$A = B = \Gamma = 0 \qquad (8d)$$

Die Gleichungen ($8c$) und ($8d$) sind Bedingungen, welche an der Erdoberfläche zu erfüllen sind.

Indem wir es unternehmen, aus den allgemeinenen Differentialgleichungen (6) einige wichtige Beziehungen abzuleiten, ist es nothwendig, denselben andere Form zu verleihen.

Zu dem Ende addiren wir zur ersten Gleichung in (6) die identisch erfüllte Gleichung

$$v\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x} - w\frac{\partial w}{\partial x} = 0$$

und zur zweiten Gleichung

$$u\frac{\partial u}{\partial y} - u\frac{\partial u}{\partial y} + w\frac{\partial w}{\partial y} - w\frac{\partial w}{\partial y} = 0$$

und endlich zur dritten Gleichung

$$u\frac{\partial u}{\partial z} - u\frac{\partial u}{\partial z} + v\frac{\partial v}{\partial z} - v\frac{\partial v}{\partial z} = 0$$

so erhalten wir

$$\frac{\partial u}{\partial t}+\frac{1}{2}\frac{\partial(u^2+v^2+w^2)}{\partial x}+v\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}-2\lambda\, sin\ \theta\right)+w\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)=\frac{\partial \psi}{\partial x}-\Xi$$

$$\frac{\partial v}{\partial t}+\frac{1}{2}\frac{\partial(u^2+{}^2v+w^2)}{\partial y}+u\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}+2\lambda\, sin\ \theta\right)+w\left(\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}+2\lambda\, cos\ \theta\right)=\frac{\partial \psi}{\partial y}-H$$

$$\frac{dw}{\partial t}+\frac{1}{2}\frac{\partial(u^2+v^2+w^2)}{\partial z}+u\left(\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}\right)+v\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}-2\lambda\, cos\ \theta\right)=\frac{\partial \psi}{\partial z}-Z$$

Indem wir die neuen Functionen einführen, die durch die Gleichungen

$$\tfrac{1}{2}(u^2+v^2+w^2)-\psi=\tfrac{1}{2}(u^2+v^2+w^2)+\Pi-G=\Phi \tag{9a}$$

$$\xi=\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}-2\lambda\, cos\ \theta \qquad \eta=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x} \qquad \zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}+2\lambda\, sin\ \theta \tag{9b}$$

definirt sind, so werden die obenstehenden Gleichungen

$$\left.\begin{aligned}\frac{\partial u}{\partial t}+\frac{\partial \Phi}{\partial x}-v\zeta+w\eta+\Xi=0\\ \frac{\partial v}{\partial t}+\frac{\partial \Phi}{\partial y}+u\zeta-w\xi+H=0\\ \frac{\partial w}{\partial t}+\frac{\partial \Phi}{\partial z}-u\eta+v\xi+Z=0\end{aligned}\right\} \tag{9c}$$

Die Grössen

$$\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z},\quad \frac{\partial u}{\partial z}-\frac{\partial w}{\partial x},\quad \frac{\partial v}{\partial x}-\frac{\partial u}{\partial y},$$

bedeuten nun bekanntlich das Doppelte der Componenten der Drehungsgeschwindigkeit eines Flüssigkeitstheilchens in Bezug auf die x—, y—, und z— Achse. Da ferner $2\lambda\ sin\ \theta$ und, $-2\lambda\ cos\ \theta$ die Componenten der Drehungsgeschwindigkeit eines Theilchens, welches mit der Erde um ihre Axe dreht in Bezug auf z—, respect im Bezug auf x— Achse bedeuten, so sind $\xi\ \eta\ \zeta$ die Componenten der aus der Rotation der Erde und eigener Rotation resultirenden Drehungsgeschwindigkeit eines Flüssigkeitstheilchens. Es ist sonach, weil die Drehungsgeschwindigkeit der Erde nirgends weiter vorkommt als in Verbindung mit den Componenten der eigenen Rotationsgeschwindigkeit, dass die Erdrotation im Grossen und Ganzen nur auf die

Rotationsgeschwindigkeit des Lufttheilchens Einfluss ausübt, dass dieser unter allen Umständen hinreichend berücksichtigt wird, sobald man zu den Componenten der Rotationsgeschwindigkeit, diejenige der Erdrotation hinzufügt.

Wir wollen nun einige Beziehungen aus den Differentialgleichungen ableiten, auf die wir später noch zurückkommen werden. Man multiplizire die erste Gleichung in $(9c)$ mit u, die zweite mit v, und die dritte mit w, und addire zusammen, so dass

$$\begin{aligned} & u\left(\frac{\partial u}{\partial t}+\frac{\partial \Phi}{\partial x}\right)+v\left(\frac{\partial v}{\partial t}+\frac{\partial \Phi}{\partial y}\right)+w\left(\frac{\partial w}{\partial t}+\frac{\partial \Phi}{\partial z}\right) \\ & -uv\zeta+wu\eta+u\Xi \\ & +uv\zeta-wv\xi+vH \\ & -uw\eta+vw\xi+wZ=0 \end{aligned}$$

d. h.

$$u\left(\frac{\partial u}{\partial t}+\frac{\partial \Phi}{\partial x}\right)+v\left(\frac{\partial v}{\partial t}+\frac{\partial \Phi}{\partial y}\right)+w\left(\frac{\partial w}{\partial t}+\frac{\partial \Phi}{\partial z}\right)=-(u\Xi+vH+wZ) \quad (10)$$

Multiplizirt man ferner die erste Gleichung mit ξ und die zweite mit η und die dritte mit ζ, so entsteht durch Addition

$$\begin{aligned} & \xi\left(\frac{\partial u}{\partial t}+\frac{\partial \Phi}{\partial x}\right)+\eta\left(\frac{\partial v}{\partial t}+\frac{\partial \Phi}{\partial y}\right)+\zeta\left(\frac{\partial w}{\partial t}+\frac{\partial \Phi}{\partial z}\right) \\ & -v\zeta\xi+w\xi\eta+\xi\Xi \\ & +u\zeta\eta-w\xi\eta+\eta H \\ & -u\eta\zeta+v\xi\zeta+\zeta Z=0 \end{aligned}$$

d. h.

$$\xi\left(\frac{\partial u}{\partial t}+\frac{\partial \Phi}{\partial x}\right)+\eta\left(\frac{\partial v}{\partial t}+\frac{\partial \Phi}{\partial y}\right)+\zeta\left(\frac{\partial w}{\partial t}+\frac{\partial \Phi}{\partial z}\right)=-(\Xi\xi+H\eta+Z\zeta) \quad (11)$$

Da es sich in der vorliegenden Aufgabe lediglich um die Strömungen der atmosphärischen Luft handelt, so werden wir dieselbe fortan als incompressibel betrachten.

§ II. *Allgemeine Differentialgleichung für die Bewegung der Atmosphäre.*

Unter "Isodynamische Fläche" verstehe ich eine Fläche deren Gleichung

$$\Phi = const. \qquad (13)$$

ist; demnach eine Fläche gleicher Energie der Windbewegung.

Dieselbe fällt immer mit der isobarischen Fläche zusammen, so bald die Geschwindigkeit der Windbewegung so ausserordentlich klein ist, dass ihre Quadrate als unendlich klein vernachlässigt werden dürfen; denn: dann ist

$$\Phi = \Pi - G = const.$$

Es können ferner diese beiden Flächen mit einander zusammenfallen, wenn die Bewegung der Atmosphäre so vor sich geht, dass überall da wo der Druck constant ist, die resultirende Geschwindigkeit constant ist oder auch, wenn die lebendige Kraft der Luftströmung durch dieselbe Funktionsform ausdrückbar ist, wie der Druck.

Die durch die Gleichung (13) definirte Oberfläche kann gefunden werden, wenn die Geschwindigkeits componenten u, v, w und die Componenten der Rotationsgeschwindigkeit eines Lufttheilchens gemäss gewisser partiellen Differentialgeleichungen gefunden worden sind, welche wir jetzt ableiten wollen.

Wir diffirentiren zunächst die erste Gleichung in ($9e$) nach y, und die zweite nach x; so entstcht durch Subtraction

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\zeta\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+u\frac{\partial \zeta}{\partial x}+v\frac{\partial \zeta}{\partial y}$$
$$-w\left(\frac{\partial \xi}{\partial x}+\frac{\partial \eta}{\partial y}\right)-\xi\frac{\partial w}{\partial x}-\eta\frac{\partial w}{\partial y}+\frac{\partial H}{\partial x}-\frac{\partial \Xi}{\partial x}=0$$

da aber, weil die Luft als incompressibel betrachtet wird,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=-\frac{\partial w}{\partial z} \quad \text{und} \quad \frac{\partial \xi}{\partial x}+\frac{\partial \eta}{\partial y}=-\frac{\partial \zeta}{\partial z}$$

so folgt mit Rücksicht auf (9b)

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + w\frac{\partial \zeta}{\partial z} - \xi\frac{\partial w}{\partial x} - \eta\frac{\partial w}{\partial y} - \zeta\frac{\partial w}{\partial z} + \frac{\partial H}{\partial x} - \frac{\partial \Xi}{\partial x} = 0 \quad (14)$$

Differentirt man die erste Gleichung nach z und die dritte nach x so entsteht durch Subtraction

$$\frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y} + w\frac{\partial \eta}{\partial z} - \xi\frac{\partial v}{\partial x} - \eta\frac{\partial v}{\partial y} - \zeta\frac{\partial v}{\partial z} + \frac{\partial \Xi}{\partial z} - \frac{\partial Z}{\partial x} = 0 \quad (15)$$

Auf ganz dieselbe Weise erhält man die dritte Gleichung.

$$\frac{\partial \xi}{\partial t} + u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y} + w\frac{\partial \xi}{\partial z} - \xi\frac{\partial u}{\partial x} - \eta\frac{\partial u}{\partial y} - \zeta\frac{\partial u}{\partial z} + \frac{\partial Z}{\partial y} - \frac{\partial H}{\partial z} = 0. \quad (16)$$

In diesen Gleichungen kommt Φ — Function nicht mehr vor. Hat man die Grössen u v w gemäss dieser Gleichungen bestimmt, so handelt es sich darum Φ selbst zu bestimmen.

Differentirt man zu dem Ende die erste Gleichung in (9c) nach x, die zweite nach y, und die dritte nach z, so entsteht, da in Folge der Bedingung der Incompressibilität

$$\frac{\partial \frac{\partial u}{\partial t}}{\partial x} + \frac{\partial \frac{\partial v}{\partial t}}{\partial y} + \frac{\partial \frac{\partial w}{\partial t}}{\partial z} = \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

ist

$$\begin{aligned} \Delta\Phi &+ u\left(\frac{\partial \zeta}{\partial y} - \frac{\partial \eta}{\partial z}\right) - v\left(\frac{\partial \zeta}{\partial x} - \frac{\partial \xi}{\partial z}\right) + w\left(\frac{\partial \eta}{\partial x} - \frac{\partial \xi}{\partial y}\right) \\ &+ \xi\left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right) + \eta\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) - \zeta\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \\ &+ \frac{\partial \Xi}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial Z}{\partial z} = 0 \end{aligned}$$

wo

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = \Delta\Phi$$

gesetzt worden ist

Weil nun in Folge der Gleichung (9 b)

$$\frac{\partial \zeta}{\partial y}-\frac{\partial \eta}{\partial z}=\frac{\partial^2 v}{\partial x \partial y}-\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 w}{\partial x \partial z}=-\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 u}{\partial z^2}+\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=-\varDelta u$$

$$\frac{\partial \zeta}{\partial x}-\frac{\partial \xi}{\partial z}=\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 u}{\partial y \partial x}-\frac{\partial^2 w}{\partial z \partial y}+\frac{\partial^2 v}{\partial z^2}=\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}-\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)=\varDelta v$$

$$\frac{\partial \eta}{\partial x}-\frac{\partial \xi}{\partial y}=\frac{\partial^2 u}{\partial z \partial x}-\frac{\partial^2 w}{\partial x^2}-\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 v}{\partial z \partial y}=-\frac{\partial^2 w}{\partial x^2}-\frac{\partial^2 w}{\partial y^2}+\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial z}\right)=-\varDelta w.$$

und ferner weil

$$\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}=-(\xi+2\lambda \cos\theta) \qquad \frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}=-\eta$$

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=(\zeta-2\lambda \sin\theta).$$

erhalten wir schliesslich

$$\begin{aligned}\varDelta\Phi - u\,\varDelta u - v\,\varDelta v - w\,\varDelta w - \xi\,(\xi+2\lambda\cos\theta)-\eta^2 \\ -\zeta\,(\zeta-2\lambda\sin\theta)+\frac{\partial \varXi}{\partial x}+\frac{\partial H}{\partial y}+\frac{\partial Z}{\partial z}=0\end{aligned} \tag{17}$$

Wie nun eine solche Gleichung integrirt werden kann, wenn u, v, w als Functionen von x, y, z und t gewäss der Gleichungen (14) (15) (16) bekannt sind, wenn ausser dem die Raumgebiete, wo Φ einer solchen partiellen Differentialgleichung genügt, gegeben sind, ist im Allgemeinen schwierige Aufgabe. Offenbar muss die Function aber einer anderen Differentialgleichung Genüge leisten, wenn der Punkt $(x\ y\ z)$ in ein anderes Raumgebiet gelangt, wo die Lufttheilchen keine andere Rotationsgeschwindigkeit hat, als die durch die Erdrotation hervorgerufene, also ein Raumgebiet, wo ein Geschwindigkeits-potential existiren würde, wenn die Erde nicht rotirte. Wir denken uns die Atmosphäre in zwei Arten Raumgebiete getheilte; in Raumgebiete, wo eine Wirbelbewegung existirt, und in Raumgebiete, wo eine solche nicht vorhanden ist; wo also die Lufttheilchen keine andere Rotation hat, als die durch Rotation der Erde veranlasste, und nennen das erstere, das Wirbelgebiet und das letztere das wirbelfreie Gebiet der Atmosphäre. Für das Wirbelgebiet gelten dann die Gleichungen (14)

(15) (16) (17). Für das wirbelfreie Gebiet aber, da in diesem Gebiete überall

$$\frac{\partial v}{\partial x}-\frac{\partial v}{\partial y}=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}=\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}=0 \qquad (18)$$

so dass

$$\xi=-2\lambda \cos\theta \qquad \eta=0 \qquad \zeta=2\lambda \sin\theta$$

ist, verwandeln sich die Gleichungen (14) (15) (16) in

$$2\lambda \cos\theta\frac{\partial w}{\partial x}-2\lambda \sin\theta\frac{\partial w}{\partial z}+\frac{\partial H}{\partial x}-\frac{\partial \Xi}{\partial y}=0$$

$$2\lambda \cos\theta\frac{\partial v}{\partial x}-2\lambda \sin\theta\frac{\partial v}{\partial z}+\frac{\partial \Xi}{\partial z}-\frac{\partial Z}{\partial x}=0$$

$$2\lambda \cos\theta\frac{\partial u}{\partial x}-2\lambda \sin\theta\frac{\partial u}{\partial z}+\frac{\partial Z}{\partial y}-\frac{\partial H}{\partial z}=0$$

oder mit Rücksicht auf (18)

$$\frac{\partial}{\partial z}(2\lambda \cos\theta u-2\lambda \sin\theta w)+\frac{\partial H}{\partial x}-\frac{\partial \Xi}{\partial y}=0$$

$$\frac{\partial}{\partial y}(2\lambda \cos\theta u-2\lambda \sin\theta w)+\frac{\partial \Xi}{\partial z}-\frac{\partial Z}{\partial x}=0$$

$$\frac{\partial}{\partial x}(2\lambda \cos\theta u-2\lambda \sin\theta w)+\frac{\partial Z}{\partial y}-\frac{\partial H}{\partial z}=0$$

Die Gleichung (17) verwandelt sich in die einfachere

$$\Delta\Phi=-\left(\frac{\partial \Xi}{\partial x}+\frac{\partial H}{\partial y}+\frac{\partial Z}{\partial z}\right)$$

da in diesem Fall

$$\Delta u=\Delta v=\Delta w=0$$

ist.

§ *III. Allgemeine Beziehungen zwischen Isodynamen, Windbahn und Wirbelaxe.*

Unter "Isodynamen" verstehe ich Linien, in denen die isodynamischen Flächen die Erdoberfläche schneiden, *d. h.* die Schnittcurven

der Oberflächengleichungen

$$\Phi = const.$$

mit der Oberfläche der Erde. Die Normale der isodynamischen Oberfläche schneidet im Allgemeinen die zur Erdoberfläche verticalen Linie, wie die Bahn des Windes, welche unmittelbar auf der Erdoberfläche überall senkrecht zu ihrer Normale stehen muss. Unter "Normale der Isodynamen" soll im Folgenden die Normale verstanden werden, welche senkrecht zu dem Curvenelemente der Isodynamen, und zugleich zu der isodynamischen Fläche steht.

Ich verstehe ferner unter "Deviationswinkel," den Winkel welchen die Bahncurve des Windes mit der Normale der Isodynamen bildet, und bezeichne denselben mit i.

Die Gleichung, durch die dieser Winkel definirt wird, ist

$$cos\ i = \frac{\left(\frac{\partial \Phi}{\partial x} u + \frac{\partial \Phi}{\partial y} v + \frac{\partial \Phi}{\partial z} w\right)}{\omega \left(\frac{\partial \Phi}{\partial n}\right)}$$

wo zur Abkürzung

$$\sqrt{u^2 + v^2 + w^2} = \omega, \qquad \sqrt{\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2 + \left(\frac{\partial \Phi}{\partial z}\right)^2} = \frac{\partial \Phi}{\partial n}.$$

gesetzt worden ist. n bedeutet die Normale der isodynamischen Fläche.

Wenn wir hierbei nur stationäre Bewegung in's Auge fassen wollen, so lässt sich leicht eine allgemeine Beziehung zwischen dem Deviationswinkel und der Windbahn ableiten.

Es folgt nämlich ans (10), indem

$$\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial w}{\partial t} = 0$$

gesetzt wird

$$u \frac{\partial \Phi}{\partial x} + v \frac{\partial \Phi}{\partial y} + w \frac{\partial \Phi}{\partial z} = -\ (u\Xi + vH + wZ)$$

Man setze

$$R = \sqrt{(\Xi^2 + H^2 + Z^2)} \qquad (18a)$$

und

$$\begin{aligned} \Xi &= R \cos (Rx) & \qquad u &= \omega \cos (\omega x) \\ H &= R \cos (Ry) & \qquad v &= \omega \cos (\omega y) \\ Z &= R \cos (Rz) & \qquad w &= \omega \cos (\omega z) \end{aligned}$$

Mithin erhält man

$$u \frac{\Phi \partial}{\partial x} + v \frac{\partial \Phi}{\partial y} + w \frac{\partial \Phi}{\partial z} = - \omega R \cos (\omega R).$$

Weil nun, wenn der Punkt, $x\ y\ z$, worauf alle hier auf tretenden Grössen sich beziehen, auf der Erdoberfläche liegt, die Richtung von R *d. h.* des resultirenden Reibungswiderstandes, mit derjenige der resultirenden horizontalen Geschwindigkeit zusammenfällt, muss

$$\cos (\omega R) = 1$$

mithin

$$u \frac{\partial \Phi}{\partial x} + v \frac{\partial \Phi}{\partial y} + w \frac{\partial \Phi}{\partial z} = - \omega R.$$

Es folgt hieraus

$$\cos i = - \frac{R}{\left(\frac{\partial \Phi}{\partial n} \right)^2} \qquad (18b)$$

und

$$\operatorname{tag} i = \sqrt{\left(\frac{1}{\cos^2 i} - 1 \right)} = \frac{\sqrt{\left(\frac{\partial \Phi}{\partial n} \right)^2 - R}}{R}$$

Wir bilden den Zähler mittelst der Gleichungen.

$$\frac{\partial \Phi}{\partial x} = v \zeta - w \eta - \Xi$$

$$\frac{\partial \Phi}{\partial y} = - u \zeta + w \xi - H$$

$$\frac{\partial \Phi}{\partial z} = u \eta - v \xi - Z$$

Es ist

$$\left(\frac{\partial\Phi}{\partial n}\right)^2 = R^2 - 2\Xi(w\eta - v\zeta) + v^2\zeta^2 + w^2\eta^2 - 2wv\eta\zeta.$$
$$- 2H(u\zeta - w\xi) + u^2\zeta^2 + w^2\xi^2 - 2uw\xi\zeta. \qquad (18d)$$
$$- 2Z(v\xi - u\eta) + v^2\xi^2 + u^2\eta^2 - 2uv\xi\eta.$$

Da nun überall längs der Isodynamen

$$\Xi = R \cos(\omega x)$$
$$H = R \cos(\omega y)$$
$$Z = R \cos(\omega z)$$

gesetzt werden kann, so heben sich die mit Ξ H Z behafteten Glieder auf; es folgt

$$\left(\frac{\partial\Phi}{\partial n}\right)^2 = R^2 + v^2\zeta^2 + w^2\eta^2 - 2uv\eta\zeta.$$
$$+ u^2\zeta^2 + w^2\xi^2 - 2uw\xi\zeta.$$
$$+ v^2\xi^2 + u^2\eta^2 - 2uv\xi\eta.$$

Um diesen Ausdruck auf eine andere Form zu bringen, addiren wir hierzu die identisch erfüllte Gleichung

$$u^2\xi^2 - u^2\xi^2 + v^2\eta^2 - v^2\eta^2 + w^2\zeta^2 - w^2\zeta^2 = 0$$

dann verwandelt sich jener Ausdruck in

$$\left(\frac{\partial\Phi}{\partial n}\right)^2 = R^2 + (\xi^2 + \eta^2 + \zeta^2)(u^2 + v^2 + w^2) - (u\xi + v\eta + w\zeta)^2.$$

Nennt man die Resultante der Drehungsgeschwindigkeit σ, so dass

$$\sigma = \sqrt{\xi^2 + \eta^2 + \zeta^2}. \qquad (18c)$$

und versteht man unter σ zugleich die Richtung der resultirenden Drehungsaxe, so kann man setzen

$$\xi = \sigma \cos(\sigma x)$$
$$\eta = \sigma \cos(\sigma y)$$
$$\zeta = \sigma \cos(\sigma z)$$

hieraus folgt.

$$\left(\frac{\partial\Phi}{\partial n}\right)^2 = R^2 + \sigma^2\omega^2 - \sigma^2\omega^2\cos^2(\sigma\omega) =$$
$$= R^2 + \sigma^2\omega^2 \sin^2(\sigma\omega)$$

Mithm erhalten wir für den Deviationswinkel den emfachen Ausdruck.

$$tag\, i = \pm \left(\frac{\omega}{R}\right) \sigma\, sin\, (\sigma\omega). \qquad (19)$$

Dieser Ausdruck kann nirgends verschwinden, so lange $\sigma\, sin\, (\sigma\omega)$ nirgends verschwindet, da $\left(\frac{\omega}{R}\right)$ überall auf der Erdoberfläche einen endlichen positiven Werth besitzen muss. Sehen wir zu, ob irgendwo $\sigma\, sin\, (\sigma\omega)$ verschwinden kann. Offenbar muss, falls irgendwo ein solches Gebiet auf der Erdoberfläche existirt, entweder $sin\, (\sigma\omega) = 0$ sein, oder $\sigma = 0$ sein.

Das erstere kann nun nicht stattfinden; weil bei einer Wirbelbewegung die Strömungslinien d. h. die Richtung der resultirenden Geschwindigkeit doch nicht mit derjenigen des Rotationsaxe zusammenfallen können. Ob aber das letztere eintreten kann, bedarf noch einer näheren Untersuchung.

Verschwindet in einem Gebiete σ *d. h.*

$$\xi = \eta = \zeta = 0$$

Dann müssen den Gleichungen (14) (15) (16) zu Folge in diesem Gebiete

$$\frac{\partial \Xi}{\partial y} = \frac{\partial H}{\partial x} \qquad \frac{\partial Z}{\partial x} = \frac{\partial \Xi}{\partial z} \qquad \frac{\partial Z}{\partial y} = \frac{\partial H}{\partial z}.$$

sein, welche Gleichungen nichts anderes aussagen, als dass die Componenten des Reibungswinderstandes als Differentialquotienten von Einer Function dargestell werden können. Dass irgend wo die Drehungsgeschwindigkeit des Luftheilchens verschwinde, führt demnach zum unmöglichen Postulate, dass in einem solchen Bewegungsgebiete dem Reibungswiderstand, welcher ein Potential nicht haben kann, ein solches zukomme. Hieraus kann man weiter den Schluss ziehen, dass *die Rotationsgeschwindigkeit die Luftheilchens in keinem Punkt der Erdatmosphäre verschwindet, wo die Componenten des Reibungs-*

widerstandes nicht verschwinden. Als eine Consequenz dieses Schlusses folgt weiter der wichtige Salz dass der *Deviationswinkel in keinem Gebiete der Erdoberfläche verschwindet, und dass daher die Windbahnen überall gegen die Normale der Isodynamen (oder Isobaren) geneigt sein müssen.*

Die Gleichung (19) gestattet uns auf die Bahncurven des Windes zu schliessen, wenn die Isodynamen bekannt sind, und umgekehrt auf die Natur der Isodynamen, wenn die Windbahn bekannt ist.

Wenn wir uns vorstellen, dass nirgends sonst, als in Polargegenden und Aequatorialgegenden Gebiete mit verticalen Strömungen bestehen, so bestehen die Isodynamen bei dem allgemeinen Kreislauf der atmosphärischen Luft von Polargegenden nach dem Aequator, und umgekehrt offenbar aus lauter um den Erdpol in sich zurücklaufenden Raumcurven. Da nun der Deviationswinkel nirgends verschwindet, *so kann die Bahncurve des Windes im allgemeinen nur eine um den Erdpol gewundene Raumspirale sein;* denn nur Spiralen können ein System von in sich zurücklaufenden Curven unter einem spitzen Winkel schneiden.

Wenn aber irgendwo auf der Erdoberfläche in einem beschränkten Gebiete eine solche Gleichgewechtsstörung der Atmosphäre eintritt, dass die Isodynamen um einen gewissen Mittelpunct ein System von geschlossenen Curven bilden, *so ist die Windbahn nothwindig auch eine Raumsirpale, welche die Erdoberfläche umwindet.*

Wir fassen die Bewegung eines Luftheilchens in's Auge, welches entlang der Erdoberfläche um eine Axe rotirt, die senkrecht zur Erdoberfläche steht und denken uns dabei i von der Richtung aus gezählt, in der die Luftströmung stattfindet, sodass i ein spitzer Winkel, ist, dann folgt, aus der Gleichung (19), da in diesem Fall $sin\ (\sigma\omega) = 1$ ist, weil die horizontale (entlang der Erdoberfläche)

Richtung des Windes überall in dem Wirbelgebiete zu der Wirbelaxe senkrecht steht,

$$tag\, i = \left(\frac{\omega}{R}\right)\sigma.$$

d. h. vermöge der Bedentung von σ

$$tag\, i = \left(\frac{\omega}{R}\right)\sqrt{\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}+2\lambda \sin\theta\right)^2+\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\varrho x}\right)^2+\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}-2\lambda\cos\theta\right)^2} \tag{20}$$

wo der Wurzelausdruck absolut zu nehmen ist. In dieser Gleichung ist ein wichtiges Gesetz euthalten; das Dove'sche Gesetz der Wirbelstürme.

Man denke sich auf der südlichen Hemisphäre ein anderes Gebiete der Wirbelbewegung, welche in allen Stücken mit demjemgen auf der nördlichen Hemisphäre übereinstimmt, sodass auch für dieses Gebiet der Wirbelbewegung der Ausdruck (20) gilt, und dem Werthe nach auch mit demjenigen der nördlichen Hemisphäre zusammenfällt. Da nun, wenn wir für die nördliche Hemisphäre, θ positiv in die Rechnung bringen, θ für die südlichen Hemisphäre negativ zu setzen ist, so können die Werthe von σ *d. i* der resultirenden Drehungsgeschwindigkeit für beide Hemisphären nur dann zusammenfallen, wenn die Componente der Drehungsgeschwindigkeit in Bezug auf die verticale Axe $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ für die südliche Hemisphäre das Vorzeichen wechselt, weil *tag i* ja vollständig unabhängig von der Lage der Coordinatenaxe und der Quotient $\frac{\omega}{R}$ eine durchaus positive Grösse ist.

Wir gelangen somit zu dem Satze, dass, wenn die Luft durch irgend welche Ursache in wirbelnde Bewegung gerathen ist; die Rotation der Lufttheilchen auf der südlichen Hemisphäre im entgegengesetzten Sinne geschehen muss, wie auf der nördlichen Hemisphäre. Weil der Wirbelsturm, wie es Beobachtungen darthun,

in einer wirbelnden Bewegung der Luftmasse besteht, so folgt aus dem eben abgeleiteten Satze unmittelbar, dass, wenn ein Wirbelsturm auf der nördlichen Hemisphäre auftritt, und der Wind in der Richtung *S O N W* um ein Centrum rotirt und wenn ein Wirbelsturm unter sonst denselben Umständen (gleiche Rotationsgeschwindigkeit, der Lufttheilchen, gleiche Windgeschwindigkeit, und gleiche Reibung auf der Erdoberfläche) auf der südlichen Hemisphäre auftritt, die Drehung des Windes in dem letzteren in der entgegengesetzten Richtung *S W N O* vor sich gehen muss.

Dieses ist das wichtige Dove'sche Gesetz der Wirbelstürme. Wann nun aber die Drehung der Lufttheilchen in einem Wirbelsturm auf der einen Hemisphäre in dem einen, oder in dem andern Sinne geschieht; das lässt sich aus der Gleichung (20) nicht entscheiden. Wir werden jedoch weiter unten sehen, indem wir die Wirbelbewegung der Luft an der Erdoberfläche in einem besonderen Fall verfolgen, dass, wenn die Wirbelbewegung durch eine vertical aufsteigende Luftströmung entsteht, die Rotation der Lufttheilchen auf der nördlichen Hemisphäre immer in Richtung *S O N W* geschieht und dass die Lufttheilchen aber in der entgegengesetzten Richtung rotiren, sobald die Wirbelbewegung einer vertical niedersteigenden Strömung ihre Entstehung verdankt.

Aus den Relationen (18a) und (19) lassen noch einige Sätze von ähnlicher Tragweite ableiten.

Wir denken uns; Es haben die Lufttheilchen keine andere Rotationsgeschwindigkeit als die durch Erdrotation veranlasste. Die Horizontalcomponente der Drehung, deren Axe mit einer Verticallinie in dem betreffenden Punkt der Erdoberfläche zusammenfällt, beträgt dem absoluten Werthe nach $\lambda \sin \theta$. Wenn diese Grösse bei der Bewegung des Lufttheilchens positiv zu nehmen ist, so geschieht die Rotationsbewegung offenbar im Sinne der Erdrotation *d. h.* in der

Richtung mit der Sonne. Ist hingegen $\lambda \sin \theta$ negativ zu nehmen so geht die Rotationsbewegung des Lufttheilchen in entgegengesetzter Richtung vor sich *d. h.* gegen die Sonne. Die erstere Art der Windbewegung nennt man bekanntlich in der neneren Meteorologie eine cyklonale, während die zweite Art eine antcyklonale Bewegung genannt wird.

Wenn nun ein Lufttheilchen eigene Rotationsgeschwindigkeit besitzt, so dass die Horizontalcomponent der Rotationsgeschwindigkeit $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ durchaus in genan demselben Sinne, wie die Rotation der Erde gerichtet ist, so hat man offenbar auch mit einer cyklonalen Bewegung zu während im entgegengesetzten Falle die Bewegung eine anticyklonale sein wünde und die Horizontalcomponente der Rotationsgeschwindigkeit $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ negativ zu nehmen wäre, wenn in den beiden Fällen der Theil der Rotationsgeschwindigkeit, welcher von der Erdrotation selbst herrührt, positiv genommen wird. Bezeichnet man mit c die cyklonale, und mit a die anticyklonale Luftströmung, so ergiebt sich für beide Fälle aus (19) als Deviation der Windbahn

$$tag\ i_c = \frac{\omega}{R}\sqrt{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + 2\lambda \sin \theta\right)^2 + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} - 2\lambda \cos \theta\right)^2}$$

$$tag\ i_a = \frac{\omega}{R}\sqrt{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - 2\lambda \sin \theta\right)^2 + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} - 2\lambda \cos \theta\right)^2}$$

Wenn wir jetzt die Annahme machen, dass so wohl die cyklonale als die anticyklonale Bewegung des gedachten Lufttheilchens unter demselben Umständen entstehen [*d. h.* gleiche Isodynamen, gleiche Geschwindigkeit und Reibung] so folgt aus den obenstchenden Gleichungen ohne Weiteres

$$tag\ i_c > tag\ i_a$$

d. i

$$i_c > i_a$$

Dass heisst mit Worten: *die cyklonale Windbahn schneidet die Isodynamen, (oder unter Umständen Isobaren) unter kleinerem Winkel, als die anticyklonale*, oder was dasselbe ist; *die Cyklonale Windbahn hat stärkere Krünmung, als die anticyklonale.*

Dieser Satz gilt ganz allgemein.*

Zu einem zweiten gleich einfachen Satz von grasser Tragweite gelangte man durch die Einführung des in der neueren Meteorologie gebränchlichen Begriffs "Gradienten," als dessen analytischer Ausdruck man mit Oberbeck † den Differentialquotienten des hydrostatischen Drucks nach der Normale der Isobaren nehmen kann. Dehnt man nun den Begriff des Gradienten dahin aus, dass man unter demselben die Differenz der Totalenergie der Luftströmung in der Richtung der schnellsten Abnahme derselben versteht, so kann $\frac{\partial \Phi}{\partial n}$ gleichfalls als der analytische Ausdruck des Gradienten im erweiterten Sinne gelten. Bezeichnet man den Gradienten in der cyklonalen Luftströmung wieder mit dem Index c, und denjenigen in der anticyklonaten mit a, so folgt aus (18a) für beide Fälle

$$\cos i_c = -\frac{R}{\left(\frac{\partial \Phi}{\partial n}\right)_c} \qquad \cos i_a = -\frac{R}{\left(\frac{\partial \Phi}{\partial n}\right)_a}$$

Weil nun unter denselben herrschenden Verhältnissen i_c immer grösser ist, als i_a; so muss

$$\frac{R}{\left(\frac{\partial \Phi}{\partial n}\right)_c} < \frac{R}{\left(\frac{\partial \Phi}{\partial n}\right)_a}$$

* Herr Dr. Sprung Assistent an der deutschen Seewarte zu Hamburg ist auf dem Weg einer vorwiegend geometrischen Betrachtuug zu demselben Satz gelangt (Archiv der deutschen Seewarte Jahrgang II. 1879. pag. 21) Die Ableitung dieses Satzes aus den hydrodynamischem Differentialgleichungen in solcher Allgemeinheit dürfte aber wohl neu sein.

† Wiedemanns Annalen Band XVII pag. 133.

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$$\left(\frac{\partial \Phi}{\partial n}\right)_c > \left(\frac{\partial \Phi}{\partial n}\right)_a$$

*d. h. Die cyklonalen Bewegungsformen in der Atmosphäre sind von grösseren Gradienten begleitet als die anticyklonalen.**

Auch dieser Satz gilt unter sonst gleichen Umständen ohne alle Einschränkung.

Haben die Lufttheilchen im gewisen Gebiet der Atmosphäre keine andere Rotationsgeschwindigkeit, als die durch Erdrotation hervorgerufene, so erhält die Gleichung (19) eine bemerkenswerthe Gestalt. Es ist unter diesem Umstande.

$$\xi = -2\lambda \cos\theta \qquad \eta = 0 \qquad \zeta = 2\lambda \sin\theta$$

sodass

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} \qquad \frac{\partial w}{\partial x} = \frac{\partial u}{\partial z} \qquad \frac{\partial v}{\partial z} = \frac{\partial w}{\partial y}$$

Dann ist

$$\sigma = \sqrt{4\lambda^2} = 2\lambda.$$

und die Axenrichtung der Drehung eines Lufttheilchens fällt selbstredend mit der Axe der Erdrotation zusammen. Bezeichnet man mit λ zugleich die Richtung der Erdaxe; so folgt aus (19) die Gleichung

$$tag\, i = 2\left(\frac{\omega}{R}\right)\lambda \sin(\lambda\omega)$$

Denkt man sich die Erdoberfläche als eine reine Kugelfläche, und bezeichnet den Azimuthwinkel der Windbahn mit χ, so kann man in dem Punkt der Erdoberfläche, dessen Breite θ ist, setzen.

$$\cos(\lambda\omega) = \cos\chi \cos\theta$$

* Vergleiche Dr. Sprung (a. o. a. O. pag 22)

Die oben abgeleitete Gleichung verwandelt sich so mit in

$$tag\ i = \frac{2\lambda\omega}{R}\sqrt{1 - cos^2\chi\ cos^2\theta}\ * \qquad (21)$$

Der Deviationswinkel ist sonach unter sonst gleichen Umständen von der Richtung des Windes abhängig. Bei reinem Ost-oder Westwinde ist

$$tag\ i = \frac{2\lambda\omega}{R} \qquad (19a)$$

also unabhängig von der Breite, während dieses bei reinem Nord oder Südwind nicht der Fall ist; denn es ist für diesen Fall

$$tag\ i = \frac{2\lambda\ \omega\ sin\ \theta}{R} \qquad (19b)$$

Die Windrichtung schliesst demnach mit den Isodynamen (oder Isobaren) im Allgemeinen grösseren Winkel, wenn sie Nord oder Süd ist, als wenn sie Ost oder West ist.

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* Insofern als nur die unterste Schichte der Atmosphäre in Betracht gezogen wird, und die Annahme gestattet ist, dass der Reibungswiderstand an der Erdoberfläche proportional den Horizontalcomponenten der Windgeschwindigkeit ist, sodan man setzen kann

$$\Xi = \kappa u \qquad H = \kappa v \qquad Z = \kappa w$$

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$$R = \kappa w$$

folglich

$$tag.\ i = \frac{2\lambda}{n}\sqrt{1 - cos^2\chi\ cos^2\theta}$$

so kann man leicht zur ungefähren Kenntniss der Constante κ gelangen, welche den Reibungswiderstand der Erdoberfläche bestimmt. Für höheren Breitengrad ist annährend

$$tag.\ i = \frac{2\lambda}{\kappa}$$

Nun beträgt der Winkel i nach den Beobachtungen von Herrn Capt. Hoffmeyer für Dänemark (Meteorolog. Zeitschrift der österreichischen Gesellschaft. Bd. XIII pag. 338.)

Windrichtung	*N W*	*N E W*	*E N E*	*E S*	*S E S*	*S S W*	*W S W*	*N W W*	im Mittel
i	77.5	74.5	67.5	61	60.5	65.5	71.5	75.5	69.2

Hieraus ergiebt sich aus der obigen Gleichung mit Zugrundlegung des Werthes $\lambda = 0{,}00007292$

$$\kappa = 0{,}00005395\ (Meter)$$

also eine Grösse von denselben Ordnung wie λ.

Man denke sich die Isodynamen so beschaffen, dass die Windbahn dieslben unter dem constanten Winkel i durchsetzt. Als Azimuthwinkel der Windbahn findet man aus (21)

$$cos\ \chi = \frac{1}{cos\ \theta}\sqrt{1 - \frac{R^2}{4\lambda^2\omega^2}\,tag^2\,i} \qquad (19d)$$

Das Azimuth ist also mit der geographischen Breite veränderlich. Wenn nun in irgend einem Punkt der Erdoberfläche, dessen Breite θ_0 heissen möge, die Luftströmung als Ost—oder Westwind auftritt, und der Quotient $\frac{\omega}{R}$ überall denselben Werth behielt, so hat man als Azimuth der Windbahn in dieser Luftströmung aus (19d) und (19a) für die Breite θ

$$cos\ \chi = \frac{1}{cos\ \theta}\sqrt{1 - \frac{R^2}{4\lambda^2\omega^2}\ \frac{4\lambda^2\omega^2}{R^2}}\ d.\ h. =0$$

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$$\chi = \frac{\pi}{2}\ \text{oder}\ -\frac{\pi}{2}$$

Wir gelangen somit zu dem Satz, *dass eine Luftströmung welche einmal als reiner Ost oder West auftritt, immer die Richtung Ost und West beihehält*—dass demnach eine dem Parallelkreise parallele Luftströmung nicht durch die Rotation der Erde von ihrer ursprünglichen Bichtung abgelenkt wird. Dieses ist nichts anderes, als das bekannte Hadley'sche Princip. Ganz anderes verhält sich mit einer Luftströmung, die ursprünglich polar oder aequatorial gerichtet war. Für diesen Fall findet man unter derselben Annahme aus (19d) und (16b)

$$cos\ \chi = \frac{1}{cos\ \theta}\sqrt{\left(1 - \frac{R^2}{4\lambda^2\omega^2}\ \frac{4\lambda^2\omega^2\ sin^2\ \theta_0}{R^2}\right)} = \frac{cos\ \theta_0}{cos\ \theta}.$$

Der Azimuth der Windbahn ist also mit der geographischen Breite veränderlich; die Luftströmung, welche ursprünglich als reiner Nord aufgetreten ist, wird von dieser Richtung nach und nach in dem Maasse abgelenkt, als sie in immer niedrigere Breite kommt. Wenn die

Luftströmung eine rein südliche ist, so wird die für den Azimuth der Windbahn gefundene Gleichung unbrauchbar—da dann $cos\ \theta_0 > cos^2\ \theta$ und $cos\ \chi$ keine reele Grösse ist. Es geht daraus herbor, dass bei einer rein südlichen Luftströmung *tag i* nicht überall denselben Werth haben kann.

Aus der Gleichung (19*d*) geht übrigens hervor dass eine Luftströmung von der bezeichneten Art so lange möglich ist, als $tag\ i < \frac{2\lambda\omega}{R}$ ist., und ferner sie nicht in jeder Breite auftreten kann; denn der grösste Werk, den $cos\ \chi$ überhanpt erhalten kann, ist ja 1. Der Parallelkreis, welcher also senkrecht von Windbahn geschnitten wird und demnach die Grenze der bezeichneten Luftströmung bildet, ist durch die Gleichung

$$cos\ \theta' = \sqrt{1 - \frac{R^2}{4\lambda^2\omega^2}\ tag^2\ i}$$

bestimmt, wenn θ' die Breite des betreffenden Parallelkreises bedeutet. Von da ab schneidet die Windbahn den Parallelkreis unter immer kleinerem Winkel, und unter dem Aequator selbst ist der Azimuth der Windbahn durch die Gleichung

$$cos\ \chi_0 = \sqrt{1 - \frac{R^2}{4\lambda^2\omega^2}\ tag^2\ i}$$

bestimmt, *d. h.* nahe zu $\chi = \frac{\pi}{2}$ wenn $\frac{R^2\ tag^2\ i}{4\lambda^2\omega^2}$ sich sehr wenig von 1 unterscheidet; die Luftströmung geschieht daher unter dem Aequator nahezu parallel dem Parallelkreis. Es ist überhaupt.

$$cos\ \theta' = cos\ \chi_0$$

d. h. Bei der in Rede stehenden Luftströmung schneiden die Windbahnen den Aequator unter demjenigen Winkel, welcher das Complement der Breite des Grenzkreises bildet.

Die ausgezogene Curve in Fig. 2 (Tafel XIII) mag den ungefähren Verlauf der Windbahn in diesem Fall darstellen.

Aehnliche Luftströmungen kommen nun in der Natur thatsächlich vor, und sind als Passatwinde wohl bekannt. Die Beobachtung, dass die Region der Passatwinde nicht bis zum Polarkreis hinaufreicht, sonderen nur auf einen Gürtel unter den Tropen beschränkt ist, ist also auch theoretisch begründbar, und zur Erklärung dieser Thatsache scheint das allmählige Niedersinken der oberen Aequatorialströmung nicht das einzig und allein Wesentliche zu sein; da die Grenz, innerhalb deren die Windbahnen unter reelem Winkel die Parallelkreise schneidet auch wesentlich durch den Werth des Bruchs $\frac{R}{2\lambda\omega}$ bestimmt ist.

Ich bemerke hierbei, dass man* das Wachsen des Azimuthwinkels der Windbahn mit abnehmender Breite wirklich im Passatgebiete des atlantischen Oceans beobachtet hat, was man auch aus der Gleichung (19 d) unmittelbar folgern könnte. Dieselbe kann sonach als ein analytischer Ausdruck gelten, für die Luftströmung in Passatregion; insoweit, als die Isodynamen, oder man darf in diesem Fall wohl dafür setzen, die Isobaren so beschaffen sind, dass der Deviationswinkel constant ist, was in der Wirklichkeit auf hoher See, wo der Einfluss der angrenzenden Continente auf den Verlauf der Isobaren verschwindet, annährend richtig sein dürfte.

Am Aequator und an der Grenzzone der Passatwinde selbst wird aber der gedachte Ausdruck wesentliche Modification erleiden müssen, einmal, weil die Regionen der Passatwinde auf den beiden Hemisphären durch einen schmahlen Gürtel der vertical aufstreigenden Luftströmung, die Kalmengürtel, getrennt ist, und die isobarische Fläche eine andere werden muss, als die, deren Normale den Windbahn einen Constanten Winkel schliesst, und das ander Mal, weil in Folge des allmäligen Niedersinkens der Aequatorial-

* Sieh. Coffin. Winds of northern hemisphere. Washington City, published by the Smithonian Institution. November 1853 pag. 172.

strömung *d. i* des oberen Possatwindes, die Greuz der unteren Passatregion früher beginnen muss als bei der durch die Gleichung (19*c*) bestimmten Breit. Das Gesetz, welches den gauzen Verlauf der reinen Polarströmung, der unteren Passatwinde beherrscht, ist nun allerdings in der Gleichung (21*a*) enthalten; die Discussion derselben setzt aber die Kemtniss der Gleichung für die Isobarische Fläche und des Quotientem $\frac{\omega}{R}$ als Function von den Coordinaten voraus, welche einen Punkt der Erdoberfläche bestimmen.

Betrachten wollen wir noch den Fall, wo die Isobaren dem Parallelkreise parallele Kreise bilden, also den Fall, wo der Deviationswinkel i gleich ist dem Azimuthwinkel.

Für diesen Fall haben wir die Gleichung

$$tag\,\chi = \frac{2\lambda\omega}{R}\sqrt{(1-cos^2\chi\,cos^2\theta)}$$

oder in den wir $\frac{2\lambda\omega}{R} = K$ setzen, und nach $cos^2\chi$ aufläsen,

$$cos^2\chi = \frac{1+K^2 \pm \sqrt{(1+K^2)^2 - 4K^2\,cos^2\theta}}{2\,cos^2\theta \cdot K^2.}$$

Wenn eine solche Bewegung entlang der Erdoberfläche möglich, und χ daher immer einen reelen Werth haben soll, so muss das untere Zeichen genommen werden, da soust. $cos^2\chi$ in Allgemeinen > 1 sein würde. Man hat sonach

$$cos^2\chi = \frac{1+K^2 - \sqrt{(1+K^2) - 4\,cos^2\theta K^2}}{2\,cos^2\theta K^2}$$

Für die Aequatrialgegenden erhält man

$$cos^2\chi_0 = 1$$

und für die Polargegend findet man leicht

$$cos^2\chi_{90} = \frac{1}{1+K^2}$$

und für die Breite. 45°

$$cos^2 \chi_{45} = \frac{1+K^2-\sqrt{1+K^4}}{K^2}$$

Der Azimuthwinkel der Windbahn nimmt also mit wachsender Breite zu, und hat um so kleineren Werth, je kleiner K, *d. h.* je unbedeutender die Rotationsgeschwindigkeit der Erde gegen den Quotienten $\frac{R}{\omega}$ in Betracht kommt. Nimmt hingegen K zu, *d. h.* wird $\frac{R}{\omega}$ klein gegen die Rotationsgeschwindigkeit der Erde, so wachst der Azimuthwinkel und für verschwindend kleinen Werth von K, wird derselbe $=\frac{\pi}{2}$; die Windbahn schneidet dann die Isobaren gar nicht, und geht parallel dem Parallelkreise.

Die in Fig. 2 (Tafel XIII) gezeichnete punktirte Spirallinie stellt die Windbahn bei solcher Luftströmung dar. Sie erinnert unwillkürlich an die zurückkehrende Aequatrialströmung; an den oberen Passatwind. Die Luft strömt polwärts vom Aequator aus mit bei wachsender Breite immer wachsendem Azimuth und die Bahncurve ist eine Spirale, welche den Meridian unter um so grösserem Winkel schneidet, deren Windungen um so enger wird, je kleiner die Strömungsgeschwindigkeit gegen den Reibungswiderstand ausfällt.

Es lässt sich ferner aus der Gleichung (11) eine bemerkenswerthe Relation zwischen der Axenrichtung der Wirbelbewegung und der Normale der isodynamischen Flächen ableiten, welche auch zu besonderes einfachen Sätzen führen. Im Falle der stationären Luftbewegung wird die gedachte Gleichung.

$$\xi \frac{\partial \Phi}{\partial x} + \eta \frac{\partial \Phi}{\partial y} + \zeta \frac{\partial \Phi}{\partial z} = -(\xi \, \Xi + \eta \, H + \zeta Z)$$

Dieses lässt sich auch leicht in

$$cos\ (n\sigma)\ \sigma \frac{\partial \Phi}{\partial n} = -R\sigma\ cos\ (\sigma R)$$

verwandeln, wo σ und R durch die Gleichungen (18c) (18a) bestimmt

sind und wo n wieder die Normale der isodynamischen Flächen bedeutet. Wir erhalten somit die Relation

$$\cos(n\sigma) = -\frac{R}{\frac{\partial \Phi}{\partial n}} \cos(\sigma R)$$

oder auch mit Rücksicht auf die Gleichung (18b)

$$\cos(n\sigma) = \cos i, \cos(\sigma R)$$

Nun soll nach der eingeführten Annahme die Richtung der Reibungsresultante R überall der Resultante der zur Erdoberfläche parallelen Geschwindigkeit parallel sein. Der Winkel (σR) welche die Wirbelaxe σ mit der Richtung der Resultante R schliesst, ist identisch mit demjenigen, welche sie mit der Richtung der resultirenden Horizontalgeschwindigkeit bildet. Es sei Θ der Neigungswinkel der Wirbelaxe gegen die Tangentialebene der Erdoberfläche von der Richtung aus gezählt, für welche R positiv ist, welche also der Richtung der resultirenden Horizontalgeschwindigkeit der Strömung entgegengesetzt ist; so ist

$$\cos \Theta = \cos(\sigma R)$$

Mithin folgt die Gleichung

$$\cos(n\sigma) = \cos i \cos \Theta \tag{21a}$$

Man denke jetzt auf der Erdoberfläche ein Wirbelsystem, welches so beschaffen ist, dass jedes Lufttheilchen in demselben unmittelbar auf der Erdoberfläche um eine zur Verticallinie parallele Axe rotirt. Dann ist $\Theta = \frac{\pi}{2}$ zu setzen; es folgt hieraus, dass auch

$$\sphericalangle (n\sigma) = \frac{\pi}{2}$$

ist. Wenn demnach in einem Wirbelsystem jedes Lufttheilchen um eine um die Verticallinie parallele Axe rotirt, so stehen die Isodynamischen Flächen auf der Erdoberfläche überall senkrecht *d. h.* die Isodynamen sind die Durchschnittscurven der zur Erdoberfläche orthonalen Flächenschaar. Es ist hieraus zu schliessen, dass wenn die

Erde als eine Kugel zu betrachten ist, die isodynamischen Flächen gewisse Kegelflächen sind, deren Spitzen sämmtlich im Mittelpunkt der Kugulfläche liegen. Wenn das Gebiet der Wirbelbewegung auf der Erdoberfläche durch Curven geschlossen ist, so sind die Isodynamen auch eine Schaar von geschlossenen Curven. Haben die Lufttheilchen hingegen keine andere Rotation, als die der Erde, so fällt die Wirbelaxe mit der Erdrotation zusammen, steht also senkrecht zur Erdoberfläche und die Isodynamen, müssen daher eine Schaar um den Erdpol geschlossener Curven sein. In der Höhe der Erdatmosphäre ist die Wirbelaxe im allgemeinen gegen die Tangentialebene oder Erdoberfläche geneigt, da $cos\,(n\sigma)$ im Allgemeinen nicht verschwindet, überall wo der Einfluss des Reibungswiderstandes nicht verschwindet. Hinsichtlich dieses Neigungswinkels lässt sich nun aus der Gleichung (21a) ein charakterischer Unterschied zwischen einem cyklonalen und anticyklonalen Wirbelsystem ableiten. Wie aus der Gleichung (18d) unmittelbar ersichtlich, haben die Gradienten auch in der höheren Luftschichte bei einer cyklonalen Bewegungsform grössere Werthe, als bei einer anticyklonalen, da die Resultante der Rotationsgeschwindigkeit σ für jene grösser als für diese sein muss, es folgt auch hieraus, dass der Deviationswinkel in der höheren Luftschichte für Cyklonalen Formen grössere sein muss, als für anticyklonale. Dieses Umstandes halber ergiebt sich die Ungleichung

$$\left(\frac{cos\,(n\sigma)}{cos\,\Theta}\right)_c < \left(\frac{cos\,(n\sigma)}{cos\,\Theta}\right)_a$$

Hierin ist ein Satz enthalten, welcher sich also aussprechen lässt: Wenn die Richtung der Wirbelaxe im cyklonalen und anticyklonalen Wirbelsystem unter sonst gleichen Umständen gleiche Neigung gegen die Tangentialchene der Erdoberfläche hat, so schneidet die Wirbelaxe im cyklonalen System die Normale der isodynamischen Fläche unter

grösseren Winkel, als im anticyklonalen System. Bildet die Wirbelaxe in den beiden Systemen unter sonst gleichen Umständen mit der Normale der isodynamischen Fläche gleichen Winkel, so ist der Neigungswinkel der Wirbelaxe im cyklonalen Wirbelsystem alle Mal kleiner als im anticyklonalen Wirbelsystem.

Dieser Satz steht nun im genauesten Zusammenhang mit der von dem scharfsinnigen Meteorologen Redfield * zuerst gemachten Bemerkung, dass, wenn ein wirbeluder Luftcylinder entlang der Erdoberfläche schreitet, derselbe durch den Reibungswiderstand am Boden nothwendig vorneigen und sich schon in der höheren Luftschichte bemerkbar machen muss, ehein der unteren Schichte wahrgenommen wird. Dabei neigt sich der Wirbelcylinder, wenn er cyklonal rotirt, weniger nach vorn, als wenn er im anticyklonalen Sinne rotirte, und die Krümmung der anticyklonalen Wirbelfäden, welche den gedachten Cylinder bilden, muss daher unter sonst denselben Umständen stärker sein, als diejenige der cyklonalen. Auch dieses wird durch die Erfahrung bestätigt, dass, wenn die Druckminima, denen, wie wir weiter in einem Beispiel sehen werden, die cyklonalen Wirbelfäden gehören, fortschreiten, dieselben bei Weitem rapideres Fallen des Barometerstandes hervorruft, als wenn im fortschreitenden Gebiete der Druckmaxima, denen, wie wir ebenfalls weiter unten sehen werden, die anticyklonalen Wirbelfäden gehören, das Barometer steigt. Es beruht also die oben gedachte Verschiedenheit der Gradienten in den beiden Wirbelsystemen einfach auf der Verschiedenheit der Krümmung der Wirbelfäden.

§ *IV. Die Integration nach dem Raum.*

Ich will jetzt einen allgemeinen Ausdruck für die Geschwindigkeitscomponenten u v w herzustellen suchen und zwar für

* O. Marbach. Physik. Lexikon. Art. Winde. pag. 984.

verschiedene Raumgebiete, wo die atmosphärische Luft im verschiedenen Bewegungszustande sich befindet. Ich denke mir den von der Atmosphäre eingenommenen Raum in Raumgebiete getheilt, theils in Raumgebiete der vertical aufsteigenden Strömung, theils in die der dadurch veranlassten horizontalen Strömung. Die Geschwindigkeit der verticalen Strömung nehme ich als gegeben an, eben so das Raumgebiet, wo sie stattfindet. In dem übrigen Raumgebiete soll die Luft nur horizontale Geschwindigkeit haben, *d. h.* nur parallel zur Erdoberfläche strömen.

Bezeichnet man nun die Componenten der gegebenen Geschwindigkeit der verticalen Strömung wieder mit A B Γ und diejenigen der horizontalen Strömung in den beiden Raumgebieten mit u' v' w'; dann sind die Componenten der Geschwindigkeit in Raumgebiete der horizontalen Strömung

$$u = u' \qquad v = v' \qquad w = w'$$

und im Raumgebiete der verticalen Strömung

$$u = u' + A \qquad v = v' + B \qquad w = w' + \Gamma.$$

wo bei es sogleich bemerkt werden möge, dass A B Γ negativ zu rechnen ist, wenn in dem betreffenden Raumgebiete die Strömung vertical niederwärts geschieht.

Die Bedingungen, denen u v w an der Erdoberfläche und an der Grenzfläche der beiden Raumgebiete zu genügen haben, sind bereits aufgestellt. Es handelt sich hier zunächst darum für u v w solchen Ausdruck zu finden, dass die Bedingungsgleichungen (9*b*) und die Bedingung der Massencontinuität

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

erfüllt werden.

Um solche u, v, w, zu finden denke ich mir, eine Function φ so bestimmt, dass sie für das innere Raumgebiete [womit das Raumgebiet der verticalen Strömung bezeichnet wird] die Differential-

gleichung

$$\Delta \varphi = -\left(\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + \frac{\partial \Gamma}{\partial z}\right) \tag{22}$$

und in dem äusseren Raumgebiete aber die Gleichung

$$\Delta \varphi = 0 \tag{22a}$$

befriedigt. Wenn wir nun im äusseren Raumgebiete setzen

$$\begin{aligned} u &= \frac{\partial W}{\partial y} - \frac{\partial V}{\partial z} + \frac{\partial \varphi}{\partial x} \\ v &= \frac{\partial U}{\partial z} - \frac{\partial W}{\partial x} + \frac{\partial \varphi}{\partial y} \\ w &= \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} + \frac{\partial \varphi}{\partial z} \end{aligned} \tag{23}$$

und im inneren Raumgebiete

$$\begin{aligned} u &= \frac{\partial W}{\partial y} - \frac{\partial V}{\partial z} + \frac{\partial \varphi}{\partial x} + A \\ v &= \frac{\partial U}{\partial z} - \frac{\partial W}{\partial x} + \frac{\partial \varphi}{\partial y} + B \\ w &= \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} + \frac{\partial \varphi}{\partial z} + \Gamma \end{aligned} \tag{24}$$

wo U, V, W gewisse Functionen von $x\ y.z$ und t sind, welche den partiellen Differentialgleichungen genügen

$$\Delta U = \frac{\partial \Gamma}{\partial y} - \frac{\partial B}{\partial z} - \xi - 2\,\lambda \cos\theta \tag{25}$$

$$\Delta V = \frac{\partial A}{\partial z} - \frac{\partial \Gamma}{\partial x} - \eta \tag{26}$$

$$\Delta W = \frac{\partial B}{\partial x} - \frac{\partial A}{\partial y} - \zeta + 2\,\lambda \sin\theta. \tag{27}$$

dann wird die Bedingung der Massencontinuität erfüllt, und es werden die Gleichung (9b) *d. i.*

$$\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = \xi + 2\,\lambda \cos\theta$$

$$\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = \eta$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta - 2\,\lambda \sin\theta$$

so wohl im inneren als im äusseren Raumgebiet erfüllt wenn U, V, W über dies die Bedingung

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}=0 \tag{28}$$

erfüllen.

Die Grössen.

$$\begin{aligned}
\frac{\partial \Gamma}{\partial y}-\frac{\partial B}{\partial z}-\xi-2\lambda\cos\theta &= \frac{\partial(\Gamma-w)}{\partial x}-\frac{\partial(B-v)}{\partial z}\\
\frac{\partial A}{\partial z}-\frac{\partial \Gamma}{\partial x}-\eta &= \frac{\partial(A-u)}{\partial z}-\frac{\partial(\Gamma-w)}{\partial x}\\
\frac{\partial B}{\partial x}-\frac{\partial A}{\partial y}-\zeta+2\lambda\cos\theta &= \frac{\partial(B-v)}{\partial x}-\frac{\partial(A-u)}{\partial y}
\end{aligned}$$

bedeuten nun nichts Anderes, als die Componenten der Drehungsgeschwindigkeit parallel der Erdoberfläche in Bezug auf die Coordinatenaxen. Bezeichnet man dieselben mit ξ' η' und ζ', und ihre Resultante mit σ', so dass

$$\frac{\partial w'}{\partial x}-\frac{\partial v'}{\partial z}=\xi' \quad \frac{\partial u'}{\partial z}-\frac{\partial w'}{\partial x}=\eta' \quad \frac{\partial v'}{\partial x}-\frac{\partial u'}{\partial y}=\zeta'$$

$$\sigma'=\sqrt{(\xi'^2+\eta'^2+\zeta'^2)} \tag{28a}$$

dann haben die Functionen U, V, W die Gleichungen zu erfüllen.

$$\begin{aligned}
\Delta U &= \xi'\\
\Delta V &= \eta' \qquad\qquad (32)\\
\Delta W &= \zeta'\\
\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}&+\frac{\partial W}{\partial z}=0
\end{aligned}$$

Haben die Lufttheilchen im äusseren Raumgebiete, keine eigene Rotation, so dass

$$\xi=-2\lambda\cos\theta \quad \eta=0 \quad \zeta=2\lambda\sin\theta$$

so müssen U, V, W derselben Differentialgleichung genügen wie φ. *d. h.*

$$\Delta U=0 \quad \Delta V=0 \quad \Delta W=0 \tag{33}$$

Wie solche Differentialgleichungen, wie von (25) bis (33) zu integriren sind, lehrt uns die Potentialtheorie. Es kann sich nun mehr darum handeln U, V, W, so zu bilden, dass, ihre nach der Normale der Trennungsfläche der verschiedenen Raumgebiete genommenen Differentialquotienten continuirlich in einander übergehen, wenn der Punkt $(x\, y\, z)$ von einem Raumgebiete zum andern übergeht. *d. h.* bezeichnet man das innere Raumgebiet mit dem Buchstaten i, und das äussere mit a, so müssen U V W und φ so gebildet werden, dass an der Trennungsfläche der verschiedenen Raumgebiete

$$\frac{\partial W_i}{\partial n}=\frac{\partial W_a}{\partial n} \quad \frac{\partial V_i}{\partial n}=\frac{\partial V_a}{\partial n} \quad \frac{\partial U_i}{\partial n}=\frac{\partial U_a}{\partial n} \quad \frac{\partial \varphi_i}{\partial n}=\frac{\partial \varphi_a}{\partial n} \tag{34}$$

ist, damit die Continuität der horizontalen Bewegung bemerkstelligt werde.

Ich muss hierbei bemerken, dass wir je nach der Natur der als gegeben betrachteten Raumgebietes der verticalen Strömung zwei wesentlich verschiedenen Strömungserscheinungen zu unterscheiden haben. Wenn das gedachte Raumgebiete durch Linien

$$A=0 \qquad B=0 \qquad \Gamma=0$$

gebildet ist, so geht die Strömung der Luft auch in verticaler Richtung continuirlich vor sich, indem die Horizontialrichtung der Strömung allmählig in die Verticalströmung übergeht. Ist das innere Raumgebiet hingeben durch Stromlinien der verticalen Strömung selbst gebildet, so werden die Lufttheilchen, welche bis an die Trennungsfläche in reiner Horizontalbewegung begriffen waren, durch Verticalströmung auch ohne jede Vermittelung vertical aufwärts, oder niederwärts gerissen. Welcher von diesen Bewegungsformen eintritt, das hängt lediglich von dem Werthe ab, welchen die gegebenen Geschwindigkeitscomponenten der verticalen Strömung an der Trennungsfläche der Raumgebiete besitzen.

Es ist nicht schwer allgemeine Integrale aufzufinden, welche die

Gleichungen (22) (22a) (29) (30) (31) und (33) befriedigen, und zwar unter der Voraussetzung, dass im äusseren Raumgebiete Wirbel nicht existiren, und in dem inneren Raumgebiete die Rotationsgeschwindigkeit der Lufttheilchen gegeben sei, und dass der Luftraum sich im Übrigen in's Unendliche erstrecke, und in der Unendlichkeit ruhe, und in der Endlichkeit durch die Erdoberfläche begrenzt sei. Dann genügt man solchen Differentialgleichungen, wie (22) (22a) (29) (30) (31) und (33) durch die Raumintegrale *

$$\varphi = \quad \frac{1}{4\pi}\iiint\left(\frac{\partial A}{\partial x}+\frac{\partial B}{\partial y}+\frac{\partial \Gamma}{\partial z}\right)\frac{dx'dy'dz'}{r}. \tag{35}$$

$$U = -\frac{1}{4\pi}\iiint\frac{\xi' dx'dy'dz'}{r} \tag{36}$$

$$V = -\frac{1}{4\pi}\iiint\frac{\eta' dx'dy'dz'}{r} \tag{37}$$

$$W = -\frac{1}{4\pi}\iiint\frac{\zeta' dx'dy'dz'}{r} \tag{38}$$

wo $dx'dy'dz'$ ein Element des wirbelerfüllten Raumgebietes der verticalen Strömung bezeichnet und r die Entfernung eines Punktes $(x\ y\ z)$ von diesem Raumelemente

$$= \sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$$

zu setzen ist

Dass diese fur U, V, W gesetzten Raumintegrale auch die Gleichung (28) befriedigt, beruht auf dem Umstande, dass auch in unserem Problem die Wirbelfäden nirgends in dem Luftraum endigen, dass sie entweder in sich zurücklaufen, oder an die starre Erdoberfläche stossen, wo sie abbrechen, da ja das Raumintegral

$$\iiint\left(\frac{\partial \xi'}{\partial x'}+\frac{\partial \eta'}{\partial y'}+\frac{\partial \zeta'}{\partial z'}\right)dx'\ dy'\ dz'$$

$$= -\iint \sigma' \cos(\sigma'\ n)\ ds.$$

* Sieh. H. Helmholtz, Über Integrale der hydrodynamischen Gleichungen, welche den Wirbelbewegungen entsprechen. Crelle's Journal. Bd. 55.

in jedem Punkt des Luftraums verschwindet, und an der durch Wirbellinien gebildeten Oberfläche des Wirbelgebietes $\sigma' = 0$ ist, oder wenigstens

$$\xi' \cos (nx) + \eta' \cos (ny) + \zeta' \cos (nz) = 0$$

Die Functionen φ, U, V, W welche durch die Gleichungen (35)(36)(37)(38) definirt sind, bleiben bekanntlich überall stetig und endlich, und verschwinden in der Unendlichkeit. Ihre Differentialquotienten nach der Normale des Raumgebietes, über welches die Integration ausgedehnt wird, haben auch die Eigenschaft, in der Unendlichkeit zu verschwinden, und gehen continuirlich in einander über, wenn der Punkt (x y z) von einem Raumgebiete in das andere herübergeführt wird *d. h.* die Bedingung der Continuität der horizontalen Strömung in den verschiedenen Raumgebieten ohne Weiteres durch die Form der Functionen φ, U, V, W erfüllt. Was die Grenzbedingung anbetrifft, welche noch an der Erdoberfläche und an der äusseren Grenzfläche der Atmosphäre zu erfüllen ist, so können ξ', y', ζ' so gegeben sein, dass die Bedingungsgleichung ($8f$) von selbst erfüllt wird, oder sie kann dadurch erfüllt werden, wie Professor Helmholtz es gezeigt hat,* dass man zu den Gleichungen (23) und (24) der Reihe nach

$$\frac{\partial P}{\partial x}, \quad \frac{\partial P}{\partial y}, \quad \frac{\partial P}{\partial z}$$

hinzugefügt, wo P eine Function ist, welche die Gleichung $\Delta P = 0$ befriedigt und in der Unendlichkeit verschwindet, und P passend bestimmt.

Wenn gleich die weitere Behandlung des Problems im Allgemeinen mit grosser Schwierigkeit verknüpft zu sein pflegt, so sieht man doch klar, wie die Theorie der Bewegung der Erdatmosphäre auf diejenige der Wirbelbewegung in einer Flüssigkeit zurückgeführt werden kann, wenn ξ', η', ζ' nur gemäss der Kräfte, welche auf die rotirenden Lufttheilchen einwirken, gegeben sind, und

* H. Helmholtz, a. o. a. O.

ferner, dass diejenigen Fälle der Wirbelbewegung, in denen die Lösung bisher gelungen ist, unmittelbar auf die Bewegungserscheinungen der Atmosphäre anwendbar sind, so wie diejenigen Fundamentalsätze über die Wirbelbewegung, welche von Helmholtz und *A.* aufgestellt worden sind, und nicht auf der Voraussetzung beruhen, dass auf die Flüssigkeitstheilchen nur Kräfte einwirken, welche Potential haben.

§ *V. Bewegungsgleichungen unter specieller Annahme.*

Es ist indessen gerade das Schwierigste in dem vorliegenden Problem, ξ, η und ζ anzugeben; denn die Geschwindigkeit der horizontalen Strömung hängt im Wesentlichen von der Geschwindigkeit der gegebenen verticalen Strömung ab, und die Componenten der Rotationsgeschwindigkeit eines Lufttheilchens, welche aus der Rotationsgeschwindigkeit der Erde einerseits und aus der eigenen Rotationsgeschwindigkeit andererseits resultirt, müssen gemäss der Differentialgleichungen (14) (15) (16) bestimmt werden, welche aber neben ξ, η, ζ auch noch u, v, w enthalten, also Functionen, welche erst dam bestimmt werden können, falls ξ, η, ζ bestimmt worden sind.

Diese Schwierigkeit kann allerdings dadurch umgangen werden, indem man statt der zu bestimmenden ξ, η, ζ ΔU, ΔV, ΔW und statt der noch unbekannten u, v, w die dafür gesetzten Ausdrücke (23) (24) in die Differentialgleichungen (14) (15) (16) einsetzt, und somit zwischen den 3 Functionen U, V, W drei partielle Differentialgleichungen dritter Ordnung herstellt. Weil die Function φ gemäss der Gleichung (22) oder (22*a*) und (34) gegeben ist, so ist die vorliegende Aufgabe gelöst, falls es gelingt, die obengedachte Differentialgleichung dritter Ordnung zwischen U, V, W aufzulösen und man solche particuläre Lösungen nimmt, welche im gegebenen Falle den Grenzbedingungen (24) Genüge leisten.

So bietet die Aufgabe dennoch Schwierigkeiten dar, die allem Lösungsversuche trotzen würden. Indem wir doch eine Lösung desselben versuchen, wollen wir die Bewegungsgleichungen durch die Annahme vereinfachen, dass das Stück der Erdoberfläche, so weit wir es in Betracht ziehen, als eine unendlich ausgedehnte Ebene angesehn werden könne, deren mittlere Breite θ sei, und deren Normale überall mit der z-Axe zusammenfalle, und dabei halten wir an der Annahme fest, dass die Verzögerung, welche die bewegten Theilchen in Folge der Reibung längs der Erdoberfläche erleiden, direct der horizontalen Resultante der Strömungsgeschwindigkeit proportional sei, was für die unmittelbar auf der Erde ruhende Luftschichte wohl angenährt richtig ist, so dass jetzt

$$\Xi = \kappa u' \qquad H = \kappa v' \qquad Z = 0$$

gesetzt werden könne, wo κ wieder eine Constante bedeutet. Die Bewegungsgleichungen (14) (15) (16) gehen dann über in :

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + w\frac{\partial \zeta}{\partial z} - \xi\frac{\partial w}{\partial x} - \eta\frac{\partial w}{\partial y} - \zeta\frac{\partial w}{\partial z} + \kappa\left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}\right) = 0$$

$$\frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y} + w\frac{\partial \eta}{\partial z} - \xi\frac{\partial v}{\partial x} - \eta\frac{\partial v}{\partial y} - \zeta\frac{\partial v}{\partial z} + \kappa\frac{\partial u'}{\partial z} = 0$$

$$\frac{\partial \xi}{\partial t} + u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y} + w\frac{\partial \xi}{\partial z} - \xi\frac{\partial u}{\partial x} - \eta\frac{\partial u}{\partial y} - \zeta\frac{\partial u}{\partial z} - \kappa\frac{\partial v'}{\partial z} = 0$$

Wir specialisiren unsere Aufgabe noch weiter durch die Annahme, dass die Rotationsgeschwindigkeit in dem Wirbelgebiete überall parallel zur unendlichen Ebene gerichtet sei, und jedes Lufttheilchen um eine Axe rotire, welche mit der z-Achse zusammenfällt, so dass

$$\xi = 0 \quad \eta = 0 \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + 2\lambda \sin\theta$$

und

$$u = u' \qquad w = w' + \Gamma.$$

Wir erhalten aus den obenstehenden Gleichungen

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + w\frac{\partial \zeta}{\partial z} - \zeta\frac{\partial w}{\partial z} + \kappa\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \qquad (39)$$

$$-\zeta\frac{\partial v}{\partial z} + \kappa\frac{\partial u}{\partial z} = 0 \qquad (40)$$

$$-\zeta\frac{\partial u}{\partial z} - \kappa\frac{\partial v}{\partial z} = 0 \qquad (41)$$

Multiplizirt man nun von diesen Gleichungen die zweite mit ζ, und die dritte mit κ, so entsteht zunächst durch Addition

$$-(\zeta^2 + \kappa^2)\frac{\partial v}{\partial z} = 0 \qquad d.\ h.\ \frac{\partial v}{\partial z} = 0$$

mithin folgt aus (40) oder (41)

$$\frac{\partial u}{\partial z} = 0$$

folglich

$$\frac{\partial \zeta}{\partial z} = \frac{\partial}{\partial z}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

Da ferner $\eta = 0$ *d. i* $\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$ ist, so folgt

$$\frac{\partial w}{\partial x} = 0$$

da ferner $\xi = 0$ ist *d. i* $2\lambda \cos\theta = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}$, so folgt

$$\frac{\partial w}{\partial y} = +\, 2\lambda\cos\theta$$

Somit erhalten wir als Bewegungsgleichungen

$$\frac{\partial v}{\partial z} = \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} = 0 \qquad \frac{\partial w}{\partial y} = +\, 2\lambda\cos\theta.$$

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial z}{\partial y} - \zeta\frac{\partial w}{\partial z} + \kappa(\zeta - 2\lambda\sin\theta) = 0 \qquad (41a)$$

d. h. die Geschwindigkeitscomponenten u und v sind nur von x und y abhängig, w hingegen nur von y und z. *d. h.* da w in Bezug auf y linear ist.

$$w = f(z) + 2\lambda\cos\theta\, y + const.$$

Die Hülfsfunctionen U, V, W, φ werden in unserem Falle

$$U=0 \qquad V=0 \qquad W=-\frac{1}{4\pi}\iiint(\zeta-2\lambda \sin\theta)\frac{dx'dy'dz'}{r}$$

$$\varphi=\frac{1}{4\pi}\iiint\frac{\partial\Gamma}{\partial z}\ \frac{dx'dy'dz'}{r}$$

und die Ausdrücke für die Geschwindigkeitscomponenten

$$u=\frac{\partial W}{\partial y}+\frac{\partial\varphi}{\partial x}$$

$$v=-\frac{\partial W}{\partial x}+\frac{\partial\varphi}{\partial y}$$

$$w=\frac{\partial\varphi}{\partial z}+\Gamma.$$

Weil nun w von x unabhängig sein soll, so muss $\frac{\partial\varphi}{\partial z}=0$ *d. h.* φ von z unabhängig sein, mithin

$$u=\frac{\partial W}{\partial y}+\frac{\partial\varphi}{\partial x}$$

$$v=\frac{\partial W}{\partial x}+\frac{\partial\varphi}{\partial y}$$

$$w=\Gamma$$

Wie es vorausgesetzt wurde, soll das Raumgebiet der verticalen Strömung einen unendlich langen Cylinder bilden, so dass Γ eine Function von z allein ist, so kann dies nur dann stattfinden wenn $2\lambda \cos\theta . y$ als unendlich klein vernachlässigt werden darf, *d. h.* ein solches aufrechtstehendes cylindrisches Raumgebiet der Verticalströmung kann nur im höheren Breitengrade bestehen; was sich übrigens von selbst versteht, da jede verticale Strömung Folge der Erdrotation eine Ablenkung nach Westen erleidet, wenn sie vertical aufwärts, und nach Osten, wenn sie vertical niederwärts geschieht. Dass die Componenten der Geschwindigkeit u und v von z unabhängig sei, kann es nicht für jeden beliebigen Ausdruck für Γ stattfinden; die Bedingung $\frac{\partial\varphi}{\partial z}=0$ verlangt, dass auch $\frac{\partial\Gamma}{\partial z}$ von z

unabhängig sei, *d. h.* Γ ist einer linearen Function von z gleich zu setzen, so dass

$$\Gamma = \gamma . z + const.$$

wo γ eine Constante ist, die leicht ermittelt werden kann, wenn die Differenz der Temperaturen in den beiden Raumgebieten gegeben ist. *

Wir erhalten sonach unter der Annahme eines cylindrischen Raumgebietes der verticalen Strömung folgende Lösungsformel des Problems

für das innere Raumgebiet

$$u = \frac{\partial W}{\partial y} + \frac{\partial \varphi}{\partial x}$$

$$v = -\frac{\partial W}{\partial x} + \frac{\partial \varphi}{\partial y}$$

$$w = \gamma z + const.$$

wo φ eine Function ist, welche im inneren Raumgebiete die Gleichung

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = -\gamma \tag{42a}$$

und im äusseren

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \tag{42b}$$

befriedigt und W eine Function, welche im wirbelerfüllten Gebiete die Gleichung

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} = -\zeta + 2\lambda \sin \theta \tag{41c}$$

* Man betrachte einen Luftcylinder von der Länge z. Innerhalb desselben soll in Folge der stärkeren Erwärmung die Dichtigkeit μ' sein, und ausserhalb aber μ. Nun steigt ein Lufttheilchen aufwärts wegen des Auftriebes, dessen Grösse

$$g(\mu-\mu')z.$$

ist, wenn g die Beschleunigung der Schwerkraft bedeutet. Die Gleichung

$$\frac{d^2 z}{dt^2} = g(\mu-\mu')z$$

ergiebt einerseits

$$z = e^{\sqrt{g(\mu-\mu')}t.}$$

und die Gleichung

$$w = \frac{dz}{dt} = \gamma z$$

andererseits

$$z = e^{\gamma t}$$

Es kommt heraus durch Vergleich

$$\gamma = \sqrt{g(\mu-\mu')}$$

und im wirbelfreien Raumgebiete aber

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} = 0 \qquad (41d)$$

erfüllt. Die Ausdrucke für die Geschwindigkeitscomponenten im äusseren wirbelfreien Raumgebiete sind dann

$$\begin{aligned} u &= \frac{\partial W}{\partial y} + \frac{\partial \varphi}{\partial x} \\ v &= -\frac{\partial W}{\partial x} + \frac{\partial \varphi}{\partial y} \\ w &= 0. \end{aligned} \qquad (41e)$$

Die allgemeine Lösung der Differentialgleichungen (42a) und (42b) ist für jedes Raumgebiet bekannt. Bezeichnet man mit $d\alpha\, d\beta$ ein Element der xy—Ebene, so genügt man den gedachten Differentialgleichungen, indem man setzt

$$\varphi = -\frac{\gamma}{2} \iint \log(\rho)\, d\alpha\, d\beta. + const \qquad (41d)$$

$$\rho = \sqrt{(x-\alpha)^2 + (y-\beta)^2}$$

wo die Integration über das ganze Gebiet der Verticalströmung auszudehnen ist.

Diese Function φ stimmt ihrem Wesen nach mit dem Potential eines mit der constanten Masse γ erfüllten Cylinders überein. Die Function hat daher einen unendlich grossen Werth; weil die Const. unendlich gross ist; ihre Differentialquotienten haben aber einen endlichen Werth, und gehen continuirlich in einander über, wenn der Punkt $x\ y$, durch die Grenzfläche des Cylinders hindurchgeht, dessen Querschnitt im Überigen ein beliebiger ist. Was die Function W aubelangt, so muss sie so bestimmt werden, dass sie die partielle Differentialgleichung (41c) und (41d) befriedigt, wobei ζ die Differentialgleichung (41a) erfülle. Indem wir den bereits oben angedeuteten Gedankengang verfolgen, setzen wir in die Gleichung (41a) statt ζ den aequivalenten Ausdruck $2\lambda \sin\theta - \Delta W.$, wobei die Operation $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ wie gebräuchlich durch Δ angedeutet werden möge.

Wir erhalten dann für das Raumgebiet der Verticalströmung als Bestimmungsgleichung von W

$$\frac{\partial \Delta W}{\partial t}+\frac{\partial \Delta W}{\partial x}\left(\frac{\partial \varphi}{\partial x}+\frac{\partial W}{\partial y}\right)+\frac{\partial \Delta W}{\partial y}\left(\frac{\partial \varphi}{\partial y}-\frac{\partial W}{\partial x}\right)+(\kappa-\gamma)\Delta W-2\lambda \sin\theta\Delta\varphi=0 \tag{42}$$

und für das äussere Raumgebiete

$$\frac{\partial \Delta W}{\partial t}+\frac{\partial \Delta W}{\partial x}\left(\frac{\partial \varphi}{\partial x}+\frac{\partial W}{\partial y}\right)+\frac{\partial \Delta W}{\partial y}\left(\frac{\partial \varphi}{\partial y}-\frac{\partial W}{\partial x}\right)+\kappa\Delta W=0 \tag{43}$$

Ich mache hier sogleich auf eine bemerkenswerthe Thatsache aufmerksam, welche unmittelbar aus diesen Differentialgleichungen gefolgert werden kann. Verschwindet nämlich in einem Raumgebiet der verticalen Strömung die Rotationsgeschwindikeit, so ist nothwendig, dass in diesem Raumgebiet auch $\Delta\varphi$ *d. i,* γ verschwindet; *d. h.* eine jede verticale Strömung bringt, indem sie entsteht, eine wirbelnde Bewegung der Luftmasse hervor—; aber nicht umgekehrt braucht überall da, wo die Lufttheilchen in einer wirbelnden Bewegung begriffen sind, eine verticale Strömung vorhanden zu sein, weil auch im äusseren Raumgebiet die Gleichung (43) bestehen kann. Wie man weiter aus dieser Gleichung schliessen kann, können die Lufttheilchen im äusseren Raumgebiete nie mit constanter Rotationsgeschwindigkeit rotiren. Denn; wenn gleich im inneren Raumgebiete die Rotationsgeschwindigkeit der Lufttheilchen

$$\zeta=+\frac{2\lambda \sin\theta.\kappa}{(\kappa-\gamma)}$$

d. h.
$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=+\frac{2\lambda \sin\theta.\gamma}{(\kappa-\gamma)}$$

also constant sein kann, kann dieses im äusseren Raumgebiete nie der Fall sein; denn die Annahme $\Delta W=const.$ führt der Gleichung (43) zu Folge unmittelbar zu dem absurden Schluss

$$\kappa=0$$

Als Bestimmungsgleichung für die isodynamische Fläche erhalten

wie aus (17) für das wirbelerfüllte innere Raumgebiet

$$\Delta\Phi - \frac{\partial \Delta W}{\partial y}\left(\frac{\partial W}{\partial y} + \frac{\partial \varphi}{\partial x}\right) + \frac{\partial \Delta W}{\partial x}\left(\frac{\partial \varphi}{\partial y} - \frac{\partial W}{\partial x}\right) - (\Delta W - 2\lambda \sin\theta)\Delta W - \kappa\gamma = 0 \tag{44}$$

weil $\frac{\partial \zeta}{\partial y} = -\Delta u \qquad \frac{\partial \zeta}{\partial x} = \Delta v \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\gamma.$

ist, und für das äussere Raumgebiet

$$\Delta\Phi - \frac{\partial \Delta W}{\partial y}\left(\frac{\partial W}{\partial y} + \frac{\partial \varphi}{\partial x}\right) + \frac{\partial \Delta W}{\partial x}\left(\frac{\partial \varphi}{\partial y} - \frac{\partial W}{\partial x}\right) - (\Delta W - 2\lambda \sin\theta)\Delta W = 0. \tag{45}$$

Als Gleichung für Deviationswinkel erhalten wir aus (20) für unserem speciellen Fall.

$$tag\ i = \frac{1}{\kappa}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + 2\lambda \sin\theta\right) \tag{46}$$

§ VI. *Wirbelfreie Luftströmung im Raumgebiet der horizontalen Bewegung.*

Haben die Lufttheilchen keine andere Rotationsgeschwindigkeit als die durch Erdrotation hervorgerufene, und ist ihre Bewegung überall parallel der unendlichen Ebene, als welche wir die Erdoberfläche betrachtet haben, so lässt sich die Bewegung der atmosphärischen Luft ganz allgemein verfolgen. * Man nehme zunächt γ in der Gleichung (42) als unendlich klein an, so dass $\Delta\varphi$ von Null sich unendlich wenig unterscheidet. Da hierbei ζ sich unendlich wenig von $2\lambda \sin\theta$ unterscheidet, so ist ΔW auch unendlich klein. Wenn wir dabei nur eine stationäre Bewegung in's Auge fassen, was wir fortan thun wollen, so verwandelt sich unsere Differentialgleichung (42) in

$$+\Delta(\kappa W - 2\lambda \sin\theta\varphi) - \gamma\Delta W = 0$$

oder mit Vernachlässigung unendlich kleiner Grösse zweiter Ordnung

$$\Delta(\kappa W - 2\lambda \sin\theta\varphi) = 0 \tag{47}$$

* Dieser Fall ist auch von Herrn Oberbeck eben so allgemein behandelt worden. Sieh seine bereits citirte Abhandlung.

Dieser Gleichung wird genügt, wenn

$$W = \frac{2\lambda \sin\theta}{\kappa} . \varphi. \tag{48}$$

gesetzt wird; eine Auflösung, die in so fern als eine allgemeine zu bezeichnen ist, als die Gleichung (47) die einzige ist, welcher W überhaupt zu genügen hat. Hierdurch erhalten wir als Geschwindigkeitscomponenten im Raumgebiet der wirbelfreien horizontalen Bewegung

$$\begin{aligned} u &= \frac{\partial\varphi}{\partial x} + \frac{2\lambda \sin\theta}{\kappa} \frac{\partial\varphi}{\partial y} \\ v &= \frac{\partial\varphi}{\partial y} - \frac{2\lambda \sin\theta}{\kappa} \frac{\partial\varphi}{\partial x} \end{aligned} \tag{49}$$

Gleichungen, welche mit den von Herrn Oberbeck auf einem anderen Wege gefundenen bis auf das Vorzeichen des zweiten Gliedes übereinstimmt.* Ganz ebenso leicht lässt sich ein Ausdruck für Isodynamen finden. Es ist nämlich in dem betrachteten Gebiete, da $\varDelta W = 0$ ist

$$\varDelta \Phi = 0$$

wie es aus der Gleichung (45) hervorgeht. Die Function Φ genügt also genau derselben partiellen Differentialgleichung, und es kann daher angenommen werden, dass man

$$\Phi = \alpha\,\varphi + const.$$

setzen kann, wo α eine Constante ist, die noch näher bestimmt werden muss. Die Annahme, dass Φ also auch eine particuläre Lösung der Gleichung $\varDelta\varphi = 0$ ist, rechtfertigt sich in der That durch die Möglichkeit, α so zu bestimmen, dass Φ der Differentialgleichungen

$$\begin{aligned} &\frac{\partial\Phi}{\partial x} - 2v\lambda \sin\theta + \kappa u = 0 \\ &\frac{\partial\Phi}{\partial y} + 2\lambda \sin\theta . u + \kappa v = 0 \end{aligned} \tag{49a}$$

* Bei Herrn Oberbeck ist das zweite Glied negativ. Dies rührt daher, weil Herr Oberbeck seine Coordinatenaxen in entgegengesetzter Richtung drehen liess, wie die Erdrotation.

genügt, Gleichungen, in die die Fundamentalgleichungen (9c) in unserem Falle übergehen.

Man bilde, um das Behauptete nachzuweisen, $\frac{\partial\Phi}{\partial x}$ und $\frac{\partial\Phi}{\partial y}$ und setze für u, v ihre Ausdrücke in (49a), so kommt

$$\alpha\frac{\partial\varphi}{\partial x} - 2\lambda\sin\theta\left(\frac{\partial\varphi}{\partial y} - \frac{2\lambda\sin\theta}{\kappa}\ \frac{\partial\varphi}{\partial x}\right) + \kappa\left(\frac{\partial\varphi}{\partial x} + \frac{2\lambda\sin\theta}{\kappa}\ \frac{\partial\varphi}{\partial y}\right) = 0$$

$$\alpha\frac{\partial\varphi}{\partial y} + 2\lambda\sin\theta\left(\frac{\partial\varphi}{\partial x} + \frac{2\lambda\sin\theta}{\kappa}\ \frac{\partial\varphi}{\partial y}\right) + \kappa\left(\frac{\partial\varphi}{\partial y} - \frac{2\lambda\sin\theta}{\kappa}\ \frac{\partial\varphi}{\partial x}\right) = 0$$

woraus weiter durch Auflösung der Klammern folgt

$$\left(\alpha + \kappa + \frac{4\lambda^2\sin^2\theta}{\kappa}\right)\frac{\partial\varphi}{\partial x} = 0$$

$$\left(\alpha + \kappa + \frac{4\lambda^2\sin^2\theta}{\kappa}\right)\frac{\partial\varphi}{\partial y} = 0$$

d. h.

$$\alpha = -\left(\kappa + \frac{4\lambda^2\sin^2\theta}{\kappa}\right)$$

Mithin erhält man

$$\Phi = -\left(\kappa + \frac{4\lambda^2\sin^2\theta}{\kappa}\right)\varphi + const. \qquad (50)$$

d. h. mit anderen Worten, dass die Isodynamen im wirbelfreien Raumgebiete der Horizontalströmung überall mit der Curve $\varphi = const$ parallel verlaufen. Als Deviationswinkel in einem solchen Gebiete folgt aus (46) ohne Weiteres

$$tag\ i = \frac{2\lambda\sin\theta}{\kappa}$$

also constant. *Im Raumgebiet der wirbelfreien Horizontalströmung, schneidet die Windbahn die Isodynamen überall unter demselben Winkel* Der Druck kann ebenfalls ganz allgemein für jeden Punkt des betrachteten Raums angegeben werden. Wir haben nämlich

$$\Phi = \tfrac{1}{2}(u^2 + v^2) + \frac{p}{\mu} - G$$

und hieraus folgt, da

$$\tfrac{1}{2}(u^2 + v^2) = \left(1 + \frac{4\lambda^2 \sin^2\theta}{\kappa^2}\right)\left\{\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2\right\}$$

ist

$$p = const + \mu G - \mu\left(\kappa + \frac{4\lambda^2 \sin^2\theta}{\kappa}\right)\varphi - \frac{\mu}{2}\left(1 + \frac{4\lambda^2 \sin^2\theta}{\kappa^2}\right)\left\{\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2\right\}$$

wie Herr Oberbeck* abgeleitet hat.

Ganz eben so leicht kann die Windbahn angegeben werden. Die allgemeine Gleichung für die Windbahn

$$dx : dy : dz = u : v : w$$

wird in unserem Fall

$$\frac{dy}{dx} = \frac{v}{u} \quad d.\,h. = \frac{\frac{\partial\varphi}{\partial y} - K\frac{\partial\varphi}{\partial x}}{\frac{\partial\varphi}{\partial x} + K\frac{\partial\varphi}{\partial y}} \tag{51}$$

wenn man zur Abkürzung $\frac{2\lambda \sin\theta}{\kappa} = K$ setzt, und kann ganz allgemein integrirt werden. Man setze zu dem Ende

$$(x - \alpha) = \rho \cos\chi$$
$$(y - \beta) = \rho \sin\chi$$

so dass

$$\rho = \sqrt{(x - \alpha)^2 + (y - \beta)^2}$$

ist, und führe dieselben in die Gleichung (51). Es ist zunächst

$$\frac{\partial\varphi}{\partial x} = \frac{\partial\varphi}{\partial\rho}\cos\chi - \frac{\partial\varphi}{\partial\chi}\frac{\sin\chi}{\rho} \qquad dx = \cos\chi\, d\rho - \rho\sin\chi\, d\chi$$

$$\frac{\partial\varphi}{\partial y} = \frac{\partial\varphi}{\partial\rho}\sin\chi + \frac{\partial\varphi}{\partial\chi}\frac{\cos\chi}{\rho} \qquad dy = \sin\chi\, d\rho + \rho\cos\chi\, d\chi.$$

so verwandelt sich die Gleichung (51), in dem wir diese einführen in:

$$d\rho\left(\sin\chi\cos\chi\frac{\partial\varphi}{\partial\rho} - \frac{\partial\varphi}{\partial\chi}\frac{\sin^2\chi}{\rho} + K\frac{\partial\varphi}{\partial\rho}\sin^2\chi + K\frac{\partial\varphi}{\partial\chi}\frac{\cos\chi\sin\chi}{\rho}\right) + (f.\,S.)$$

* o. a. Ort. pag. 136.

$$+d\chi\left(\rho\frac{\partial\varphi}{\partial\rho}cos^2\chi-\frac{\partial\varphi}{\partial\chi}sin\chi\,cos\chi+K\frac{\partial\varphi}{\partial\rho}sin\,\chi\,cos\,\chi\,\rho+K\frac{\partial\varphi}{\partial\chi}cos^2\chi\right)$$

$$=d\rho\left(\rho\,sin\chi\,cos\chi\frac{\partial\varphi}{\partial\rho}+\frac{\partial\varphi}{\partial\chi}\frac{cos^2\chi}{\rho}-K\frac{\partial\varphi}{\partial\rho}cos^2\chi+K\frac{\partial\varphi}{\partial\chi}\frac{cos^2\chi\,sin^2\chi}{\rho}\right)$$

$$+d\chi\left(-\rho\frac{\partial\varphi}{\partial\rho}sin^2\chi-sin\,\chi\,cos\,\chi\frac{\partial\varphi}{\partial\chi}+K\frac{\partial\varphi}{\partial\rho}sin\,\chi\,cos\,\chi\cdot\rho-K\frac{\partial\varphi}{\partial\chi}sin^2\chi\right)$$

d. h. $$d\rho\left(-\frac{\partial\varphi}{\partial\chi}\frac{1}{\rho}+K\frac{\partial\varphi}{\partial\rho}\right)+d\chi\left(\rho\frac{\partial\varphi}{\partial\rho}+K\frac{\partial\varphi}{\partial\chi}\right)=0$$

oder $$K\left(\frac{\partial\varphi}{\partial\rho}d\rho+\frac{\partial\varphi}{\partial\chi}d\chi\right)=\frac{\partial\varphi}{\partial\chi}\frac{d\rho}{\rho}-\rho\frac{\partial\varphi}{\partial\rho}d\chi. \tag{52}$$

Die linke Seite ist ein totales Differential einer Function φ und dass die rechte Seite auch ein solches ist, lässt sich leicht nachweisen. Man bilde $\frac{\partial^2\varphi}{\partial x^2}$, $\frac{\partial^2\varphi}{\partial y^2}$, indem man diese in Polarcoordinaten ausdrückt. Man findet

$$\frac{\partial^2\varphi}{\partial x^2}=\frac{\partial^2\varphi}{\partial\rho^2}cos^2\chi-2\frac{\partial^2\varphi}{\partial\rho\partial\chi}\frac{sin\,\chi\,cos\,\chi}{\rho}+\frac{\partial\varphi}{\partial\rho}\frac{sin^2\chi}{\rho}$$

$$+\frac{2}{\rho^2}\frac{\partial\varphi}{\partial\chi}sin\,\chi\,cos\,\chi+\frac{1}{\rho^2}\frac{\partial^2\varphi}{\partial\chi^2}sin^2\chi$$

$$\frac{\partial^2\varphi}{\partial y^2}=\frac{\partial^2\varphi}{\partial\rho^2}sin^2\chi+2\frac{\partial^2\varphi}{\partial\rho\partial\chi}\frac{sin\,\chi\,cos\,\chi}{\rho}+\frac{\partial\varphi}{\partial\rho}\frac{cos^2\chi}{\rho}$$

$$-\frac{2}{\rho^2}\frac{\partial\varphi}{\partial\chi}sin\,\chi\,cos\,\chi+\frac{1}{\rho^2}\frac{\partial^2\varphi}{\partial\chi^2}sin^2\chi$$

Mithin entsteht durch Addition

$$\Delta\varphi=\frac{\partial^2\varphi}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\varphi}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\varphi}{\partial\chi^2}=0$$

Dies lässt sich auch so schreiben.

$$\rho\frac{\partial\rho\frac{\partial\varphi}{\partial\rho}}{\partial\rho}+\frac{\partial^2\varphi}{\partial\chi^2}=0$$

oder auch

$$\frac{\partial \frac{\partial \varphi}{\partial \log \rho}}{\partial \log \rho} + \frac{\partial^2 \varphi}{\partial X^2} = 0$$

d. h.

$$\frac{\partial^2 \varphi}{\partial \log \rho^2} = -\frac{\partial^2 \varphi}{\partial X^2} \tag{53}$$

Die Gleichung (52) lässt sich nun auch so schreiben.

$$K d\varphi = \frac{\partial \varphi}{\partial X} d \log \rho - \frac{\partial \varphi}{\partial \log \rho} d X \tag{54}$$

Wenn nun die rechte Seite ein totales Differential ist, so lässt sich jederzeit eine Function U finden, welche die Bedingung befriedigt

$$\frac{\partial \varphi}{\partial X} d \log \rho - \frac{\partial \varphi}{\partial \log \rho} d X = \frac{\partial U}{\partial \log \rho} d \log \rho + \frac{\partial U}{\partial X} d X$$

d. h.

$$\frac{\partial U}{\partial \log \rho} = \frac{\partial \varphi}{\partial X} \qquad \frac{\partial U}{\partial X} = -\frac{\partial \varphi}{\partial \log \rho}.$$

Eine solche Function U lässt sich aber auch finden, wenn φ die Bedingung erfüllt

$$\frac{\partial^2 U}{\partial \log \rho \, \partial X} = \frac{\partial^2 U}{\partial X \, \partial \log \rho}.$$

d. h.

$$\frac{\partial^2 \varphi}{\partial X^2} = -\frac{\partial^2 \varphi}{\partial \log \rho^2}$$

welche Bedingung φ in der That der Gleichung (53) zu Folge erfüllt.

Die Integration der Differentialgleichung.

$$K d\varphi = \frac{dU}{\partial \log \rho} d \log \rho + \frac{\partial U}{\partial X} dX = dU.$$

ergiebt unmittelbar als allgemeine Gleichung der Windbahn im Gebiete der wirbelfreien horizontalen Bewegung

$$K \varphi - U = const. \tag{54}$$

wie auch Herr Oberbeck mittelst der complexen Variabeln gefunden hat.

Die Function U lässt sich indessen leicht als eine von φ abhängige Integralfunction angeben.

Man setze

$$U = -\int_c^X \frac{\partial \varphi}{\partial \log \rho} dX.$$

und bestimme die willkürliche Constante c so, dass

$$\frac{\partial \varphi}{\partial X} \qquad \text{für } X = c$$

verschwindet.

Dann genügt dieser Ausdruck für U der gestellten Bedingung, der U genügen muss. Es ist

$$\frac{\partial U}{\partial \log \rho} = -\int_c^X \frac{\partial^2 \varphi}{\partial \log \rho^2} dX$$

hieraus folgt vermöge der Gleichung (53)

$$\frac{\partial U}{\partial \log \rho} = \int_c^X \frac{\partial^2 \varphi}{\partial X^2} dX = \frac{\partial \varphi}{\partial X}$$

Es ist ferner ohne Weiteres

$$\frac{\partial U}{\partial X} = -\frac{\partial \varphi}{\partial \log \rho}$$

Man sieht, dass der für U angenommene Ausdruck wirklich der gestellten Bedingung Genüge leistet.

Wir erhalten somit als allgemeine Gleichung der Windbahn im wirbelfreien Gebiet der Horizontalströmung

$$\int_c^X \frac{\partial \varphi}{\partial \log \rho} dX + \frac{2\lambda \sin \theta}{\kappa} . \varphi = const \qquad (55)$$

Diese Gleichung beruht indessen auf der Voraussetzung dass die Beschaffenheit der Function φ es gestatte, eine Constante c so zu finden, dass $\frac{\partial \varphi}{\partial X}$ für $X = c$ bei jedem Werthe von ρ verschwindet. Hat aber die Function φ nicht diese Beschaffenheit, so kann man allen gestellten Bedingungen genügen, indem man setzt

$$U = -\int_0^X \frac{\partial \varphi}{\partial \log \rho} dX + \int \left(\frac{\partial \varphi}{\partial X}\right)_{X=0} d \log \rho.$$

Es ist nämlich, da $\left(\frac{\partial \varphi}{\partial X}\right)_{X=0}$ eine Function von ρ allein ist,

$$\frac{\partial U}{\partial \log \rho} = -\int_0^X \frac{\partial^2 \varphi}{\partial \log \rho^2} + \left(\frac{\partial \varphi}{\partial X}\right)_{X=0}$$

oder

$$= \frac{\partial \varphi}{\partial X} - \left(\frac{\partial \varphi}{\partial X}\right)_{X=0} + \left(\frac{\partial \varphi}{\partial X}\right)_{X=0} = \frac{\partial \varphi}{\partial X}$$

und

$$\frac{\partial U}{\partial X} = -\frac{\partial \varphi}{\partial \log \rho}$$

wie oben. Die Gleichung der Windbahn in diesem Fall ist daher

$$\int_0^X \frac{\partial \varphi}{\partial \log \rho} dX - \int \left(\frac{\partial \varphi}{\partial X}\right)_{X=0} d \log \rho + \frac{2\lambda \sin \theta}{\kappa} \varphi = const. \qquad (55a)$$

Mittelst der Formeln (55) oder (55a) sind wir in den Stand gesetzt, für jede particuläre Lösung der Gleichung $\Delta \varphi = 0$ die Windbahn zu berechnen. Eine particuläre Lösung ist z. B.

$$\varphi = a(x^2 - y^2)$$

wo a eine Constante ist. Setzt man wieder Polarcoordinaten ein, so wird

$$\varphi = a\rho^2(\cos^2 X - \sin^2 X) = a\rho^2 \cos 2X.$$

dann ergiebt unsere allgemeine Gleichung.

$$2a\rho^2 \int_c^X \cos 2X dX + \frac{2\lambda \sin \theta}{\kappa} a\rho^2 \cos 2X = const.$$

c ist hier $=0$ zu setzen. Die Ausführung der Integration ergiebt

$$a\rho^2\left(\sin 2X + \frac{2\lambda \sin \theta}{\kappa} \cos 2X\right) = const.$$

oder indem man wieder zum rechtwinkligen Coordinatensystem zurückkehrt.

$$2xy + \frac{2\lambda \sin \theta}{\kappa}(x^2 - y^2) = \frac{const}{a}$$

d. h. eine Schaar Hyperbeln.

Wir wollen unsere Gleichung auf einen etwas complizirteren Fall anwenden, als der soeben betrachtete. Eine particuläre Lösung der Gleichung $\Delta\varphi = 0$ ist,

$$\varphi = \gamma_1 \log \rho_1 + \gamma_2 \log \rho_2$$

wo γ eine Constante ist, und

$$\rho_1 = \sqrt{(x - \alpha_1) + (y - \beta_1)^2} \qquad \rho_2 = \sqrt{(x - \alpha_2)^2 + (y - \beta_2)^2}$$

zu setzen ist. Diese Lösung entspricht dem Fall, in dem zwei wirberfüllte cylindrische Raumgebiete vorhanden sind, deren Mittelpuncte durch $\alpha_1\ \beta_1$ resp. $\alpha_2\ \beta_2$ bestimmt sind, und deren Dimension als unendlich klein gegen ihre gegenseitige Entfernung betrachtet werden darf. Um in diesem Fall die Windbahn zu finden, setzen wir

$$x - \alpha_1 = \rho_1 \cos X_1 \qquad x - \alpha_2 = \rho_2 \cos X_2$$
$$y - \beta_1 = \rho_1 \sin X_1 \qquad y - \beta_2 = \rho_2 \sin X_2$$

so dass

$$\varphi = \gamma_1 \log \rho_1 + \gamma_2 \log \rho_2.$$

Die Gleichung der Windbahn wird für diesen Fall

$$\gamma_1 \int d\, X_1 + \gamma_2 \int d\, X_2 + \gamma_1 K \log \rho_1 + \gamma_2 K \log \rho_2 = const.$$

d. i

$$\gamma_1 X_1 + \gamma_2 X_2 + K \log (\rho_1^{\gamma_1} \rho_2^{\gamma_2}) = const$$

wo $\dfrac{2\,\lambda \sin\theta}{\kappa}$ wieder $= K$ gesetzt worden ist.

Wesentlich einfach wird diese Gleichung, wenn wir den Coordinatenanfang in die Mitte des Abstandes der beiden Raumgebiete verlegen, so dass.

$$\alpha_1 = -\alpha_2 = \alpha$$
$$\beta_1 = -\beta_2 = 0$$

und die Annahme machen, dass die Geschwindigkeit der verticalen Strömung in den beiden Raumgebieten dieselbe Grösse habe; dann wird die obenstehende Gleichung für die Windbahn

$$X_1 + X_2 + K \log \rho_1 \rho_2 = const.$$

Wenn wir jetzt zum rechtwinkligen Coordinatensystem zurückkehren, mittelst der Gleichungen

$$arctag \frac{y}{x-\alpha} = X_1 \qquad arctag\left(\frac{y}{x+\alpha}\right) = X_2$$

so erhalten wir

$$arctag\left(\frac{2\,x\,y}{x^2-y^2-\alpha^2}\right) + K \log \sqrt{[y^2+(x-\alpha)^2]\,[y^2+(x+\alpha)^2]} = const$$

Die vollständige Discussion der in dieser Gleichung enthaltenen Spiralen ist mit grosser Schwierigkeit verbunden. Indessen kann man dieselben unschwer auf dem Weg der Construction erhalten, weil die Isodynamen $\Phi = const$ oder $\varphi = const$ *d. h.* $\rho_1\,\rho_2 = const.$ aus den Cassini'schen Curven bestehen, und die Windbahn vermöge der Gleichung (50*a*) dieselben under dem constanten Winkel *arct.K* schneiden muss. Die Figur 3 (Tafel XIII) veranschaulicht den ungefähren Verlauf der Windbahnen in diesem Fall

Eine andere particuläre Lösung der Gleichung $\Delta\,\varphi = 0$ ist.

$$\varphi = \gamma \log \rho_1 - \gamma \log \rho_2$$

wo ρ_1 und ρ_2 die nämliche Bedeutung haben, wie oben. Dieser Lösung entspricht der Fall, wo zwei gegen ihren gegenseitigen Abstand unendlich kleine cylindrische Raumgebiete vorhanden sind, von denen das eine von vertical aufsteigender und das andere von vertical niedersteigender Strömung gebildet ist. Die Gleichung der Isodynamen ist;

$$\varphi = const \;\; d.\,h.\, \frac{\rho_1}{\rho_2} = const.$$

eine Gleichung, welche einer Schaar Kreislinien entsprechen, die über dem Abstand zweier Punkte als Durchmesser beschrieben sind, welche zu den Punkten $\alpha_1\,\beta_1$ und $\alpha_2\,\beta_2$, *d. i.* zu den Mittelpunkten der beiden Gebieten der verticalen Strömung harmonisch liegen. Die Gleichung der Windbahn ist für diesen Fall.

$$X_1 - X_2 + K \log\left(\frac{\rho_1}{\rho_2}\right) = const$$

oder indem wir zum rechtwinkeligen Coordinatensysteme zurückkehren

$$arctag\left(\frac{2\alpha y}{x^2 + y^2 - \alpha^2}\right) + K \log \sqrt{\frac{y^2 + (x - \alpha)^2}{y^2 + (x + \alpha)^2}} = const.$$

Auch diese Spirale kann auf dem Wege der Construktion leicht gewonnen werden und ist in Fig. 4 (Tafel XIII) dargestellt.

Einen besonders in Bezug auf das Nächstzufolgende wichtigen Fall bildet die particuläre Lösung der Gleichung $\Delta\varphi = 0$

$$\varphi = -\frac{\gamma}{2} R^2 \log \rho. \qquad \rho = \sqrt{(x - \alpha)^2 + (y - \beta)^2} \tag{56}$$

ein Fall, wo nur ein Raumgebiet der Verticalströmung vorhanden ist, welche einen Kreiscylinder vom endlichen Radius R bildet, und mit der Geschwindigkeit $w = \gamma z$ je nach dem Varzeichen von γ emporsteigt, oder niederwärts stürzt. Wir wollen für diesen Fall die Rechung vollständig durchführen.

Als Componenten der Geschwindigkeit erhalten wir aus (49) ohnes Weiteres, da

$$\frac{\partial\varphi}{\partial x} = \frac{d\varphi}{d\rho}\frac{(x-\alpha)}{\rho} \qquad \frac{\partial\varphi}{\partial y} = \frac{d\varphi}{d\rho}\frac{(y-\beta)}{\rho}$$

ist

$$\left.\begin{aligned} u &= -\frac{\gamma}{2}\frac{R^2}{\rho^2}\left[(x-\alpha) + \frac{2\lambda \sin\theta}{\kappa}(y-\beta)\right] \\ v &= -\frac{\gamma}{2}\frac{R^2}{\rho^2}\left[(y-\beta) - \frac{2\lambda \sin\theta}{\kappa}(x-\alpha)\right] \end{aligned}\right\} \tag{57}$$

Die resultirende Geschwindigkeit ist daher

$$\omega = \frac{\gamma}{2}\frac{R^2}{\rho^2}\sqrt{\rho^2 + \frac{4\lambda^2 \sin^2\theta}{\kappa^2}\cdot\rho^2}$$
$$= \frac{\gamma}{2}\frac{R^2}{\rho}\sqrt{\left(1 + \frac{4\lambda^2 \sin^2\theta}{\kappa^2}\right)}\,*$$

* Bei Herrn Oberbeck steht statt des Factors $\frac{R^2}{\rho}$, $\frac{R^2}{\rho^2}$, was offenbar auf einem Druckfehler beruhen kann. Sieh. Oberbeck. a d. b. a Ort pag. 142. Gleichungen (29). Ebenso in seinem Ausdruck der Geschwindigkeitsresultante im inneren wirbelerfüllten Gebiete fehlt der Factor ρ [pag. 143. Gleichungen (30)].

Die durch diese Gleichungen dargestellte Bewegung des Lufttheilchens ist eine rotatorische, und ihre Richtung ist verschieden, je nach dem die Breite die nördliche oder südliche ist. Erinneren wir uns, dass wir bei der Ableitung der Fundamentalgleichungen die Coordinatenaxen so gelegt haben, dass die positive y—Achse gegen Osten und die positive x—Achse gegen Süden gekehrt ist, so werden u und v für $x = \alpha$ negativ *d. h.* die Componenten sind in diesem Ort nach West und N gerichtet. Für $y = \beta$ ist hingegen u negativ und v positiv; die Componenten der Windgeschwindigkeit nach N und O gerichtet und so fort mit der Bewegung in den beiden anderen Quadranten. Die Lufttheilchen kreisen demnach auf der nördlichen Hemisphäre in der Richtung $S\ O\ N\ W$, also cyklonal.

Auf der südlichen Hemisphäre ist $\sin\,\theta$ negativ zu setzen, so dass

$$u = -\frac{\gamma}{2}\frac{k^2}{\rho^2}\Big((x-\alpha) - \frac{2\lambda \sin\theta}{\kappa}(y-\beta)\Big)$$

$$v = -\frac{\gamma}{2}\frac{k^2}{\rho^2}\Big((y-\beta) + \frac{2\lambda \sin\theta}{\kappa}(x-\alpha)\Big)$$

u ist dann für $x = \alpha$ positiv, also nach S gerichtet und v hingeben negativ, hat desswegen die Richtung W. Für $y = \beta$ ist u negativ und auch v *d. h.* u ist nach N gerichtet, und v nach W und ihre Resultante ist daher $N\ W$ und so fort. Die Lufttheilchen kreisen daher um den Punkt $(\alpha\beta)$ in der Richtung $N\ O\ S\ W$, also in der entgegengesetzten Richtung wie auf der nördlichen Hemisphäre. Ist hingegen die Verticalströmung eine niedersteigende, so dass γ negativ zu nehmen ist, so ist die Bewegung der Lufttheilchen ausserhalb des Gebietes der Verticalströmung gerade eine umgekehrte:—die Lufttheilchen kreisen um den Mittelpunkt $(\alpha\ \beta)$ in der Richtung $N\ O\ S\ W$ auf der nördlichen Hemisphäre und auf der südlichen dagegen in der Richtung $S\ O\ N\ W$ *d. h.* anticylonal.

Als Windbahn erhält man unmittelbar aus der Gleichung (55),

indem man für φ ihren Werth einsetzt

$$X + \frac{2\,\lambda \sin\theta}{\kappa} \log\rho = const.$$

oder
$$\rho = const.\; e^{-\frac{\kappa}{2\lambda \sin\theta} X} \tag{58}$$

d. h. jedes Lufttheilchen beschreibt dabei eine logarithmische Spirale, und zwar eine, welche nach innen gewunden ist. Die Isodynamen sind in unserem Fall concentrische Kreise; denn ihre Gleichung ist

$$\Phi = \left(\kappa + \frac{4\,\lambda^2 \sin^2\theta}{\kappa}\right)\frac{\gamma R^2}{2} \log\rho = const. \tag{59}$$

Hieraus folgt als Gleichung für den Druck

$$p = const + \mu\left(\kappa + \frac{4\,\lambda^2 \sin^2\theta}{\kappa}\right)\frac{R\gamma^2}{2} \log\rho - \frac{\mu}{2}\left(1 + \frac{4\,\lambda^2 \sin^2\theta}{\kappa^2}\right)\frac{\gamma^2 R^4}{4\rho^2} \tag{60}$$

Die Isobaren $p = const.$ fallen demnach in diesem Fall mit den Isodynamen zusammen und auch sie werden überall von der Windbahn unter einem constanten Winkel geschnitten, dessen trigonometrische Tangente $= \frac{2\,\lambda \sin\theta}{\kappa}$ ist.

Es hat keinerlei Schwierigkeit, die Coordinaten eines Lufttheilchens als Functionen der Zeit darzustellen. Man fasse ein Lufttheilchen in's Auge, dessen Lage zu Zeit $t = 0$ durch die Polarcoordinaten $\rho_0\ X_0$ gegeben ist.

Weil nun

$$u = \frac{dx}{dt} \qquad\qquad v = \frac{dy}{dt}$$

oder indem man setzt; $(x-\alpha) = \rho \cos X \quad (y-\beta) = \rho \sin X$

$$u = \frac{d\rho}{dt}\cos X - \rho \sin X \frac{dX}{dt}$$

$$v = \frac{d\rho}{dt}\sin X + \rho \cos X \frac{dX}{dt}$$

so ist

$$u\,(x-\alpha) + v\,(y-\alpha) = \rho\,\frac{d\rho}{dt}$$

Mithin folgt durch Einsetzung der Werthe für u und v.

$$-\frac{\gamma}{2}R^2=\rho\frac{d\rho}{dt.}$$

d. h.

$$\rho=\sqrt{\rho_0-\gamma R^2 t.} \tag{61}$$

Es ist ferner.

$$v\,(x-\alpha)-(y-\beta)\,u=\rho^2\frac{dX}{dt.}$$

d. i

$$\frac{\gamma\,\lambda\sin\theta}{\kappa}R^2=\rho^2\frac{dX}{dt.}$$

oder wenn man für ρ^2 den oben ermittelten Werth einführt so kommt

$$X+X_0=\frac{\gamma\,\lambda\sin\theta}{\kappa}R^2\int_0^t\frac{dt}{(\rho_0^2-\gamma R^2 t)}$$

d. h.

$$X+X_0=-\frac{\lambda\sin\theta}{\kappa}\log\,(\rho_0^2-\gamma R^2 t) \tag{62}$$

womit die Bewegung der Luftheilchen in dem in Rede stehenden Falle in allen Stücken vollständig bestimmt worden ist.

§ *VII. Kreisförmige Cyklonen und Anticyklonen.*

Eine solche allgemeine Betrachtung, wie sie oben in Bezug auf das wirbelfreie Gebiet der horizontalen Strömung durchgeführt worden ist, lässt sich in Bezug auf das wirbelerfüllte Gebiet der verticalen Strömung nicht anstellen, wegen der Unmöglichkeit, die Differentialgleichung (42) allgemein zu integriren.

Der Fall, wo zunächst eine vollständige Lösung des Problems bemerkstelligt werden kann, ist derjenige, wo das wirbelerfüllte Raumgebiet der verticalen Strömung kreisförmig begrenzt ist, und das äussere Raumgebiet der Horizontalströmung entweder wirbelfrei oder von verticalen Wirbelfäden erfüllt ist.

Ich bemerke hier sogleich, dass ein Theil dieses Problems bereits von Herrn Oberbeck gelöst worden ist und dass die Formeln, zu denen er auf einem anderen Wege überhaupt gelangt ist, mit den von mir aus der Grundgleichung (42) entwickelten übereinstimmen.

Wir nehmen an: das Raumgebiet der verticalen Strömung bilde einen unendlich langen Kreiscylinder vom Radius R, sodass es durch die Gleichungen

$$(x-\alpha)^2+(y-\beta)^2=R^2 \qquad z=\infty$$

bestimmt wird. Die Luft soll in diesem cylindrischen Raum entweder emporsteigen mit der Geschwindigkeit

$$w=\gamma z.$$

oder niederwärts strömen mit der Geschwindigkeit

$$w=-\gamma z.$$

Sie soll aber ausserhalb dieses Raums in der Unendlichkeit ruhen so dass dort

$$u=v=0$$

ist.

Die Function φ ist sowohl für einen Punkt ausserhalb als innerhalb des Gebietes der verticalen Strömung bekannt. Es ist für einen inneren Punkt.

$$\varphi_i=-\frac{\gamma}{4}[R^2(2 \log R-1)+\rho^2]$$

wenn die Strömung vertical aufwärts, und

$$\varphi_i=\frac{\gamma}{4}[R^2(2 \log R-1)+\rho^2]$$

wenn sie vertical niederwärts geschieht. Für einen äusseren Punkt hat man den beiden Fällen entsprechend, entweder

$$\varphi_a=-\frac{\gamma}{2} R^2 \log \rho.$$

oder

$$\varphi_a=\frac{\gamma}{2} R^2 \log \rho$$

zu setzen, sodass

$$\left.\begin{aligned}\frac{\partial \varphi_i}{\partial n} &= \frac{\partial \varphi_i}{\partial \rho} = \pm \frac{\gamma}{2} R \\ \frac{\partial \varphi_a}{\partial n} &= \frac{\partial \varphi_a}{\partial \rho} = \pm \frac{\gamma}{2} R\end{aligned}\right\} \quad \text{für} \quad \rho = R$$

und folglich die eine Bedingung der Continuität der Bewegung in den beiden Raumgebieten

$$\frac{\partial \varphi_i}{\partial n} = \frac{\partial \varphi_a}{\partial n}$$

erfüllt ist.

Da nun die Luftbewegung rings um die Axe des cylindrischen Raumgebietes vollkommen symmetrisch ist, so kann sowohl W als ΔW offenbar als eine Funktion von ρ allein dargestellt werden, so dass also

$$\frac{\partial W}{\partial x} = \frac{dW}{d\rho} \frac{(x-\alpha)}{\rho} \qquad \frac{\partial W}{\partial y} = \frac{dW}{d\rho} \frac{(y-\beta)}{\rho}$$

wird. Die Differentialgleichungen (42) und (43) werden durch diesen Umstand sehr einfach.

Wir betrachten zunächst den Fall, wo die verticale Strömung vertical aufwärts geschieht, und bilden W zuerst für das äussere Raumgebiet. Die Differentialgleichung (43) nimmt in unserem Fall die Form an

$$\frac{d\Delta W}{d\rho} \frac{(x-\alpha)}{\rho} \left(\frac{dW}{d\rho} \frac{(y-\beta)}{\rho} + \frac{d\varphi}{d\rho} \frac{(x-\alpha)}{\rho} \right) + \frac{d\Delta W}{d\rho} \frac{(y-\beta)}{\rho} \left(\frac{d\varphi}{d\rho} \frac{(y-\beta)}{\rho} - \frac{dW}{d\rho} \frac{(x-\alpha)}{\rho} \right)$$
$$+ \kappa \Delta W = 0$$

oder,

$$\frac{d\Delta W}{d\rho} \left[\frac{(x-\alpha)^2 + (y-\beta)^2}{\rho^2} \right] \frac{d\varphi}{d\rho} + \kappa \Delta W = 0$$

d. h.

$$\frac{d\Delta W}{d\rho} \frac{d\varphi}{d\rho} + \kappa \Delta W = 0 \tag{63}$$

Da nun aber $\frac{d\varphi}{d\rho} = -\frac{\gamma}{2}\frac{R^2}{\rho}$ ist, so folgt.

$$-\frac{\gamma}{2}\frac{R^2}{\rho}\frac{d\Delta W}{d\rho} + \kappa \Delta W = 0$$

Die Integration ergiebt hieraus

$$\Delta W = Ce^{+\frac{\kappa\rho^2}{\gamma R^2}}$$

wo C die willkürliche Integrationsconstante ist.

Diese Lösung ist offenbar absurd, wenn C nicht verschwindet; denn; es geht aus dieser Gleichung hervor, dass ΔW, mithin auch W, folglich u und v in der unendlich grossen Entfernung vom Gebiet der verticalen Strömung unendlich gross werden müsste, was der Voraussetzung wiederspricht. Es muss daher $C=0$ gesetzt werden. *d. h.*

$$\Delta W = 0$$

ist die Lösung für einen äusseren Punkt.

Es können hiernach Luftwirbel wohl innerhalb des Gebietes der vertical aufsteigenden Strömung vorhanden sein, nicht aber ausserhalb desselben; denn die Gleichung $\Delta W = 0$ ist ja gleichbedeutend mit $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$. Hier entsteht daher nur eine wirbelfreie reine horizontale Strömung der Luft, und jedes Lufttheilchen würde geradlinig nach dem Gebiet der Verticalsttrömung fliessen, wenn die Erde nicht rotirte. Die Ausdrück (57) (58) (59) (60) (61) (62) gelten daher für die Luftbewegung in diesem äusseren Raumgebiet.

Die Differentialgleichung für das wirbelerfüllte innere Raumgebiet (42) wird in unserem Fall, indem man die Gleichung $\Delta\varphi = -\gamma$ berücksichtigt

$$\frac{d\Delta W}{d\rho}\frac{(x-\alpha)}{\rho}\left(\frac{dW}{d\rho}\frac{(y-\beta)}{\rho}+\frac{d\varphi}{d\rho}\frac{(x-\alpha)}{\rho}\right)+\frac{d\Delta W}{d\rho}\frac{(y-\beta)}{\rho}\left(\frac{d\varphi}{d\rho}\frac{(y-\beta)}{\rho}+\frac{dW}{d\rho}\frac{(x-\alpha)}{\rho}\right)$$
$$+ (\kappa - \gamma)\Delta W + 2\Delta\, sin\,\theta\gamma = 0$$

oder

$$\frac{d\Delta W}{d\rho}\frac{d\varphi}{d\rho} + (\kappa - \gamma)\Delta W + 2\lambda\, sin\,\theta.\gamma = 0 \qquad (64)$$

Da nun aber für das in Rede stehende Raumgebiet

$$\frac{d\varphi}{d\rho} = -\frac{\gamma}{2}\rho$$

so folgt die ebenfalls leicht integrirbare Gleichung

$$-\frac{\gamma}{2}\frac{d\Delta W}{d\rho}\rho + (\kappa - \gamma)\Delta W + 2\lambda \sin\theta.\gamma = 0 \qquad (65)$$

Das Integral hiervon ist

$$\Delta W = C\rho^{\frac{2(\kappa-\gamma)}{\gamma}} - \frac{2\lambda \sin\theta.\gamma}{(\kappa-\gamma)}$$

unter der Voraussetzung, dass $\kappa > \gamma$ ist. C ist eine willkürliche Constante, welche dazu verwendet werden kann, die Grenzbedingung, welcher W noch zu genügen hat, zu erfüllen. Um $\frac{dW}{d\rho}$ zu finden, bemerke man, dass

$$\Delta W = \frac{\partial^2 W}{dx^2} + \frac{\partial^2 W}{\partial y^2} = \frac{d^2 W}{d\rho^2} + \frac{1}{\rho}\frac{dW}{d\rho} = \frac{1}{\rho}\frac{d\rho\frac{dW}{d\rho}}{d\rho}$$

ist. Es folgt hieraus

$$\rho\frac{dW}{d\rho} = \int\left(C\rho^{\frac{2(\kappa-\gamma)}{\gamma}+1} - \frac{2\lambda\sin\theta.\gamma}{(\kappa-\gamma)}\rho\right)d\rho$$

d. h.

$$\frac{dW}{d\rho} = \frac{C\gamma}{2\kappa}\rho^{\frac{2\kappa}{\gamma}-1} - \frac{\lambda\sin\theta.\gamma\,\rho}{(\kappa-\gamma)} + \frac{C_1}{\rho}$$

die zweite Constante C_1 muss offenbar $=0$ gesetzt werden, da sonst u und v für jeden Werth von κ und γ im Mittelpunkt des Wirbels unendlich gross werden müsste. Mithin erhalten wir

$$\frac{dW}{d\rho} = \frac{C\gamma}{2\kappa}\rho^{\frac{2\kappa}{\gamma}-1} - \frac{\lambda\sin\theta.\gamma\,\rho}{(\kappa-\gamma)}$$

Die Integrationsconstante C bestimmt sich aus der Grenzbedingung

$$\frac{dW_i}{\partial n} = \frac{\partial W_a}{\partial n}$$

d. h.

$$\frac{dW_i}{d\rho} = \frac{\partial W_a}{d\rho} \qquad \text{für } \rho = R.$$

Weil, wie wir gesehen haben, ausserhalb dieses inneren Raumgebietes Wirbel nicht existiren können; so ist für das äussere Raumgebiet

$$W_a = \frac{2\,\lambda \sin\theta}{\kappa}\varphi_a$$

Mithin ist

$$\frac{dW_a}{d\rho} = \frac{2\,\lambda \sin\theta}{\kappa}\,\frac{d\varphi_a}{d\rho} = -\frac{\lambda \sin\theta.}{\kappa}\,\gamma.\,R. \qquad \text{für } \rho = R.$$

Die obenstehende Grenzbedingung ergiebt

$$\frac{C\gamma}{2\kappa}(R)^{\frac{2\kappa}{\gamma}-1} - \frac{\lambda \sin\theta}{(\kappa-\gamma)}.\,\gamma R = -\frac{\lambda \sin\theta}{\kappa}\gamma\,R.$$

voraus dann folgt.

$$C = \frac{2\,\lambda \sin\theta.\gamma}{(\kappa-\gamma)}\left(\frac{1}{R}\right)^{2\left(\frac{\kappa}{\gamma}-1\right)}$$

Wir erhalten somit schliesslich

$$\frac{dW_i}{d\rho} = -\frac{\lambda \sin\theta\gamma.\rho}{(\kappa-\gamma)}\left[1-\frac{\gamma}{\kappa}\left(\frac{\rho}{R}\right)^{2\left(\frac{\kappa}{\gamma}-1\right)}\right]$$

Das vorliegende Problem ist hiermit vollständig gelöst. Als Componenten der Geschwindigkeit in dem inneren Raumgebiet hat man aus (41*b*)

$$\left.\begin{aligned} u &= -\frac{\gamma}{2}\left([x-\alpha] + \frac{2\,\lambda \sin\theta}{(\kappa-\gamma)}\left[1-\frac{\gamma}{\kappa}\left(\frac{\rho}{R}\right)^{2\left(\frac{\kappa}{\gamma}-1\right)}\right][y-\beta]\right) \\ v &= -\frac{\gamma}{2}\left([y-\beta] - \frac{2\,\lambda \sin\theta}{(\kappa-\gamma)}\left[1-\frac{\gamma}{\kappa}\left(\frac{\rho}{R}\right)^{2\left(\frac{\kappa}{\gamma}-1\right)}\right][x-\alpha]\right) \\ w &= \gamma\, z. \end{aligned}\right\} \quad (66)$$

Die beiden ersten Gleichungen stellen eine rotatorische Bewegung der Luft dar, etwa einen von Piddington als Cyklonen bezeichneten Wirbelsturm: denn wenn wir die Bewegung eines Lufttheilchens näher in Bezug auf die vier Cardinalhimmelsgegenden verfolgen, so finden wir genau dasselbe Gesetz der Winddrehung wieder, welches

man als das den Cyklonen eigenthümlich bezeichnen kann; nämlich, dass die Wirbelbewegung der Luft auf der nördlichen Hemisphäre in dem Sinne *S O N W S* geschieht, und auf der südlichen Hemisphäre aber in dem entgegengesetzten Sinne *S W N O S*. Wir schreiben, um dieses einzusehen, die Ausdrücke für die Geschwindigkeitscomponenten im äusseren wirbelfreien Raumgebiet noch einmal hin:

$$\left.\begin{aligned} u &= -\frac{\gamma}{2}\frac{R^2}{\rho^2}\left[(x-\alpha) + \frac{2\lambda \sin\theta}{\kappa}(y-\beta)\right] \\ v &= -\frac{\gamma}{2}\frac{R^2}{\rho^2}\left[(y-\beta) - \frac{2\lambda \sin\theta}{\kappa}(x-\alpha)\right] \end{aligned}\right\} \quad (67)$$

und erinneren uns dabei wieder des Umstandes, dass wir bei der Herleitung dieser Gleichungen die Coordinaten so gelegt haben, dass die positive y—Axe gegen Osten, und die positive x Achse parallel dem Mittagskreise gerichtet ist. Wir bezeichnen weiter in dem Ausdrücke für die Geschwindigkeitscomponenten u und v in den beiden Gebieten den Factor von $(x - \alpha)$ mit a und denjenigen von $(y - \beta)$ mit b., so dass wir für beide Gebiete

$$u = -a(x-\alpha) - b(y-\beta)$$
$$v = -a(y-\beta) + b(x-\alpha)$$

schreiben können.

Nun werde $x - \alpha = 0$. Die Componenten der Geschwindigkeit in diesem Punkt sind

$$u = -b(y-\beta)$$
$$v = -a(y-\beta)$$

Die Geschwindigkeitscomponenten sind sonach für $y - \beta > 0$ nach *W* und *N* gerichtet; ihre Resultante selbst hat daher die Richtung *N W*. Für $(y - \beta) < 0$ werden u und v positiv, und ihre Resultante ist nach *S O* gerichtet. In dem Punkt $(y - \beta) = 0$ werden die Componenten

$$u = -a(x-\alpha)$$
$$v = +b(x-\alpha)$$

Für die Punkte $(x - \alpha) > 0$ und $(x - \alpha) < 0$ hat die Resultante dieser Geschwindigkeitscomponenten die Richtung $N\ O$, respectiv $S\ W$.

Jedes Lufttheilchen dreht sich sonach auf der nördlichen Hemisphäre um das Centrum des cylindrischen Wirbelraums im Sinne $S\ O\ N\ W\ S$, also von Rechts nach Links, im entgegengesetztem Sinne, wie der Uhrzeiger, ganz so, wie das bekannte Gesetz der Cyklonen es verlangt.

Dass auf der südlichen Hemisphäre die Drehungsgeschwindigkeit eines jeden Lufttheilchens in Bezug eine zur Erdoberfläche senkrechte Axe unter nämlichen Umständen entgegengesetzte Richtung haben muss, wie auf der nördlichen Hemisphäre, glaube ich bereits im Allgemeinen nachgewiesen zu haben. Demgemäss stellen die Gleichungen (66) und (67) in der That eine rotatorische Bewegung der Luft in dem Sinne $S\ W\ N\ O\ S$ dar, wenn man dieselben auf die südliche Hemisphäre anwendet. Hier hat man nämlich θ negativ zu nehmen, wobei so wohl die positive y—Achse, als die positive x—Axe ihre Richtung nach O und S unverändert beibehält. Da b in den oben stehenden Ausdrücken für die Geschwindigkeitscomponenten $\sin\ \theta$ als Factor enthält, so muss b auf der südlichen Hemisphäre negativ gesetzt wenden. Diese Bemerkung ergiebt für die südliche Hemisphäre.

$$u = -a(x - \alpha) + b(y - \beta)$$
$$v = -a(y - \beta) - b(x - \alpha)$$

Die durch diese Ausdrücke bestimmte Drehung erfolgt in der That in der entgegengesetzten Richtung, wie diejenige auf der nördlichen Hemisphäre. Es ist nämlich im Punkt $(x - \alpha) = 0$

$$u = +b(y - \beta)$$
$$v = -a(y - \beta)$$

Die Componente u ist nach Süden, oder Norden und v nach West, oder nach Ost gerichtet, je nachdem $y - \beta > 0$ oder < 0 ist; ihre

Resultante hat die Richtung nach S W respect N O. Für $(y-\beta)=0$ werden

$$u = -a(x-\alpha)$$
$$v = -b(x-\alpha)$$

Die Resultante dieser Componenten hat die Richtung nach N W respect. S O. je nachdem $(x-\alpha)>0$ oder <0 ist.

Jedes Lufttheilchen in der Cyklone kreist demnach auf der südlichen Hemisphäre in der Richtung S W N O S, in demselben Sinne, wie der Uhrzeiger, ganz so, wie das bekannte Dove'sche Gesetz es verlangt.

Die Gleichung für die Isodynamen für das äussere Gebiet haben wir bereits ermittelt; sie sind concentrische Kreise, wie die Isobaren. Die Gleichung für die Isodynamen für das innere wirbelerfüllte Gebiet lässt sich auch leicht bestimmen. Da

$$\frac{\partial \Delta W}{\partial y}\frac{\partial \varphi}{\partial x}+\frac{\partial \Delta W}{\partial x}\frac{\partial \varphi}{\partial y}=\frac{\partial \Delta W}{\partial \rho}\frac{d\varphi}{d\rho}\left(-\frac{(x-\alpha)(y-\beta)}{\rho^2}+\frac{(x-\alpha)(y-\beta)}{\rho^2}\right)=0$$

$$-\frac{\partial \Delta W}{\partial y}\frac{\partial W}{\partial y}-\frac{\partial \Delta W}{\partial x}\frac{\partial W}{\partial x}=-\frac{d\Delta W}{d\rho}\frac{dW}{d\rho}\left(\frac{(y-\beta)^2+(x-\alpha)^2}{\rho^2}\right)=-\frac{d\Delta W}{d\rho}\frac{dW}{d\rho}$$

ist, so verwandelt sich die Gleichung (44) in

$$\Delta\Phi-\frac{d\Delta W}{d\rho}\frac{dW}{d\rho}-(\Delta W-2\lambda\sin\theta)\Delta W-\kappa\gamma=0$$

Weil in unserem Fall die Bewegung der Luft überall symmetrisch ist in Bezug auf das Wirbelcentrum, so darf angenommen werden, dass auch Φ eine Function von ρ allein ist. Diese Bemerkung giebt

$$\Delta\Phi=\frac{1}{\rho}\frac{d\rho\frac{d\Phi}{d\rho}}{d\rho}$$

Mithin erhält man durch zweimalige Integration

$$\Phi=const+\frac{\kappa\gamma\rho^2}{4}+\int\frac{d\rho}{\rho}\int\rho d\rho\left(\frac{d\Delta W}{d\rho}\frac{dW}{d\rho}+\Delta W^2-2\lambda\sin\theta\Delta W\right)$$
$$+C\log\rho.$$

Die willkürliche Constante C muss offenbar $=0$ gesetzt werden, weil Φ sonst im Wirbelcentrum unendlich gross werden müsste. Das zweifache Integral lässt sich leicht ausführen. Wir betrachten zu dem Ende das Integral

$$\int \rho d\rho \frac{d\Delta W}{d\rho} \frac{dW}{d\rho} + \int \rho d\rho (\Delta W)^2$$

Indem wir das erste Integral partiell integriren, kommt

$$\int \rho d\rho \frac{d\Delta W}{d\rho} \frac{dW}{d\rho} = \Delta W \rho \frac{dW}{d\rho} - \int d\rho \Delta W \frac{d\rho \frac{dW}{d\rho}}{d\rho}$$

da aber

$$\Delta W = \frac{1}{\rho} \frac{d}{d\rho}\left(\rho \frac{dW}{d\rho}\right)$$

ist, so folgt

$$\int \rho d\rho \frac{d\Delta W}{d\rho} \frac{dW}{d\rho} = \Delta W \rho \frac{dW}{d\rho} - \int \rho d\rho (\Delta W)^2$$

Hieraus folgt weiter

$$\int \rho d\rho \left(\frac{d\Delta W}{d\rho} \frac{dW}{d\rho} + \Delta W^2 \right) = \Delta W \rho \frac{dW}{d\rho}$$

Mithin kommt

$$\Phi = const + \frac{\kappa.\gamma \rho^2}{4} + \int d\rho \Delta W \frac{dW}{d\rho} - 2\lambda \sin\theta \int \frac{d\rho}{\rho} \int \rho \Delta W d\rho.$$

Da ferner erstens.

$$\int \Delta W \rho d\rho = \int \frac{d\rho \frac{dW}{d\rho}}{d\rho} d\rho = \rho \frac{dW}{d\rho}$$

und zweitens

$$\Delta W = \frac{d^2 W}{d\rho^2} + \frac{1}{\rho} \frac{dW}{d\rho}$$

mithin

$$\Delta W \frac{dW}{d\rho} = \frac{1}{2} \frac{d\left(\frac{dW}{d\rho}\right)^2}{d\rho} + \frac{1}{\rho}\left(\frac{dW}{d\rho}\right)^2$$

so folgt schliesslich

$$\Phi = const + \frac{\kappa\gamma\rho^2}{4} - 2\lambda \sin\theta . W + \frac{1}{2}\left(\frac{dW}{d\rho}\right)^2 + \int \frac{d\rho}{\rho}\left(\frac{dW}{d\rho}\right)^2 \tag{68}$$

In unserem Fall ist

$$W = const - \frac{\lambda \sin\theta . \gamma}{(\kappa - \gamma)} \int \rho d\rho \left[1 - \frac{\gamma}{\kappa}\left(\frac{\rho}{R}\right)^{2\left(\frac{\kappa}{\gamma}-1\right)}\right]$$

d. i.

$$= const - \frac{\lambda \sin\theta \, \gamma \rho^2}{2(\kappa-\gamma)} \left[1 - \left(\frac{\gamma}{\kappa}\right)^2 \left(\frac{\rho}{R}\right)^{2\left(\frac{\kappa}{\gamma}-1\right)}\right]$$

oder indem man $\dfrac{\kappa}{\gamma} = m$ setzt

$$W = const - \frac{\lambda \sin\theta \rho^2}{2(m-1)}\left[1 - \frac{1}{m^2}\left(\frac{\rho}{R}\right)^{2m-2}\right]$$

und

$$\int \frac{d\rho}{\rho}\left(\frac{dW}{d\rho}\right)^2 = \frac{\lambda^2 \sin^2\theta \rho^2}{(m-1)^2}\left[\frac{1}{2} - \frac{1}{m^2}\left(\frac{\rho}{R}\right)^{2m-2} + \frac{1}{2m^2(2m-1)}\left(\frac{\rho}{R}\right)^{4m-4}\right]$$

Hieraus berechnet sich als Gleichung der Isodynamen

$$\Phi = const + \frac{\kappa\gamma\rho^2}{4} + \frac{\lambda^2 \sin^2\theta \rho^2}{m(m-1)^2}\left[m^2 - 2\left(\frac{\rho}{R}\right)^{2m-2} + \frac{1}{(2m-1)}\left(\frac{\rho}{R}\right)^{4m-4}\right] \tag{69}$$

auch diese Gleichung stimmt mit der von Herrn Oberbeck gefundenen überein.*

Die Isodynamen sind demnach concentrische Kreise, wie die Isobaren. Nennt man die resultirende Geschwindigkeit wieder ω., so ist in dem inneren Gebiet

$$\omega^2 = u^2 + v^2 + w^2$$

$$= \frac{\gamma^2}{4}\rho^2\left(1 + \frac{4\lambda^2 \sin^2\theta}{(\kappa-\gamma)^2}\left[1 - \frac{\gamma}{\kappa}\left(\frac{\rho}{R}\right)^{2\left(\frac{\kappa}{\gamma}-1\right)}\right]^2\right) + \gamma^2 z^2$$

da nun

$$\Phi = \frac{\omega^2}{2} - G + \frac{p}{\mu}.$$

* Oberbeck. a. o. a. O. pag. 144. Gleichung (31).

ist, folgt als Gleichung für den Luftdruck im inneren Gebiete

$$p = const + G - \frac{\mu\gamma^2}{2}z^2 + \mu\gamma^2\frac{\rho^2}{8}(2m-1) + \mu\frac{\lambda^2 \sin^2\theta\rho^2}{2m(m-1)^2}\Big[m(2m-1) + \frac{1}{m(2m-1)}\Big(\frac{\rho}{R}\Big)^{4m-4} - 2\Big(\frac{\rho}{R}\Big)^{2m-2}\Big] \qquad (70)$$

Die Gleichung für den Deviationswinkel findet man leicht aus (46) Man berechne zu dem Ende ζ *d. h.* $2\lambda \sin\theta - \Delta W$

Da

$$\Delta W = -\frac{2\lambda\sin\theta}{(\kappa-\gamma)}\Big[1 - \Big(\frac{\rho}{R}\Big)^{2(\frac{\kappa}{\gamma}-1)}\Big]$$

ist, so folgt

$$\zeta = 2\lambda\sin\theta\Big[1 + \frac{\gamma}{\kappa-\gamma} - \frac{\gamma}{\kappa-\gamma}\Big(\frac{\rho}{R}\Big)^{2(\frac{\kappa}{\gamma}-1)}\Big]$$

$$= \frac{2\lambda\sin\theta.\kappa}{(\kappa-\gamma)}\Big[1 - \frac{\gamma}{\kappa}\Big(\frac{\rho}{R}\Big)^{2(\frac{\kappa}{\gamma}-1)}\Big]$$

Die Substitution dieses Ausdrucks im (46) ergiebt

$$tag\ i = \frac{2\lambda\sin\theta}{(\kappa-\gamma)}\Big[1 - \frac{\gamma}{\kappa}\Big(\frac{\rho}{R}\Big)^{2(\frac{\kappa}{\gamma}-1)}\Big] \qquad (66a)$$

Man kann im Übrigen zu derselben Gleichung durch die Überlegung gelangen, dass die Isodynamen in dem in Rede stehenden Fall concentrische Kreise sind. Wegen dieses Umstandes hat man für den Deviationswinkel i die Gleichung

$$u(x-\alpha) + v(y-\beta) = \rho\sqrt{(u^2+v^2)}\cos i$$

da entlang der Erdoberfläche $w = \gamma z$ verschwindet, folglich

$$tag\ i = \sqrt{\frac{1}{\cos^2 i} - 1} = \sqrt{\frac{\rho^2(u^2+v^2) - [u(x-\alpha)+v(y-\beta)]^2}{u(x-\alpha)+v(y-\beta)}}$$

Hieraus berechnet sich für das wirbelerfüllte innere Gebiet

$$tag\ i = \frac{2\lambda\sin\theta}{(\kappa-\gamma)}\Big[1 - \frac{\gamma}{\kappa}\Big(\frac{\rho}{R}\Big)^{2(\frac{\kappa}{\gamma}-1)}\Big]$$

wie oben, während für das wirbelfreie äussere Gebiet sich ergiebt

$$tag\ i = \frac{2\lambda\sin\theta}{\kappa}.$$

Um die Bewegung der Luft vollständig zu bestimmen, bleibt noch übrig, die Gleichung der Windhahn im Wirbelgebiet abzuleiten, und schliesslich den Ort eines Lufttheilchens, dessen Coordinaten zur Zeit $t = 0$ $x_0\, y_0\, z_0$ waren, zur Zeit t zu bestimmen. Auch dieses alles hat nicht die mindeste Schwierigkeit.

Die Gleichung der Horizontalprojection der Windbahn erhält man durch Integration der Gleichung

$$u\, d\, y = v\, d\, x. \tag{71}$$

Wir führen zu dem Ende die Polarcoordinaten ρ, χ ein, und bemerken sogleich, dass χ, der Azimuthwinkel der Windbewegung, in *dem* Sinne wachsend genommen werden muss, wie die Rotationsgeschwindigkeit des Lufttheilchens gerichtet ist, so dass χ in unserem Falle auf der nördlichen Hemisphäre positiv, aber auf der südlichen Hemisphäre negativ genommen werden muss. Man setze.

$$(x - \alpha) = \rho \cos \chi$$

$$(y - \beta) = \rho \sin \chi$$

Dann erhält man für das Wirbelgebiet aus (71)

$$\frac{2\, \lambda \sin \theta}{(\kappa - \gamma)} \left[1 - \frac{\gamma}{\kappa}\left(\frac{\rho^2}{R}\right)^{\left(\frac{\kappa}{\gamma} - 1\right)}\right] d\, \rho + \rho\, d\, \chi = 0$$

Mithin folgt durch Integration, als Gleichung der Windbahn

$$\frac{2\, \lambda \sin \theta}{(\kappa - \gamma)} \left[\log \rho - \frac{\gamma^2}{2\kappa(\kappa - \gamma)}\left(\frac{\rho^2}{R}\right)^{\left(\frac{\kappa}{\gamma} - 1\right)}\right] + \chi = const.$$

d. h. die Horizontalprojection der Windbahn im Wirbelgebiet ist eine links nach innen gewundene transcendente Spirale. Die Fig. 5 (Tafel XIII) stellt den ungefähren Verlauf der Windbahn in diesem Fall dar.

Der Ort eines Lufttheilchens $(x_0\ y_0\ z_0)$ zur Zeit t lässt sich ebenfalls leicht finden. Man bilde

$$(x - \alpha)u + (y - \beta)v = (x - \alpha)\frac{dx}{dt} + (y - \beta)\frac{dy}{dt}$$

und führe die Polarcoordinaten ein, so findet man

$$\rho = \rho_0 e^{-\frac{\gamma}{2}t} \tag{72}$$

Führt man ferner die Polarcoordinaten in die Gleichung

$$(x-\alpha)v - (y-\beta)u = (x-\alpha)\frac{dy}{dt} - (y-\beta)\frac{dx}{dt}$$

ein, so kommt.

$$\rho^2 \frac{d\chi}{dt} = \Big[(x-\alpha)^2 + (y-\beta)^2\Big]\frac{\gamma}{2}\Big[1 - \frac{\gamma}{\kappa}\Big(\frac{\rho}{R}\Big)^{2\left(\frac{k}{\gamma}-1\right)}\Big]\frac{2\lambda \sin\theta}{(\kappa-\gamma)}$$

oder, indem man den für ρ bereits aufgefundenen Ausdruck einsetzt und integrirt

$$\chi + \chi_0 = \Big[\frac{\gamma t}{2} - \frac{\gamma^2}{2\kappa}\Big(\frac{\rho_0}{R}\Big)^{2\left(\frac{k}{\gamma}-1\right)}\int_0^t e^{-(k-\gamma)t}\Big]\frac{2\lambda \sin\theta}{(\kappa-\gamma)}$$

d. h.

$$= \frac{\gamma\lambda \sin\theta}{(\kappa-\gamma)}\Big[t + \frac{\gamma}{\kappa(\kappa-\gamma)}\Big(\frac{\rho_0}{R}\Big)^{2\left(\frac{k}{\gamma}-1\right)} e^{-(k-\gamma)t}\Big] \tag{73}$$

Entsprechende Ausdrücke für das äussere wirbelfreie Gebiet haben wir bereits aufgestellt.

Wenn wir nun noch den aus der Gleichung

$$w = \frac{dz}{dt} = \gamma . z.$$

fliessenden Ausdruck

$$z = z_0 e^{\gamma t}.$$

hinzuschreiben, so ist auch die Bewegung der Luft innerhalb eines kreisförmigen Wirbelgebietes in allen Stücken vollständig bestimmt und die Bewegung selbst lässt sich etwa, wie folgt, charakterisiren: In einem kreisförmig begrenzten Gebiete auf der als eine unendliche Ebene gedachten Erdoberfläche strömt die Luft in Folge der stärkeren Erwärmung mit der Geschwindigkeit $w = \gamma z$ vertical aufwärts. Um dadurch entstandene Druckdifferenz auszugleichen, stürzen die Lufttheilchen der Umgebung von allen Seiten herein und

veranlassen dadurch längs der Erdoberfläche eine Strömung parallel derselben. Die Bahnen, auf denen die Lufttheilchen von Unendlichkeit her mit wachsender Geschwindigkeit gegen das Centrum des Wirbelgebietes fliessen, sind logarithmische Spiralen und schneiden die Isobaren, wie den Kreisrand des Wirbelgebietes unter einem constanten Winkel [Gleich. (50*a*)] wo ihre Geschwindigkeit den im äusseren Raumgebiete grösstmöglichen Werth $\sqrt{u^2+v^2} = \frac{\gamma R}{2}\sqrt{\left(1+\frac{4\lambda^2 \sin^2\theta}{\kappa^2}\right)}$ erreicht.

Unmittelbar da, wo die Lufttheilchen in das Wirbelgebiet eintritt, werden sie in Folge der dort herrschenden Verticalströmung emporgerissen, und wirbeln hinauf Schraubenbahn beschreibend, deren horizontale Projectionen eine transcendende Spiralen sind, welche die Eigenschaft hat, dass ihre Windungen mit abnehmender Entfernung vom Wirbelcentrum im viel stärkeren Verhältniss zunimmt, als bis bei der einfachen logarithmischen Spirale, weil die Windbahn die Concentrischen Kreise unter um so kleineren Winkel schneidet, je kleiner der Abstand vom Centrum wird. Im Centrum selbst wird der Deviationswinkel Maximum zwar

$$tag\, i = \frac{2\lambda \sin\theta}{(\kappa-\gamma)}$$

so dass i sich um so mehr $\frac{\pi}{2}$ nährt, je kleiner $\kappa-\gamma$ wird. Was die Geschwindigkeit anbetrifft, so nimmt ihre Horizontalcomponente immur mehr ab, je näher das Centrum liegt, vorausgesetzt selbstverständlich, dass $\kappa > \gamma$ sei., und im Centrum selbst verschwindet sie ganz, wobei der Druck seinen minimalen Werth $const + G - \frac{\mu\gamma^2 z^2}{2}$ erreicht.

Es war bisher vorausgesetzt, dass $\kappa > \gamma$ sei. Wir finden aber dass γ über einen gewissen Grenzwerth nicht hinaufsteigen darf, *d. h.* soll die durch die Gleichungen (66) u. s. w. dargestellte Bewegung

eintreten, so muss die Geschwindigkeit der aufsteigenden Strömung innerhalb eines gewissen durch den Reibungswiderstand bedingten Grenzwerthes bleiben. Wird nun $\kappa = \gamma$, so werden die Ausdrücke für die Geschwindigkeitscomponenten im Wirbelgebiete unendlich gross. Wird dagegen $\kappa < \gamma$, so wurden sie im Allgemeinen im Wirbelcentrum unendlich gross. Das Verhältniss zwischen der Geschwindigkeit der verticalaufsteigenden Strömung und dem Reibungswiderstand der Erdoberfläche *d. h.* das Verhältniss $\frac{\kappa}{\gamma}$ muss innerhalb eines gewissen Grenzwerthes bleiben, damit die Voraussetzung der Continuität der Bewegung erfüllt bleibe.

Wir fassen zunächst den Fall in's Auge, wo $\kappa = \gamma$ wird und gehen dabei von der Gleichung (61) aus. Diese wird unter dem gedachten Umstande

$$\frac{d\Delta W}{d\rho} \cdot \frac{d\varphi}{d\rho} = -2\lambda \sin\theta . \gamma,$$

oder

$$\frac{\gamma}{2} \frac{d\Delta W}{d\rho} \cdot \rho = 2\lambda \sin\theta . \gamma$$

Das Integral hiervon ist

$$\Delta W = 4\lambda \sin\theta \log\rho + C$$

wo C eine willkürliche Constant ist. Eine nochmalige Integration ergiebt

$$\frac{dW}{d\rho} = 2\lambda \sin\theta\, \rho \log\rho - \lambda \sin\theta\, \rho + \frac{C\rho^2}{2}$$

Diese Function wird im Wirbelcemtrum ($\rho = 0$) nicht unendlich gross; denn $\log\rho$ ist zwar bei verschwindem ρ negativ unendlich gross, aber unendlich klein gegen $\frac{1}{\rho}$, so dass

$$\lim (\rho \log \rho) = \lim \left(\frac{\log\rho}{\frac{1}{\rho}} \right)$$

verschwindet. Der Fall $\kappa = \gamma$ muss daher noch eine mögliche

stationäre Bewegung darstellen.

Die Function W bleibt für das äussere wirbelfreie Gebiet unverändert; sie ist hier wiederun zu setzen

$$W_a = -\frac{2\,\lambda\,sin\,\theta}{\kappa}\frac{\gamma}{2}R^2\,log\,\rho.$$

Die Bedingung, welche an dem Randkreis des Wirbelgebietes zu erfüllen ist, ergiebt

$$C = -\,4\,\lambda\,sin\,\theta\,log\,(R).$$

Es folgt hieraus

$$\frac{d\,W}{d\,\rho} = -\,\lambda\,sin\,\theta\,\rho\left[1-2\,log\left(\frac{\rho}{R}\right)\right]$$

und

$$W = const - \lambda\,sin\,\theta\,\rho^2\left[1-log\left(\frac{\rho}{R}\right)\right]$$

Wir erhalten somit als Componenten der Geschwindigkeit

$$u = -\,\frac{\gamma}{2}(x-\alpha)-\lambda sin\,\theta\left[1-2\,log\left(\frac{\rho}{R}\right)\right](y-\beta)$$

$$v = -\,\frac{\gamma}{2}(y-\beta)+\lambda sin\,\theta\left[1-2\,log\left(\frac{\rho}{R}\right)\right](x-\alpha)$$

Als Deviationswinkel findet man für diesen Fall, da $\kappa = \gamma$ ist

$$tag\,i = \frac{2\,\lambda\,sin\,\theta}{\kappa}\left[1-2\,log\left(\frac{\rho}{R}\right)\right]$$

Am Begrenzungskreis des Wirbelgebietes wird dieses

$$tag\,i = \frac{2\,\lambda\,sin\,\theta}{\kappa}$$

und im Wirbelcentrum

$$tag\,i = \infty\ d.\,h.\quad i = \frac{\pi}{2}$$

d. i. am Wirbelcentrum schneidet die Windbahn die Isodynamen gar nicht; jedes Theilchen muss daher dort in kreisförmiger Rotation begriffen sein. Als Gleichung der Windbahn findet man

$$\chi + const + \frac{2\,\lambda\,sin\,\theta}{\kappa}\int\left(1-2\,log\frac{\rho}{R}\right)\frac{d\rho}{\rho} = 0$$

d. h.

$$\chi + const + \frac{2\lambda \sin\theta}{\kappa} \log\rho\left(1 - \log\frac{\rho}{R}\right) = 0$$

oder wenn man über die willkürliche Constante passend verfügt.

$$\rho = const.\ e^{\pm\sqrt{\frac{\kappa\chi}{2\lambda\sin\theta}}}$$

also eine Art logarithmische Spirale. Von den beiden Vorzeichen des Radicandus ist das negative zu wählen.

Betrachten wollen wir noch den Fall, wo $\kappa < \gamma$ ist. In diesem Fall erhält man unmittelbar aus (66) und (66*a*)

$$u = -\frac{\gamma}{2}\left[(x-\alpha) + \frac{2\lambda\sin\theta}{(\gamma-\kappa)}\left\{\frac{\gamma}{\kappa}\left(\frac{R}{\rho}\right)^{2\left(1-\frac{\kappa}{\gamma}\right)} - 1\right\}(y-\beta)\right]$$

$$v = -\frac{\gamma}{2}\left[(y-\beta) - \frac{2\lambda\sin\theta}{(\gamma-\kappa)}\left\{\frac{\gamma}{\kappa}\left(\frac{R}{\rho}\right)^{2\left(1-\frac{\kappa}{\gamma}\right)} - 1\right\}(x-\alpha)\right]$$

und als Deviationswinkel

$$tag\ i = \frac{2\lambda\sin\theta}{(\gamma-\kappa)}\left[\frac{\gamma}{\kappa}\left(\frac{R}{\rho}\right)^{2\left(1-\frac{\kappa}{\gamma}\right)} - 1\right]$$

und als Gleichung der Windbahn

$$\chi + \frac{2\lambda\sin\theta}{(\gamma-\kappa)}\left[\frac{\gamma^2}{2\kappa(\gamma-\kappa)}\left(\frac{R}{\rho}\right)^{2\left(1-\frac{\kappa}{\gamma}\right)} - \log\left(\frac{\rho}{R}\right)\right] = const.$$

Die Geschwindigkeit im Wirbelcentrum wird im Allgemeinen unendlich gross, wenn $\gamma > 2\kappa$ ist. Ist dagegen $\gamma < 2\kappa$, und dabei jedoch $\gamma > \kappa$, so ist $2\left(1 - \frac{\kappa}{\gamma}\right)$ ein echter Bruch, und die Grösse wie $\left(\frac{R}{\rho}\right)^{2\left(1-\frac{\kappa}{\gamma}\right)} \cdot \rho$ wird im Wirbelcentrum nicht unendlich gross, sondern unendlich klein wie $\rho^{1-2\left(1-\frac{\kappa}{\gamma}\right)} = \rho^{\frac{2\kappa}{\gamma}-1}$. Eine continuirliche Bewegung der Luftmasse ist daher möglich und die Horizontalprojection der Windbahn ist Spirale, deren Windungen mit abnehmendem Abstand vom Wirbelcentrum zunehmen und zwar um so schneller, je mehr $2\kappa - \gamma$ sich der Null nährt.

Überschreitet γ den oben angegebenen Grenzwerth, so bleibt die Continuität der Strömung an dem Begrenzungskreise der beiden Gebiete ungestört; allein die Geschwindigkeit wird in der Nähe des Wirbelcentrums unendlich gross, und der Druck dabei negativ unendlich gross [vergleiche Formel (70).] Da in den Ponderabilien, wie Luft und Wasser, der Druck bekanntlich unter einen gewissen negativen Grenzwerth nicht hinab sinken kann, ohne dass die Flüssigkeit zerreisst, so wird die Luftmasse, wenn $\gamma > 2\kappa$ ist, praktisch irgendwo in dem inneren Gebiete zerreissen, und um das Wirbelcentrum ein luftleerer oder wenigstens luftverdünnter Raum entstehen, dessen Dimension im Allgemeinen um so grösser sein wird, je grösser $\frac{\gamma}{2\kappa}$ ausfällt.

So entsteht in der Erdatmosphäre jedesmal eine wirbelnde Strömung der Luft im cyklonalen Sinne, wenn irgendwo in Folge der stärkeren Erwärmung eine verticale aufsteigende Strömung entsteht, und so in einer Fläche Druckdifferenz erzeugt.

Wir haben uns bisher mit dem Fall beschäftigt, wo auf der Erdoberfläche ein kreisförmig begrenztes Gebiet der vertical aufsteigenden Strömung vorhanden ist. Es liegt uns nun ob, einen anderen Fall zu untersuchen, wo irgendwo auf der wieder als eine unendliche Ebene gedachten Erdoberfläche ein durch den Kreiscylinder vom Radius R begrenztes Raumgebiet vorhanden ist, in welchem die Luft vertical niederwärts mit der Geschwindigkeit

$$w = -\gamma . z.$$

strömt.

Dieser Fall erfordert eine andere selbstständige Lösung, als der vorhergehende, was daraus hervorgeht, dass jeder für den Fall der

vertical aufsteigenden Strömung aufgestellte Ausdruck im Mittelpunkt des Gebietes der verticalen Strömung bei jedem Verhältniss von $\frac{\kappa}{\gamma}$ unendlich gross, darum unbrauchbar wird, wenn man überall statt γ, $-\gamma$ einführt.

Wir setzen wieder

$$u = \frac{\partial W}{\partial y} + \frac{\partial \varphi}{\partial x}$$

$$v = -\frac{\partial W}{\partial x} + \frac{\partial \varphi}{\partial y}$$

wo φ in dem vorliegenden Fall für das innere Raumgebiet

$$\varphi_i = \frac{\gamma}{4}[R^2(1 - 2 \log R) + \rho^2]$$

und für das äussere

$$\varphi_a = \frac{\gamma R^2}{2} \log \rho.$$

zu setzen ist, so dass die Gleichungen

$$\varDelta \varphi_i = \gamma$$

$$\varDelta \varphi_a = 0$$

befriedigt werden.

Um W_i zu bestimmen, gehen wir von der Gleichung (64) aus indem wir voraussetzen, dass auch in diesem Fall im inneren Raumgebiete Wirbel existirten, und setzen statt γ, $-\gamma$ ein, sodass wir zunächst für das innere Raumgebiet haben

$$\frac{d\varDelta W_i}{d\rho}\frac{d\varphi_i}{d\rho} + (\kappa + \gamma)\varDelta W_i - 2\lambda \sin \theta \gamma = 0$$

oder, da $\frac{d\varphi_i}{d\rho} = \frac{\gamma}{2}\rho.$ ist so folgt.

$$\frac{\gamma}{2}\frac{d\varDelta W_i}{d\rho}\rho + (\kappa + \gamma)\varDelta W_i - 2\lambda \sin \theta . \gamma = 0$$

Das Integral hiervon ist

$$\varDelta W_i = \frac{C}{\rho^{\frac{2(\kappa + \gamma)}{\gamma}}} + \frac{2\lambda \sin \theta}{(\kappa + \gamma)} \cdot \gamma.$$

Wäre nun die willkürliche Constante nicht $=0$, so müsste $\frac{dW_i}{d\rho}\rho$, mithin u und v für $\rho = 0$ *d. i.* im Wirbelcentrum bei jedem Werthe von γ unendlich gross sein. Es muss daher in dem vorliegenden Fall ganz allgemein $C = 0$ sein. Mithin folgt

$$\Delta W_i = \frac{2\lambda \sin\theta}{(\kappa+\gamma)} = -\zeta + 2\lambda \sin\theta$$

d. h.

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{2\lambda \sin\theta}{(\kappa+\gamma)}$$

Es erhellt hieraus, dass innerhalb des inneren Gebietes vom kreisförmigen Querschnitte nur Wirbel mit constanter Geschwindigkeit vorhanden sein können. Wir schreiben wieder $\Delta W = \frac{1}{\rho}\frac{d\rho\frac{dW}{d\rho}}{d\rho}$, und erhalten durch eine nochmalige Integration

$$\frac{dW_i}{d\rho} = \frac{\lambda \sin\theta \cdot \gamma'}{(\kappa+\gamma)} \cdot \rho.$$

die willkürliche Constante muss $=0$ sein, wenn $\frac{dW_i}{d\rho}$ im Wirbelmittelpunkt verschwinden soll.

Bezeichnet man die Function W für das äussere Gebiete wieder mit W_a, so muss an dem Grenzkreis die Bedingung erfüllt werden, dass

$$\frac{dW_a}{d\rho} = \frac{dW_i}{d\rho} \quad \text{für } \rho = R$$

Die für das äussere wirbelfreie Gebiet giltige Auflösung

$$W_a = \frac{2\lambda \sin\theta}{\kappa}\varphi_a$$

$$\varphi_a = \frac{\gamma}{2}R^2 \log \rho.$$

genügt aber dieser Bedingung nicht. Da ausserhalb des Gebietes der verticalen Strömung Wirbel mit constanter Geschwindigkeit nicht

existiren, so folgt, dass jetzt im Gebiet der Horizontalströmung Wirbel mit variabler Geschwindigkeit vorhanden sein müssen. Diese Bemerkung ergiebt aus (63) für unseren Fall die Differentialgleichung.

$$\kappa \Delta W_a + \frac{d\Delta W_a}{d\rho}\frac{d\varphi_a}{d\rho} = 0$$

oder, da $\dfrac{d\varphi_a}{d\rho} = \dfrac{\gamma}{2}\dfrac{R^2}{\rho}$

ist.

$$\kappa \Delta W_a + \frac{\gamma}{2}\frac{R^2}{\rho}\frac{d\Delta W}{d\rho} = 0$$

Eine Integration ergiebt hieraus

$$\Delta W = C e^{-\frac{\kappa}{\gamma}\left(\frac{\rho}{R}\right)^2} = -\zeta + 2\lambda \sin\theta.$$

d. h.

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = C e^{-\frac{\kappa}{\gamma}\left(\frac{\rho}{R}\right)^2} \tag{74}$$

Die Wirbel, welche im äusseren Raumgebiete existiren, müssen sonach mit wachsender Entfernung vom Grenzkreis sehr rasch verschwinden.

Durch eine nochmalige Integration kommt

$$\frac{dW_a}{d\rho} = \frac{C e^{-\frac{\kappa}{\gamma}\left(\frac{\rho}{R}\right)^2} + C'}{\rho}$$

C und C' sind die willkürlichen Integrationsconstanten, von denen die letztere dadurch bestimmt wird, dass die Wirbel mit wachsendem ρ sehr rasch verschwinden und die Lufttheilchen in unendlicher Entfernung keine andere Rotationsgeschwindigkeit hat, als die durch Erdrotation hervorgerufene, *d. h.* für ein unendlich grosses ρ

$$\Delta W_a = 0 \qquad \frac{dW_a}{d\rho} = \frac{2\lambda \sin\theta}{\kappa}\frac{d\varphi_a}{d\rho}$$

Diese Bemerkung ergiebt

$$C' = \frac{\lambda \sin\theta . \gamma}{\kappa} R^2$$

so dass man erhält

$$\frac{dW_a}{d\rho} = \frac{Ce^{-\frac{\kappa}{\gamma}\left(\frac{\rho}{R}\right)^2}}{\rho} + \frac{\lambda \sin\theta\, \gamma\, R^2}{\kappa\, \rho}$$

Die zweite Constante C kann dazu benutzt werden, die Bedingung, welche an dem Grenzkreise des inneren Gebietes zu erfüllen ist, zu befriedigen, *d. h.*

$$\frac{dW_a}{d\rho} = \frac{dW_i}{d\rho} \qquad \text{für } \rho = R$$

Hieraus folgt

$$\frac{Ce^{-\frac{\kappa}{\gamma}\left(\frac{\rho}{R}\right)^2}}{R} + \frac{\lambda \sin\theta . \gamma\, R}{\kappa} = \frac{\lambda \sin\theta : \gamma}{(\kappa+\gamma)} R$$

mithin

$$C = -\frac{\lambda\, R^2 \sin\theta\, \gamma}{\kappa(\kappa+\gamma)} e^{\frac{\kappa}{\gamma}}$$

Wir erhalten schliesslich

$$\frac{dW_a}{d\rho} = \frac{\lambda \sin\theta\, R^2 \gamma}{\kappa . \rho}\left[1 - \frac{\gamma}{(\kappa+\gamma)} e^{-\frac{\kappa}{\gamma}\left[\left(\frac{\rho}{R}\right)^2 - 1\right]}\right]$$

eine Function, welche allen gestellten Redingungen Genüge leistet.

Als Componenten der Geschwindigkeit finden wir sonach für das innere Raumgebiet

$$\begin{aligned} u &= \frac{\gamma}{2}\left[(x-\alpha) + \frac{2\lambda \sin\theta}{(\kappa+\gamma)}(y-\beta)\right] \\ v &= \frac{\gamma}{2}\left[(y-\beta) - \frac{2\lambda \sin\theta}{(\kappa+\gamma)}(x-\alpha)\right] \end{aligned} \tag{75}$$

und für das äussere

$$\begin{aligned} u &= \frac{\gamma}{2}\frac{R^2}{\rho^2}\left[(x-\alpha) + \frac{2\lambda \sin\theta}{\kappa}\left(1 - \frac{\gamma}{(\kappa+\gamma)} e^{-\frac{\kappa}{\gamma}\left(\frac{\rho^2}{R^2}-1\right)}\right)(y-\beta)\right] \\ v &= \frac{\gamma}{2}\frac{R^2}{\rho^2}\left[(y-\beta) - \frac{2\lambda \sin\theta}{\kappa}\left(1 - \frac{\gamma}{(\kappa+\gamma)} e^{-\frac{\kappa}{\gamma}\left(\frac{\rho^2}{R^2}-1\right)}\right)(x-\alpha)\right] \end{aligned} \tag{76}$$

Die Bewegung, welche durch diese Gleichungen dargestellt ist, entspricht einer anticyklonalen Luftströmung. Die Lufttheilchen sind in Rotation begriffen, und zwar geht dieselbe auf der nördlichen Hemisphäre in der Richtung *W N O S* vor sich, und auf der südlichen Hemisphäre aber in der Richtung *W S O N*, also genan im umgekehrten Sinne, wie die Luftströmung in der Bewegungserscheinung, welche einer aufsteigenden Strömung ihre Entstehung verdankt.

Als Deviationswinkel findet man leicht für das innere Gebiet

$$tag\ i = \frac{2\,\lambda\, sin\, \theta}{(\gamma+\kappa)} \tag{77}$$

also constant, und für das äussere Gebiet aber

$$tag \quad = \frac{2\,\lambda\, sin\,\theta}{\kappa}\left[1-\frac{\gamma}{\kappa+\gamma}\,e^{-\frac{\kappa}{\gamma}\left(\frac{\rho^2}{R^2}-1\right)}\right] \tag{78}$$

Die Gleichung der Windbahn findet man gleichfalls leicht. Sie ist für das innere Gebiet

$$\rho = Ce^{\frac{\gamma+\kappa}{2\lambda\, sin\,\theta}\chi} \tag{79}$$

also eine nach Aussen gewundene logarithmische Spirale, deren Windungszahl im inneren Gebiet um so geringer wird, je grösser γ *d. h.* die Geschwindigkeit der niederstürzenden Strömung ist. Die Windbahn in dem äusseren Gebiete hat eine bei Weitem verwickeltere Gestalt; ihre Gleichung ist

$$\chi - \frac{2\,\lambda\, sin\,\theta}{\kappa}\left(log\,\rho - \frac{\gamma}{\kappa+\gamma}\int_R^\rho \frac{e^{-\frac{\kappa}{\gamma}\left(\frac{\rho^2}{R^2}-1\right)}d\rho}{\rho}\right) = const. \tag{80}$$

also jedenfalls eine nach Aussen gewundene Spirale, welche die Eigenschaft hat, dass ihre Windungen mit wachsender Entfernung einem Maximum asymptotisch zustreben, und die Spirale selbst sich in die gewöhnliche nach Aussen gewundene logarithmische Spirale verwandelt. In Fig. 6 (Tafel XIII) ist die Bahncurve in diesem Fall gezeichnet.

Es hat ferner nicht die mindeste Schwierigkeit die Coordinaten eines Lufttheilchens zur Zeit t anzugeben.

Man findet für das innere Gebiet

$$\rho = \rho_0 e^{\frac{\gamma}{2}t} \qquad \chi + \chi_0 = \frac{\lambda \sin\theta . \gamma}{(\kappa+\gamma)} t \tag{81}$$

und für das äussere Gebiet

$$(82) \begin{cases} \rho = \sqrt{\rho_0 + \frac{\gamma R^2 t}{2}} \\ \chi + \chi_0 = \frac{2\lambda \sin\theta}{\kappa} \left\{ \log\left(\rho_0^2 + \frac{\gamma R^2 t}{2}\right) - \frac{\gamma^2 R^2}{2(\kappa+\gamma)} e^{-\frac{\kappa}{\gamma}\left(\frac{\rho_0^2}{R^2}-1\right)} \int \frac{e^{-\frac{\kappa}{2}t}\, dt}{\left(\rho_0^2 + \frac{\gamma R^2 t}{2}\right)} \right\} \end{cases}$$

Die Isodynamen sind sowohl für das innere, als für das äussere Gebiet concentrische Kreise. Ihre Gleichung für das innere Gebiet findet man aus (68)

$$\Phi = const - \frac{\kappa\gamma\rho^2}{4}\left[1 + \frac{4\lambda^2 \sin^2\theta}{(\kappa+\gamma)^2}\right]$$

und für das äussere Gebiet

$$\Phi = const - \frac{2\lambda^2 \sin^2\theta R^2 \gamma}{\kappa}\left[\log\left(\frac{\rho}{R}\right) - \frac{\gamma}{\kappa+y}\int_R^\rho \frac{f(\rho)d\rho}{\rho}\right]$$
$$+ \frac{\lambda^2 \sin^2\theta R^4 \gamma^2}{\kappa^2}\left[\frac{1}{2\rho^2}\left(1 - \frac{\gamma}{\kappa+\gamma}f(\rho)\right)^2 + \int_R^\rho \left(1 - \frac{\gamma}{\kappa+\gamma}f(\rho)\right)^2 \frac{d\rho}{\rho^3}\right]$$

indem man zur Abkürzung

$$e^{-\frac{\kappa}{\gamma}\left(\frac{\rho^2}{R^2}-1\right)} = f(\rho)$$

setzt.

Hieraus fliesst weiter als die Gleichung für den Druck für das innere Gebiet

$$p_i = const + \mu G - \frac{\mu\gamma^2 z^2}{2} - \frac{\mu\gamma\rho^2}{4}\left(\kappa + \frac{\gamma}{2}\right)\left(1 + \frac{4\lambda^2 \sin^2\theta}{(\kappa+\gamma)^2}\right) \tag{83}$$

und für das äussere Gebiet

$$\begin{aligned} p_a = const + \mu G - 2\,\frac{\mu\,\lambda^2 \sin^2\theta R^2 \gamma}{\kappa}\left[\log\left(\frac{\rho}{R}\right) - \frac{\gamma}{\kappa+\gamma}\int_R^\rho \frac{f(\rho)}{\rho}\,d\rho.\right] \\ - \frac{\gamma^2 R^4}{8\rho^2}\left[1 - \frac{8\,\lambda^2 \sin^2\theta\,\rho^2}{\kappa^2}\int_R^\rho \left[1 - \frac{\gamma}{\kappa+\gamma} f(\rho)\right]^2 \frac{d\rho}{\rho^2}\right] \end{aligned} \tag{84}$$

p_i ist demnach am grössten, wenn ρ verschwindet und nimmt stetig gegen den Begrenzungskreis ab. Es herrscht also hier im Wirbelcentrum ein Maximaldruck, dessen Grösse jedoch um so kleiner ausfällt, je grösser die Geschwindigkeit der niederstürzenden Strömung ist. Dabei giebt es im ganzen inneren Raumgebiet unmittelbar über der Erdoberfläche, wo $z = 0$ zu setzen ist, nirgends Orte, wo p_i bei endlichem Werth von γ, negativ über alle Grenzen hinans wachsen würde. Die Bewegung, welche durch die Gleichungen (75)—(84) dargestellt ist, steht überhaupt zur Cyklone im bemerkenswerthen Gegensatze, und entspricht derjenigen Erscheinung der Atmosphäre, welche man fast immer, wo hinreichende über weite Strecken der Erdoberfläche ausgedehnte synchronische Beobachtungen vorlagen, als Begleiterin der Cyklone nachgewiesen, und mit dem Namen Anticyklone belegt hat.

In einer solchen Wirbelbewegung gehn die Drehungen der Windrichtung immer im umgekehrten Sinne vor sich, wie in der eigentlichen Cyklone, und der Luftdruck im centralen Theile ist höher als in ihrer Umgehung, als in der eigentlichen Cyklone. Die Windbahn ist nach diesem Punkt des Maximaldrucks spiralig gewunden, und zwar in der umgekehrten Richtung, wie die spiralige Windbahn der Cyklone. Alle diese Charakteristiken der Anticyklone, können aus den Gleichungen (75)—(84) unmittelbar abgelesen werden.

Die Anticyklonen entstehen demnach immer da, wo in Folge der stärkeren Abkühlung oder sonstiger Ursachen eine verticale

Strömung niederstürzt, und so eine Druckdifferenz erzeugt. Ist das Gebiet der verticalen Strömung ein kreisförmiges, wie in dem behandelten Fall, so wirbeln die Lufttheilchen auf Schraubenlien, deren Horizontalprojection eine gewöhnliche logarithmische Spirale ist, niederwärts, und fahren aus dem Grenzkreis heraus auf einer in der Nähe desselben äusserst complicirten Spiralbahn. Vergleicht man den Deviationswinkel in der Anticyklone mit demjenigen in der Cyklone, so stellt sich der bemerkenswerthe Gegensatz heraus, dass die Windbahn innerhalb der Anticyklone mit der Isodynamen, oder was hier auch dasselbe ist, mit den Isobaren unter sonst gleichen Verhältnissen, [dieselbe Geschwindigkeit der verticalen Strömuug, dieselbe geographische Breite, und Reibung] überall grösseren Winkel einschliesst, als diejenige innerhalb der Cyklone, ganz so, wie wir früher allgemein abgeleitet haben.

(Fortsetzung folgt.)

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On the Formation of the Germinal Layers in Chelonia.*

By

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and

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With Plate XIV, XV, XVI, and XVII.

In the spring of 1884 we made the acquaintance of Mr. Hattori, the proprietor of a large fish-hatching establishment in Honjō, a suburb of Tōkyō. His father before him, and he, had succeeded in making the snapping turtle—Trionyx Japonicus, Schlegel—breed freely and naturally in captivity, and thus in furnishing the market with a constant and large supply of its delicate flesh. In his farm hundreds of these trurtles are annually hatched, and if the eggs are marked as they are laid the exact age of any given deposit can be determined with great precision, even to minutes in many cases. Such an opportunity for the investigation of Reptilian development seemed to us too good to be thrown away, especially as nobody had, so far as we were aware at the time, worked on the embryology of

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Chelonia since the days of Agassiz and Clark, and therefore with mordern methods of investigation. Mr. Hattori kindly consenting, we went to his farm daily during the breeding season of 1884 and of 1885, and succeeded in collecting a fairly complete series of the Tryonix embryos, beginning with the time when the eggs are deposited, and ending with their hatching out. The present paper gives the results of our study on the formation of the germinal layers. Papers on other points and later stages of development will follow from time to time since the investigation is being continued, as the pressure of other duties permit us.

We wish to return our warmest thanks to Mr. Hattori for cheerfully acceding to our numerous demands on his good nature, and for furthering greatly our work with his intelligent assistance. Thanks are also due to the authorities of the University of Tōkyō for the payment of necessary expenses attending the investigation, and for the use of instruments, reagents, &c. Finally, we wish to express our deep obligations to Dr. Isao Jijima for valuable suggestions in regard to the methods of investigation.

We made many interesting observations on the breeding habits of Trionyx, but we reserve these for some other occasion, as foreign to the purpose of this paper. We simply mention that the Trionyx eggs are nearly spherical in shape, and have a hard brittle shell like that of the fowl, and not leathery, as in some Chelonia. Their size is very variable, the smallest we measured being 10 mm., the largest 23 mm., the most usual size about 21—22 mm. in diameter. This difference in size seems to be due mostly to the size of the parent. With this we pass on at once to the consideration of the subject proper of the present communication.

The earliest stage of which we will give a detailed description is taken from an egg opened directly after its deposition. Our attempts

to obtain still earlier stages by opening pregnant females have proved but partially successful. In almost every case, with only some doubtful exceptions, the eggs we found in the oviduct were unfortunately fully as much advanced as those just laid.

On opening an egg directly after its deposition the blastoderm is always found at the pole turned above. The embryonic shield, with the pellucid area around it, stands out conspicuously as a small, nearly circular spot, on the yellow surface of the yolk. The general appearance of the embryonic shield at this stage is represented in figs. 1 *a* and *b*, enlarged about thirty diameters. Fig. 1 *a*, shows it as seen from the dorsal side, and fig. 1 *b*, as seen from the ventral side after the removal of the shield from the egg. The embryonic shield does not lie in the centre of the area pellucida (*ap.*), but is placed excentrically nearer its hind end, so that here it is continuous with the area opaca (*ao.*). The ectoblast nas already spread itself over a large part of the egg, although we did not determine its exact limits (see fig. 16). On the dorsal view the blastopore (*bl.*, fig. 1 *a*) forms the most conspicuous feature; it is seen as a wide transverse slit across the posterior part of the embryonic shield, occupying considerably more than one third of the breadth across. From the blastopore a passage leads obliquely forward and ventralward, and opens about in the centre of the ventral surface with a circular opening (*v. o.*). The walls of the ventral opening are posteriorly quite high, but become gradually lower and lower toward the front, until they sink to the general level of the ventral surface. For the sake of brevity this passage, leading from the blastopore dorsally and opening below, we shall hereafter call the blastoporic passage. It becomes eventually the neurenteric canal. Returning to the dorsal surface, the shield in front of the blastopore presents a broad flat expanse, in which are seen indistinctly three opaque lines radiating from behind forward, like the prongs of a trident. On referring to the

ventral side we see that the two lateral opaque lines correspond to the thickenings which from the walls of the inferior opening of the blastoporic passage. Accordingly they are thickest posteriorly, and gradually thin out toward the front. The middle prong of the trident corresponds to the roof of the blastoporic passage and its continuation to the front edge of the embryonic shield. It is, in fact, the chorda entoblast, which is still in the process of formation in front, as will be made clear by sections. The remaining parts of the ventral surface not taken up by these three thickenings present the appearance of a honeycomb. Of this we shall speak later on. Coming back to the dorsal surface again, the area behind the blastopore, especially the median longitudinal space, is on a lower level than the parts in front. This, the sections show us, is the line of the primitive streak. At the part where the embryonic shield posteriorly joins the area opaca there is a considerable transverse thickening (*sl.*), shown both in the dorsal and ventral views—in the latter covered with yolk matter. This undoubtedly corresponds to the "sichel" or "sickle" which Kupffer describes in a similar Lacerta embryo (No. 5, Taf. i, fig. 1, *sl.*). We should add that these differences in level become much more conspicuous after the embryonic shield has been removed and treated in reagents than when it is stretched over the yolk, and also that the embryos of this stage vary considerably in their surface views, especially when they are hardened.

Figs. 7—15 are selected from the series of transverse sections obtained from the embryo represented in figs. 1 *a* and *b*. The figures are arranged in order from behind forward. Figs. 7—9 pass through the part behind the blastopore, figs. 10, 11 through the blastoporic passage, and figs. 12—15 through the part in front of the blastopore.

In fig. 7, the most posterior section represented, the ectoblast extends over the whole, being two or three layers of cells thick in the

embryonic shield, but gradually thinning out to a single layer of flat cells toward both sides. The yolk occupies the entire lower stratum. Nuclei (*n. n.*) are visible in it. The space between the ectoblast and the yolk is occupied by a mass of mesoblast cells which is here distinctly separate from both the ectoblast and the yolk.

In fig. 8 (which by the way is taken from another embryo of the same deposit, as the section corresponding to this in the first series is unfortunately injured) the ectoblast is continuous in the median line with the mesoblast, i. e. it is very actively proliferating and giving off cells abundantly to the mesoblast.

Fig. 9 passes through the region directly behind the blastopore. The ectoblast is distinct laterally, but toward the median line, and at some distance from it, passes gradually into a mass of cells in which no layers can be distinguished. Different from fig. 8 where the ectoblast cells, although continuous in the median line with the mass below, still maintain their columnar shape over the whole dorsal surface and thus give an impression of the ectoblast extending entirely across, the ectoblast is in this section fused into the median mass without retaining the slightest trace of the columnar arrangement, and the median mass of cells thus expose their surface to the exterior for a short space in the axial line (*yk. p.*). We wish to emphasize the fact that this part directly behind the blastopore is neither at this nor at any subsequent time until considerably later (if ever at all), covered by the ectoblast of the general surface of the dody. This area we consider to be the remnant of the yolk-plug of Rusconi found in the Amphibian embryos. This will become clear in the later stages. From the axial mass, where the layers are indistinguishable, there extends toward each side a thick mesoblastic wing under the ectoblast. The yolk seems to be distinct from the mass above, although, throughout this region, protoplasmic threads seem to

connect the two.

Fig. 10 passes just in front of the dorsal lip of the blastopore where the ectoblast reflects downward and forward to become continuous with the axial strip of the entoblast or chorda-entoblast (compare fig. 16). The blastoporic passage (*bl. p.*) seen as a transverse space is still open on the left to the exterior. The floor of the passage is formed by a mass of cells continuous with the yolk-plug; in fact we may consider this a part of the plug. At a lower level the mesoblast (*mes.*) stretches out laterally as two wings from the median mass. The relations of the yolk are the same as in fig. 9.

So far the sections seem to have passed through the part known as the "sickle."

The next section represented (fig. 11) evidently passes through what may be called the neck or isthmus, i. e. the point from which the three prongs of the trident referred to in the surface view radiate (compare fig. 1 *b*). Accordingly, the entoblast is found only in the median line as a thickening constituting the walls of the blastoporic passage (*bl. p.*), which is now irregularly circular in section. The roof and the sides of the passage are formed by a columnar epithelium two or three cells thick (*enc.*). This is continuous with the ectoblast at the dorsal lip of the blastopore (compare figs. 16 and 10). It is the chorda-entoblast of Hertwig (No. 6). The floor of the passage and the lower part in general is made up of irregularly scattered cells. This is not only the continuation of the yolk-plug but also of the yolk itself, which occupied the lowest stratum in figs. 7—10, and which has been in the last two or three sections gradually merging itself into the floor of the blastoporic passage. Thus, although it does not appear in any single transverse section, the three germinal layers are fused in the region behind the blostopore. Laterally the entoblast is very thin and passes gradually into the yolk. The section is out of

the region of the "sickle," and there is no longer a mesoblastic wing on each side.

Fig. 12 passes through the posterior part of the lower opening of the blastoporic passage. The thickenings which form the lateral walls of the opening are therefore still quite thick (compare fig. 1 *b*). The columnar chorda-entoblast is found as before forming the roof and the sides of the passage, which is now open below. Towards the lower part of the side walls the columnar arrangement is lost and the cells are irregularly scattered. Further out at the sides the cells form a loose network, and then at the edge of the embryonic shield passes into the yolk.

Fig. 13 passes through the anterior part of the ventral opening of the blastoporic passage, which has now flattened itself out into a shallow groove in the median line. Its roof is still formed by the distinctly columnar chorda-entoblast. Laterally, the chorda-entoblast gradually passes into a mass of cells arranged in an irregular loose network, which in its turn is replaced by the yolk at the edge of the embryonic shield.

Passing forward, the chorda-entoblast begins gradually to confine itself more and more to the ventral median surface, until in the seventh section from fig. 13 it has the appearance presented in fig. 14. Here the columnar shape is confined to a few cells in the ventral median line. They pass above gradually into the loose network of cells which has now extended itself entirely across. The meshes of the network have also become larger than in the previous sections. It is evidently this loose network that produced the appearance of a honeycomb in fig. 1 *b*.

Fig. 14 passes in front of the ventral opening of the blastoporic passage, and indicates that the loosely scattered lower layer cells are here arranging themselves into the chorda-entoblast in the ventral

median line of this region. i. e. along the front part of the middle prong of the trident apparent in the surface views (figs. 1*a* and *b*).

Fig. 15 passes near the front end of the embryonic shield. There is no longer any trace of the chorda-entoblast; the entire entoblast is an irregular stratum of stellate cells not thick enough to form a network. It passes into the yolk at the sides.

We may here call attention to the appearances which are seen in some embryos of this stage. Round the edge of the lower opening of the blastoporic passage, especially toward the front, there is a shelf-like extension of the entoblast into the archenteric cavity somewhat like the velum of a hydromedusa. Fig. 14*a*, Pl. XVI, represents such an appearance. The section is well in front, so that the shelf-like extension is continuous across and divides a small space above from the main digestive cavity below. In sections posterior to this, the small space opens below. We do not know what the significance of this is, unless we suppose that the embryo is younger than that given in fig. 1, and therefore the ventral opening of the blastoporic passage is not yet entirely clear.

Fig. 16 is the median longitudinal section of an embryo taken from the same lot as that represented in fig. 1. The blastoporic passage is very distinct. On its dorsal lip, the ectoblast is reflected forwards and downwards and becomes continuous with the chorda-entoblast which passes in front into a loose network of cells with wide meshes, and finally, into the yolk at the edge of the embryonic shield. At the posterior wall of the blastoporic passage, the three layers, the ectoblast, the mesoblast, and the yolk (i. e. the entoblast) are merged into one another; in other words, the ectoblast and the entoblast are here fused and from the fused place a mass of mesoblast cells extends posteriorly. The three layers become independent a short distance behind the blastopore. As the cross-sections of this region (figs. 8, 9, and

10) show that the mesoblastic mass is similarly extending to each side, we may conclude that, in addition to the primitive streak (fig. 8), the mesoblast is being given off from the posterior wall of the blastoporic passage or at least from its upper part, in all posterior directions for an arc of 180°, somewhat in the shape of an open fan; and this posterior unpaired mesoblastic mass causes the swelling known as the "sickle." Examining the ectoblast of the posterior part more in detail, we find it gradually losing its columnar character as we approach the blastopore from behind, but the space where the fused median mass of cells is dorsally exposed to the exterior, viz. the yolk-plug (compare fig. 9) is not as conspicuous in the longitudinal section as in the later stages. The entoblastic part of this fused mass extends quite forward. This corresponds to the cells seen in the floor of the blastoporic passage in fig. 11. A slight projection from its extreme tip is, we imagine, the remnant of the shelf-like structure mentioned in reference to fig. 14*a*.

The principal facts brought out by the study of this stage may be summed up as follows :

1. There is a passage which, beginning with the blastopore on the posterior part of the dorsal surface, takes a forward and downward course to the ventral surface, opening in about the middle part of the latter by a circular opening.

2. At the dorsal lip of the blastopore the ectoblast is reflected and becomes continuous with the chorda-entoblast.

3. In front of the blastopore there are as yet only two primary layers, the ectoblast and the entoblast.

4. The entoblast is having its axial part arranged into a columnar epithelium to form the chorda-entoblast. This process proceeds from behind forward.

5. At the posterior wall (i. e. floor) of the blastoporic passage the

ectoblast and the entoblast are fused, and from the point of fusion the mesoblast is being given off posteriorly in all directions for the space of 180°.

6. Also, behind the place where the two primary layers are thus fused, the ectoblast is giving off cells to the mesoblast along the median line (fig. 8). This is the line of the primitive streak. It is very short and is present in only two or three sections.

7. The mesoblastic mass derived from the two sources mentioned in (5) and (6) is unpaired and constitutes the transverse swelling in the posterior part of the embryonic shield, " the sickle." This is the only place where the mesoblast is present at this stage.

8. The median mass formed by the fusion of the three layers at the posterior wall of the blastoporic passage appears for a short space on the dorsal surface (fig. 9)—the remnant of the yolk-plug of Rusconi.

Formation of the Mesoblast and of the Chorda Dorsalis.

In the previous stage, the mesoblast was found only in the region behind the blastopore. We may now proceed to describe its formation in front of the blastopore. We call attention first to the embryo represented in figs. 2 *a* and *b*. It was taken out exactly forty-eigh hours after the deposition, but as the weather was unusually cold for the season during the interval, it has made very little progress in development, and is not as far advanced as many thirty-six hours old. As before, a dorsal and a ventral view of the embryonic shield is given, although these are not taken in this case from the same embryo. The shape of the shield has not changed materially from the previous stage. In the dorsal view (fig. 2*a*) the blastopore has assumed a horseshoe shape, and is more of a slit than before. Occupying the concavity of

the horseshoe is the rudimentary yolk-plug. Round the blastopore, and along the median line in front of it, there is an opacity. This seems to be due simply to the fact that the cell layers are thicker in this region than elsewhere. In the middle of this opacity in front of the blastopore and apparently starting from the latter there is a shallow median groove, which probably corresponds to the "Primitivrinne" or "Rückenrinne" described by Hertwig in the Triton embryo (No. 6). On the ventral side (fig. 2 *b*), we wish to call especial attention to the ventral opening of the blastoporic passage (*v. o.*). In the previous stage, it was a circular opening without definite limits. In this stage it has acquired well-defined limits on all sides except towards the front, where it is only faintly bounded. Along the median line of the roof of the recess thus formed, a wide low ridge is visible and is continued in some specimens in front of this area. This is undoubtedly the chorda-entoblast. In the ventral view, the posterior part is concealed by a mass of yolk which has accumulated here in the process of removing the shield from the egg.

As we are going to describe somewhat in detail the next stage, we may omit the description of the sections of this, except one through the ventral opening of the blastoporic passage. Eig. 17 is such a section. It passes through the front part of the lower opening. There is in the median line a slight notch in the ectoblast which corresponds to the groove seen in the surface view. In the entoblast we see the axial chorda-entoblast formed as usual of columnar cells. Laterally, it passes on each side into a mass of polygonal cells—the darm-entoblast of Hertwig (No. 6)—which becomes in its turn continuous with the yolk at the edge of the embryonic shield. At the point where the chorda-entoblast and the darm-entoblast meet each other, the darm-entoblast projects as a ridge into the digestive cavity and thus constitutes one of the lateral edges which bound the ventral

opening of the blastoporic passage (compare fig. 2*b*). Conforming to the groove in the ectoblast, the chorda-entoblast projects downwards in the median line. This corresponds no doubt to the ridge seen in the surface view within the lower opening of the blastoporic passage. This section also shows that the roof of the well-defined area which forms the lower opening of the blastoporic passage is formed by the chorda-entoblast and that the latter thus occupies by itself a special recess of the digestive cavity. From just where the chorda-entoblast and the darm-entoblast join each other, there goes out laterally on each side a string of cells (*mes.*) placed dorsally to the darm-entoblast and ventrally to the ectoblast and distinct from both. This is the commencing mesoblast. The mesoblast is therefore not continuous from the first across the median line.

The surface view of the next stage is represented in fig. 3. The embryonic shield has become pear-shaped, the broader end being the front end. The blastopore is horseshoe shaped, as in figs. 2*a* and *b*, and occupying its concavity is the yolk-plug. The head-fold has just begun, and is seen as the posterior of the two semilunar curves found near the front end of the embryonic shield. The anterior curve is probably the commencing amniotic fold. Between the head-fold and the blastopore there is seen in the median line an opaque streak, which is narrowest in the middle, and becomes broader anteriorly and posteriorly. This is the chorda, which is nearly completed in the middle, but still unfinished toward each end. The area pellucida is, as before, found only toward the front and the sides. The pear-shape of the embryonic shield seems to have been produced mainly by its posterior part having lengthened.

Figs. 18—23 are selected from a series of cross-sections obtained from an embryo of this stage, and are arranged from behind forwards.

Fig. 18, the most posterior section represented, goes through the lateral limbs of the horseshoe-shaped blastopore and the yolk-plug occupying its concavity. The ectoblast, which consists of only a single layer of cells at the sides, becomes gradually thicker towards the median line, which it does not, however, reach. At a short distance from the latter, and at the lips of the blastopore, the ectoblast turns ventralward, and becomes lost in the mass of cells found in the axial line. It retains, however, its columnar character for some distance downwards. The considerable space between the two lateral lips of the blastopore is filled almost entirely by a plug (*yk. p.*) of considerable size, which projects upwards from the axial mass of cells as far as the level of the general surface of the embryo. The difference between the ectoblast and this plug is at once unmistakeable and striking. While the cells in the ectoblast are columnar and always arranged perpendicularly to the surface, the cells in the plug are polygonal and without any definite arrangement. We shall return to the discussion of this structure directly.

As just stated, the ectoblast turns downwards near the median line, and loses itself in the axial mass. All the germinal layers are, in fact, fused here, for the entoblast, although it has some appearances of being differentiated, is not entirely distinct, and the mesoblast also stretches away from this mass on each side. Toward the sides the entoblast is yet undifferentiated; it consists of an abundant protoplasmic network with numerous nuclei, and is full of yolk-spheres and granules. There is no question whatever that laterally the mesoblast receives cells from the entoblast or yolk. Especially along one line (*a*, figs. 18 and 20) nuclei are heaped in a special mass, from which cells are being given off to the mesoblast. This contribution to the mesoblast from the germinal wall is only in the posterior part, as it is no longer observable in fig. 23, and as the germinal wall

itself, even in a more advanced stage, is found only round the posterior part as a horseshoe-shaped ridge (fig. 5).

Having gone over the description of the various parts of this section, let us return to the discussion of the plug (*yk. p.*) which sticks out to the external surface between the lateral lips of the blastopore. When we compare our figure 18 with the frontal section through the yolk-plug of a Triton embryo, which Hertwig (No. 6) gives in his fig. 9, Taf. ii, we think nobody will hesitate long before concluding that the plug in our figure is homologous with the yolk-plug of Rusconi found in the Amphibian eggs. Allowing for the differences between a holoblastic and a meroblastic egg, the relations in the two figures are almost exactly alike, part for part. If the slits between the plug and the lateral lips of the blastopore extended in our figure a little more into the midst of each mesoblastic mass the resemblance would be complete; but even for the Amphibian eggs the slits do not always extend as far as represented in fig. 9, as Hertwig himself mentions (No. 6, p. 14). At any rate, in each case there is an axial mass of cells, (1) into which the ectoblast turns down at the lateral lips of the blastopore, (2) in which the entoblast is not to be distinguished, (3) from which the mesoblastic masses start away toward each side, and (4) which sends a plug upwards between the lips of the blastopore. If we compare the longitudinal section of the plug in Trionyx (fig. 24, *yk. p.*) with the sagittal section of the Amphibian yolk-plug (Hertwig, No. 6, Taf. ii, fig. 4), we see again that the relations of different parts are alike. It is true that the plug in Trionyx is not bounded posteriorly by a groove, and passes directly into the ectoblast of the primitive streak, but when we consider that the plug in Trionyx is only rudimentary, this is not to be wondered at, and is of little significance.

We think we are justified, on these grounds, in concluding that

we have in the mentioned structure of Trionyx the remnant of the yolk-plug, which appears conspicuously in the Amphibian egg. Strahl describes the same structure (compare No. 13, ser. iii, figs. 0, 0.1, 0.2; ser. iv, figs. 0, 0.1; ser. v, figs. 0, 0.1, 0.2, 0.3; ser. vi, figs. 0, 0.1; ser. vii, fig. 0.1, also No. 9, Taf. i, figs. 6, 7, 14, and 15; and No. 10, figs. 2 and 3), but, so far as known to us, has never explained its nature. Kupffer describes the "Zapfen" occupying the horseshoe-shaped blastopore of Lacerta (No. 5, Taf. i, figs. 2 and 3, *z*), but does not state its homology. He mentions that in Coluber Aesculapii the plug is sometimes divided into two parts by a median fissure (No. 5, Taf. iv, fig. 40, *f* and *g*). We have also observed a similar appearance in some of the earlier embryos of Trionyx, but we are satisfied that there is no true median fissure. What appears to be such is the optical expression of the primitive streak, along which the ectoblast is proliferating, and giving cells to the mesoblast below. Even in the earliest embryos with this appearance it is doubtful if it ever extends to the extreme tip of the plug. As far as we are aware, the only author who mentions what seems to be the yolk-plug in an amniotic Vertebrate is Gasser, who observed it in an abnormal fowl embryo (No. 4, Taf. x, figs. 4—7). The reason why the yolk-plug in Trionyx is more conspicuous at this stage than earlier stands, we think, in close connection with the fact that the blastopore has become a much better defined horseshoe-shaped slit.

We return now from this long digression to the description of the embryo before us. The sections behind fig. 18 show that immediately behind the yolk-plug, which persists distinctly in only one more section after fig. 18, the ectoblast extends over the whole surface as shown by the characteristic columnar cells. For a short space, however, the ectoblast is proliferating in the median line and is continuous with the mesoblast below. This is seen in only three

sections after which the ectoblast becomes independent. The entoblast seems, however, to be connected with the mesoblast for a greater length and to be actively contributing cells to the latter. This is the region where the germinal wall makes a horseshoe-shaped bend round the posterior part of the embryo (fig. 5). Except in this last detail, the relations of the various parts behind the blastopore are exactly as in the stage represented in fig. 1.

Going forward, fig. 19 passes through the blastoporic passage. As it is directly in front of the dorsal lip of the blastopore, the ectoblast is still continuous for a little space with the chorda-entoblast, which as usual vaults over the passage. The columnar cells extend to the sides also, but on the floor of the passage the cells are polygonal, so that this part which is the continuation of the yolk-plug differs in its appearance from the roof and the sides. To this part, too, the darm-entoblast (*end.*) is attached. From the entire side of the axial mass the mesoblastic sheet goes out on each side.

Fig. 20 passes slightly in front of the ventral opening of the blastoporic passage. In the median line the chorda-entoblast (*enc.*) forms directly the roof of the digestive cavity, without the intervention of the darm-entoblast (*end.*) which stops at a short distance from the axial line. On the left side of the section, more clearly than on the right, the darm-entoblast is seen to make a fold at its innermost point where it abuts against the chorda-entoblast and then to turn outside again to be lost in the mesoblast. The mesoblast is therefore partly continuous with the chorda-entoblast and partly with the darm-entoblast. In other words, it starts from the point where the chorda- and the darm-entoblast meet each other. The mesoblast cells in this region show a peculiar arrangement. Those cells next the ectoblast are columnar and look like the continuation of the chorda-entoblast. The cells placed ventrally to these are polygonal and without any

definite arrangement. Laterally cells are being added to the meso-blast in the whole region of the germinal wall, but especially at *a*; proliferation seems to take place in the posterior region even from the outer part of the darm-entoblast, as in this section. This is, however, confined to the part which still consists of two or three layers of cells, and never extends to the inner part which has only a single layer of cells, and constitutes the well differentiated darm-entoblast.

We pass over for the present figs. 21 and 22, and come to the most anterior section represented (fig. 23). It is in the region of the head-fold as shown by a notch (*h. f.*) on one side in the ectoblast. The darm-entoblast, which is laterally quite thick and consists of columnar cells, is internally very thin and becomes continuous with the chorda-entoblast near the median line. From the point of junction as well as from the sides of the chorda-entoblast mesoblastic cells are budding off on each side. There is in this section a small mass of mesoblast cells outside of the head-fold which is distinct from the main mass. This isolation has been brought about by the ectoblast folding downward as the head-fold; more posteriorly the lateral mass fuses with the main mass. In this section the germinal wall is absent, and thus no additions are made laterally to the mesoblast from the entoblast.

Returning to the middle region of the body, figs. 21 and 22 serve to show the first steps in the formation of the notochord. The chorda-entoblast which in fig. 20 passed laterally without any interruption into the mosoblast, is in fig. 21 marked off from the meso-blast, at least in the upper part. The cells at the border between the two are turned away from one another; thus the cells of the chorda are directed inwards and downwards, while the contiguous cells of the mesoblast are directed outwards and downwards. The mesoblast is still united with the darm-entoblast. As yet, the chorda is only a

mass of columnar cells. In fig. 22, five sections in front of fig. 21, the chorda has become rounded in outline and considerably smaller in section. The most dorsal and median cells alone are columnar, and the remaining cells are arranged as if the more lateral cells have folded inwards and downwards from the two sides and met in the median line. The mesoblast is now distinctly separated from both the chorda and the darm-entoblast. The last abuts against the chorda, but seems separate from it. This is as far as the formation of the chorda has advanced in this stage. In front of fig. 22 the chorda becomes wider again, until in the region of the head-fold it is as represented in fig. 23 ; a similar arrangement is found at the posterior end of the embryo.

Fig. 24 is a longitudinal section of another embryo from the same deposit of eggs as the one represented in fig. 3. It passes very nearly in the median line. The blastoporic passage is considerably narrower than in fig. 16. Its angle of inclination to the surface of the ectoblast is now greater, approaching more nearly a right angle ; hence it has become also much shorter than before. At the dorsal lip of the blastopore the ectoblast is reflected and becomes continuous with the chorda-entoblast. Owing to the fact that the chorda is most developed and therefore narrower in the middle region of the embryo than in front or behind, and perhaps also to the fact that the section is slightly oblique, the mesoblast (*mes.*) appears for a short space (∗—*c.*) in this section. The entoblast, which is very thick in front, especially in the head-fold (*h. f.* marked by a notch in the ectoblast), becomes suddenly reduced at the point *c* into a thin ventral layer (*end.*) which stretches posteriorly as far as the point marked with a ∗, where it seems to unite with the chorda-entoblast. From the point of junction and also continued forward from the chorda-entoblast, the mesoblast sheet stretches forwards as far as *c*, above the darm-ento-

blast (*end.*) and beneath the ectoblast. Behind the blastoporic passage there is a large mass of cells projecting downward (the Endwulst.) On the dorsal surface, directly behind the passage, columnar cells are absent for a short space. This is the longitudinal section of the yolk-plug. Following it, the ectoblast cells appear, but cannot at first be separated from the large mesoblastic mass, for this is the region of the primitive streak where the ectoblast is giving off cells below. Very soon, however, it becomes an independent sheet. The continuation downwards of the yolk-plug forms the whole posterior wall of the blastoporic passage, and is therefore seen as its floor in cross-sections. The entoblast is continuous with it at the extreme front of the "Endwulst," but becomes a distinct layer on the ventral surface. The mesoblast, utterly indistinguishable from the yolk-plug, stretches away posteriorly. Behind the blastopore the three germinal layers are thus fused. The mesoblast, which is separate from the entoblast on the ventral surface of the "Endwulst," is receiving more posteriorly additions from the yolk or germinal wall.

In the next stage which we figure (figs. 4 *a* and *b*), the head-fold has considerably advanced, and the amnion (*am.*) covers it already so that it is not visible from the dorsal side. The medullary folds have touched each other. At the posterior end the yolk plug is included between the diverging medullary folds.

The sections through the head region of this stage show beautifully, and in an unmistakeable and conclusive manner, the mode of the formation of the mesoblast and of the chorda dorsalis. Figs. 25—28 are selected to illustrate these points.

Fig. 25 is the most anterior section represented. It goes through the posterior part of the head. The amnion is closed over it, but the digestive cavity is still widely open below. The darm-entoblast formed by columnar cells does not reach the chorda-entoblast, but is

separated from it by an interval where cells are most actively proliferating and giving rise to the mesoblastic mass. Fig. 29 is a similar section from another embryo of the same stage. Here also the chorda-entoblast, instead of passing directly into the darm-entoblast, is separated from it on each side by a space where cells are actively dividing and giving rise to the mesoblast. This figure shows also more naturally than fig. 25 that the mesoblastic mass consists of spindle-shaped and stellate cells arranged in such a way as to give an impression of having radiated from their origin.

Figs. 26—28 show clearly the mode of the formation of the notochord. Fig. 26 is two sections behind fig. 25. The mesoblastic masses have separated from the chorda- and darm-entoblast. The chorda-entoblast is arcuate. The darm-entoblast abuts against it but is distinctly separate from it. In fig. 27, the third section behind fig. 26, the chorda-entoblrst has become a cord-like mass, against the more ventral side of which the darm-entoblast of both sides is applied. This corresponds to fig. 22 of the previous stage. In fig. 28, the third section behind fig. 27, the darm-entoblast has passed under the notochord from both sides, and united so as to form a continuous sheet across. The formation of the notochord is thus completed.

As in the previous stage, the notochord is finished only in the middle region of the embryo. Toward the posterior region, in front of the ventral opening of the blastoporic passage, the chorda is in the process of formation. The mode of formation is exactly as at the front end. Figs. 30—34 from an embryo of nearly the same stage as that represented in figs. 4*a* and *b*, are introduced to illustrate this process.

Fig. 30 is the most posterior section given. It is slightly in

front of the ventral opening of the blastoporic passage, which is still visible as a groove in the median line. The darm-entoblast (*end*), which is distinct laterally, does not reach the chorda-entoblast, but passes into a zone from which the mesoblastic sheet spreads away laterally and which in its turn becomes continuous with the chorda-entoblast. This corresponds to fig. 25 or fig. 29 of the anterior region, or to fig. 20 of the previous stage.

In fig. 31 the chorda-entoblast is beginning to be marked off from the mesoblast, which is, however, still united with the darm-entoblast, at least on the left side. This corresponds to fig. 21 of the previous stage.

In fig. 32 the mesoblast has become entirely separated from both the chorda-and the darm-entoblast (excepting a little spot on the left). The chorda-entoblast is now a compact mass by itself, against the sides of which the darm-entoblast is applied. This is more clearly shown on the right side than on the left. This corresponds to fig. 26 of the anterior region, and to fig. 22 of the previous stage.

In fig. 33 the darm-entoblast has passed some way under the chorda which has almost the appearance of the finished structure. Thie corresponds to fig. 27.

In fig. 34 the darm-entoblast has passed completely under the chorda and forms a continuous sheet across, and the formation of the notochord is finished. This corresponds to fig. 28.

The formation of the chorda at the anterior region comes to an end much earlier than in the posterior region, where it is continued on until considerably later, and where the growth in length of the embryo seems mainly to take place.

It remains now to state the fate of the blastoporic passage. In an embryo taken out two days later than that given in figs. 4 *a* and *b*,

from the same deposit of eggs,—in an embryo, therefore, five days old with five or six mesoblastic somites—the passage is no longer dorsally open. The medullary canal has completely closed over it and the blastoporic passage has been changed to the neurenteric canal. Figs. 35 *a*—*d*, will show the relations of the germinal layers round the passage. In *a*, the most anterior section given, the darm-entoblast, the notochord, the mesoblast, and the medullary canal are all separate. In *b*, the chorda has fused above with the walls of the medullary canal, appears for a little space in the median line on the roof of the digestive cavity, and divides the darm-entoblast of the two sides which seem to rest against it. In *c*, the canal opens below into the digestive cavity. The mesoblast is now continuous with the darm-entoblast and the walls of the neurenteric canal at the junction of the two. In *d*, the posterior part of the neurenteric canal has been cut. In the next section (not figured), the cells in the axial region are only more compact than elsewhere, and show that the posterior wall of the canal is reached. Thus from the mass behind the blastoporic passage (i. e. the "Endwulst"), the posterior wall of the neurenteric canal seems to have been developed in situ. From this mass the mesoblast is extending laterally on each side. It is not possible for us to state exactly how the yolk-plug disappears. A part of it which formed the posterior wall of the blastoporic passage is no doubt changed into the posterior wall of the neurenteric canal. A part placed more dorsally is perhaps changed directly to the ectoblast of the general surface of the embryo.

In an embryo six days old, i. e. one day older than that of fig. 35, the neurenteric canal still persists. In an embryo seven days old it is no longer found. We are not in a position to state how its disappearance is brought about.

To state briefly the principal facts brought out by our observa-

tions on the formation of the mesoblast and of the notochord :

In the embryo represented in figs. 1 *a* and *b*, the mesoblast was found only in the region behind the blastoporic passage, radiating in the shape of an open fan from the posterior wall of the passage, as well as from the ectoblast along the primitive steak, and constituting the structure called the "sickle." In further course of development the mesoblast becomes extended into the region in front of the blastoporic passage. Here it arises as a paired mass, and its point of origin is invariably at the junction of the chorda-entoblast with the darm-entoblast. In other words, one part of the mesoblast is always continuous with the chorda-entoblast, while the other part passes into the darm-entoblast.

Besides this source the mesoblast receives large contributions of cells from the germinal wall, and even from the outermost part of the darm-entoblast contiguous with the germinal wall.

The notochord is formed out of the chorda-entoblast. It is completed first in the middle, and then extends both backward and forward. Its mode of formation is the same, both in front and behind. First, at the point of the origin of the mesoblast the connection of the three structures that meet there, viz. the mesoblast, the chorda-, and the darm-entoblast, is loosened. The mesoblast is then found as two separate masses, one on each side of the median line. The darm-entoblast rests with its free edges against the sides of the chorda-entoblast ; it, however, passes gradually under the chorda-entoblast, until finally the darm-entoblast of two sides fuses in the median line, and forms a continuous sheet over the digestive cavity. In the meantime the chorda-entoblast has arranged itself into the finished chorda dorsalis.

The formation of the Blastoporic Passage.

There are differences of opinion among previous writers on the subject in regard to the formation of the blastoporic passage in Reptilia. Balfour (No. 2, p. 424-5) says: "After the segmentation and the formation of the embryonic shield (area pellucida) the blastoderm becomes distinctly divided into epiblast and hypoblast. At the hind end of the shield a somewhat triangular primitive streak is formed by the fusion of the epiblast and hypoblast, with a number of cells between them, which are probably derived from the lower rows of the segmentation cells. At the front end of the streak a passage arises, open at both extremities, leading obliquely forwards through the epiblast to the space below the hypoblast." Here Balfour does not say how this passage arises. In his 'Comparative Embryology' (vol. ii, p. 168) he says: "At the front end of the primitive streak an epiblastic involution appears, which soon becomes extended into a passage open at both extremities, leading obliquely forwards through the epiblast to the space below the hypoblast." Kupffer (No. 5) is of substantially the same view. Weldon (No. 14, p. 136) says: "At a point (*bp.*), however, the position of the future blastopore, these layers are replaced by a mass of closely-packed cells (*pr.*), exhibiting no division into layers, and forming the primitive streak, which may, in some cases at least, extend backwards as far as the commencement of the area opaca. The blastopore commences at the anterior end of this streak as a pit, open above and closed below . . . . . The floor of this pit presently breaks up, and the blastopore assumes its normal conditon, forming a communication between the archenteron and the exterior, its anterior wall forming a communication between the epiblast and the lower layer cells. From this time a change in the character of the lower layer cells takes place, begin-

ning from the anterior wall of the blastopore, where they pass into the epiblast, and proceeding forwards. Instead of being large, irregular, full of yolk, as in the previous stages, they become columnar, lose their yolk, arrange themselves in a definite layer several cells deep, and take on the characters of normal hypoblast. . . . . This process is evidently an invagination comparable to that which takes place in an Elasmobranch. It especially resembles the process described by Scott and Osborne in the newt." Strahl gave his views first in an article published in 1882 (No. 8), and again in a later writing (No. 13, p. 55). His views, as expressed in the latter, are briefly as follows:—Before the neurenteric canal is present the germinal disc consists throughout only of ectoblast and entoblast, except in the region of the primitive streak, which is oval or pear-shaped, or nearly triangular in form. In such a disc three processes, which may be independent of one another, now take place.

1. Under the primitive streak the entoblast is differentiated, so far as it has not done so already.

2. In the middle of the primitive streak the canalis neurentericus is sunk, at first perpendicularly below and then horizontally forward.

3. In the region of the primitive streak the ectoblast differentiates from the mesoblast. This differs in the regions before and behind the neurenteric canal. In front of the canal the whole mass is differentiated into the ectoblast and the mesoblast (i. e. mesoblast according to his views: we would call it the chorda-entoblast). In the region behind the canal, only the epidermal layer of the ectoblast is differentiated, the differentiation of the remaining cells into the structures for which they are destined: viz. the extreme end of the medullary canal, of the chorda, &c., takes place at a much later date.

As we stated before, we did not succeed in obtaining the stages

earlier than fig. 1. We will try, however, to reason back from our earliest stages and to deduce what processes have given rise to such a form. Of course, such à priori reasoning is liable to mistakes, and we offer the following remarks merely as suggestions which need verification by future investigations. If the blastoporic passage really commences as an epiblastic invagination, it seems to us that Kupffer is quite right in considering the invaginated sac as the gastrula cavity much reduced in size (No. 5, p. 2). But apart from the inherent improbability that the bottom of the archenteron should afterward give way and the archenteron should become connected with some cavity beyond itself, we think we have another sufficient reason in rejecting the view of an epiblastic invagination in this fact that directly behind the passage, when it is established, there is an area which is not covered by the ectoblast, i. e. the yolk-plug. We think then that what really takes place must be very much as Weldon and Strahl describe it, for these two writers differ after all, when we leave out minor points, only in this, that the former thinks the passage arises at the front end, and the latter at the middle of the primitive streak. Our views, then, on these earliest stages are as follows:

At the end of the segmentation the blastoderm becomes divided into two primary layers, the ectoblast above consisting of columnar cells, and the entoblast below consisting of irregularly shaped cells without any definite arrangement. At the region of the future blastopore and primitive streak, this process of differentiation is somewhat modified from what takes place elsewhere. When the differentiation of the ectoblast has proceeded backward and come to the future dorsal lip of the blastopore, it does not extend further in the median line over the blastodermic surface, but becomes reflected downward and continuous with the axial strip of the lower layer

cells which acquire the columnar character from this point forward in the median line of the future embryo, and arranged themselves into the chorda-entoblast. This process has proceeded to the front end of the embryonic shield in the embryo represented in fig. 1. Whether there is any actual invagination of cells from the dorsal lip of the blastopore we cannot tell, but this is of no moment so long as the ectoblast becomes continuous with the axial strip of the entoblast at the dorsal lip, and the arrangement of the lower layer cells into the chorda-entoblast proceeds from here towards the front. We can conceive the blastoporic passage itself arising in this way. As the cells arrange themselves into the chorda-entoblast, these columnar cells separate from the cells directly behind them and thus a fissure or canal is produced just at the same rate as the cells arrange themselves into the chorda-entoblast. The posterior wall of this canal would thus be composed of undifferentiated cells, as it actually is.

While the differentiation of the ectoblast thus stops, in the median line, at the dorsal lip of the blastopore, and the above-mentioned changes leading to the formation of the blastoporic passage are going on, we can suppose that the differentiation of the ectoblast is at the same time proceeding actively in the more lateral parts and is extending backwards and meeting in the median line again slightly behind the blastopore (see fig. 6). There would thus be left behind the blastopore a small space not covered by the ectoblast. This is the yolk-plug, which is of course continuous with the undifferentiated cells forming the posterior wall of the blastoporic passage. From the ectoblast in the median line behind the yolk-plug, cells begin to proliferate and constitute the primitive streak. This may happen before the blastoporic passage is completed (see Strahl, No. 8, Taf. xiv, fig. 11). Proliferation begins also from the posterior wall of the blastoporic passage. We shall then have a stage exactly like

that given in figs. 1 *a* and *b*. When we make a careful study of the latter embryo, some such series of changes as we have sketched out will become an absolute necessity. Our views are in the main like those of Weldon and of Strahl, but we think we have filled in more details. Strahl, it is true, says that the passage begins in the middle of the primitive streak. We are inclined to think that in his figs. 8 and 9, Taf. xiv (No. 8), he has stages in which the differentiation of the ectoblast from the entoblast has not proceeded as far as the dorsal lip of the blastopore. In our view, the 2nd ond 3rd processes given in his account have the closest relations to each other. Our hypothesis also makes what takes place in Reptilia harmonise well with the development of lower forms, especially of the Amphibia.

Discussion of the Results of our Observations.

In an article published as early as 1875 Balfour (No. 1, p. 208) states that "Amphioxus is the Vertebrate whose mode of development in its earliest stages is the simplest, and the modes of development of other Vertebrates are to be looked upon as modifications of this, due to the presence of food material in their ova." In the same article, as well as in several subsequent publications (Nos. 2 and 3), he endeavoured to work out the comparison of the vertebrate development with the idea given in the above quotation for its foundation. Above all, he has insisted that the mesoblast always arises as paired masses, one on each side of the median line, and that these two masses are to be regarded as paired diverticula of the alimentary canal. Recently O. Hertwig (No. 6), in connection with the "Cœlomtheorie" of himself and his brother, has worked out this idea very completely in Amphibia, and has also shown, from the investigations of other workers, how the same idea could be carried

out through other classes of Vertebrata. We need hardly say that our investigations most completely bear out Balfour's and Hertwig's view. In fact, the agreement between the development of Amphioxus and Amphibia on one side, and of Reptilia on the other, as shown by our work, is as complete as could be desired, when we make due allowance for the fact that on one side is a holoblastic and on the other a meroblastic egg. Let us examine more in detail.

When we compare our fig. 16 with Hertwig's fig. 4 (Taf. ii, No. 6) of Triton, we are at once struck with the close similarity between the two, allowing for the fact that the latter represents a whole egg, and the former only a small part of it. There is in both a passage connecting the cavity which becomes the future alimentary canal with the exterior. This is, according to Hertwig's nomenclature, "die enger Theil der Darmhöhle (*dh.*)," according to ours "the blastoporic passage." At the dorsal lip of this passage the ectoblast in both is reflected, and becomes continuous with the chorda-entoblast. In the region in front of the passage the embryo consists of only the ectoblast and entoblast. In both there is the yolk-plug behind the passage, and contiguous with it the two primary layers are fused, and from the fused point there stretches backward an unpaired mesoblastic mass. Hertwig's fig. 11, Taf. v, and figs. 7 and 10, Taf. vi, of Rana, are essentially alike.

Hertwig's fig. 17 (Taf. iv) is the frontal section through the line *a—b* of fig. 4, Taf. ii. It passes through the beginning of the unpaired mass of mesoblast. It presents an appearance very similar to our fig. 8 of the corresponding region. The ectoblast is proliferating in the median line, and giving cells to the mesoblast. In our figure the entoblast and mesoblast are separate, but we have shown already that they become continuous further forward. Hence exactly the same relations hold in this region in Triton and

Trionyx. Compare also fig. 2, Taf. vi, and fig. 5, Taf. viii, given by Hetrwig of the corresponding region in Rana.

Hertwig's fig. 9, Taf. ii, is the frontal section through the line *c—d* of fig. 4, Taf. ii. It is substantially the same as our fig. 9, although there is a closer resemblance between it and our fig. 18, as we have already shown.

Unfortunately Hertwig does not give a cross section of the front region of an embryo which has not yet developed the mesoblast; but we are sure it will be essentially like our figs. 13 and 14, although we cannot expect to find the lateral parts composed of a network of cells.

Now, as to the origin of the mesoblast, our results agree with Hertwig's account as completely as could be desired. In the region behind the blastopore he says the mesoblast arises as an unpaired mass in the Amphibia. Such is the case with Trionyx, as shown in our figs. 7, 8,16, and 24. In front of the blastopore the mesoblast arises as paired masses separated from each other in the median line by the chorda-entoblast. For this point compare our figs. 17 and 20, or, best of all, figs. 25 and 29, with Hertwig's figs. 1 and 2 (Taf. iii) of Triton. In the latter the chorda-entoblast passes into the parietal layer of the mesoblast, while the darm-entoblast is reflected just where it abuts against the chorda-entoblast, and passes into the visceral layer of the mesoblast, thus constituting what amounts to a pair of diverticula from the alimentary canal, one on each side of the chorda, repeating what is seen in Amphioxus. Hertwig has marked the entrance to these rudimentary diverticula with a star (*) in his figures. We have also marked in our figures what we consider to be the corresponding spots with the same mark (*). We think that morphologists will not find any difficulty in recognising in Trionyx the relations closely similar to those in Amphibia. In Trionyx the

mesoblastic mass becomes continuous on each side with the entoblast, just at the point where the chorda- and the darm-entoblast meet each other. The cells being much smaller in Trionyx than in Triton, it is not possible to distinguish the parietal from the visceral layer of the mesoblast; but if both the chorda- and the darm-entoblast pass into the mesoblastic mass, the relations found here amount to the same thing as found in Triton and in Amphioxus. We think our figs. 25 and 29 ought to convince the most sceptical on this point. It is significant that at one time (fig. 17) the chorda-entoblast occupies a recess of the alimentary canal by itself, and from the two sides of this recess the mesoblastic masses stretch out—a relation which recalls vividly the development of Amphioxus.

Our fig. 19 may prove a stumbling block to some in the way of comparison with Amphibia. But we think this figure is soon reduced to the general rule. We have already pointed out that the cells forming the floor of the blastoporic passage in this figure are different from those of the roof and the sides. If we consider the chorda-entoblast as extending on each side to the spot marked with the star, and this spot as corresponding with the similarly marked spot in fig. 15, Taf. iv, of Hertwig, which passes through the corresponding part of Triton, the comparison will become easy. The apparent difficulty is brought about by the cells of the floor being many layered in Trionyx.

There is another point on which we wish to touch. Although there is no doubt that the mesoblastic masses arise as what morphologically amount to diverticula of the alimentary canal, the development in Trionyx has so far changed from the primitive method that the mesoblast no longer forms an epithelium as in Amphioxus or Triton or even compact masses throughout, but at places appear as only loose masses of spindle and stellate cells (figs. 25

and 29). This fact will, we think, answer Kölliker's objection, based upon the shape of cells in the mesoblast, against the epithelial origin of the mesoblast. (We have not access to Kölliker's original paper but take his views as given in Hetrwig's paper, No. 6, p. 105). Kölliker is no doubt correct in supposing that such forms are due to very rapid proliferation.

As to the formation of the chorda, it is only necessary to compare our figs. 25—28 with Hertwig's figs. 3—6 (Taf. iii) of Triton, and figs. 8—11 (Taf. viii) of Rana, in order to be convinced of the similarity of the process in Reptilia and Amphibia. Our figs. 25 and 29 correspond with figs. 1 or 2 (Taf. iii, Hertwig) of Triton. Our fig. 26 with fig. 4 (Taf. iii) of Triton, our fig. 27 with fig. 5 (Triton), and, finally, our fig. 28 with fig. 6 (Triton).

As to the contribution to the mesoblast from the germinal wall, there is of course no equivalent in the holoblastic egg of Amphioxus or Amphibia. It seems to us that phylogenetically this source is not of much significance and is brought about wholly by adaptation. Sarasin's (No. 15) researches on the Reptilian egg have brought out the fact that new cells are added on from the yolk to the blastoderm by a process very similar to budding. Why could we not suppose that this process goes on until considerably later, and that the addition of cells to the mesoblast from the germinal wall is but the continuation of this process?

We should like to add another suggestion. In Trionyx the primitive streak is continuous with the lateral edges of the blastopore, enclosing the yolk-plug (see fig. 6). Have we not here a case where a part of the original blastopore lips has met in the median line and formed the primitive streak, while the rest of the edge of the blastopore has retained its original condition?

We think we have succeeded in showing that the development

of Reptilia harmonises completely with that of Amphibia. Our observations confirm the conclusions which Hertwig formed in regard to the Reptilian development, basing his judgment on the observations of other workers (No. 6, Theil ii), but we hope we have filled in many details not before noticed. We dissent strongly from Strahl, who in two separate publications (Nos. 11 and 13) oppose Hertwig's views. We think Strahl is singularly unfortunate in the interpretation of his sections.

We think it hardly necessary to go over other papers on the germinal layers of Reptilia (Strahl, Nos. 7, 8, 9, 10, 11, 12, 13; Kupffer, No. 5; Weldon, No. 14; Hoffmann, No. 16), and point out the points of similarity and dissimilarity between those workers and ourselves. The reader must refer to the original papers themselves.

We conclude, expressing the hope that our investigations will furnish a necessary intermediate step in establishing firmly the views of Balfour and Hertwig in higher Vertebrates.

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EXPLANATION OF PLATES XIV, XV, XVI, & XVII.

List of Reference Letters.

ao. Area opaca. *ap.* Area pellucida. *am.* Amnion. *a.* Line along which nuclei are specially heaped up in the germinal wall (Figs. 18 and 20). *bl.* Blastopore. *bl. p.* Blastoporic passage. *ch.* Notochord. *ect.* Ectoblast. *en.* Entoblast. *enc.* Chorda entoblast. *end.* Darm entoblast. *g. w.* Germinal wall. *h. f.* Head-fold. *mes.* Mesoblast. *n.* Nuclei in the yolk. *sl.* "Sickle." *v. o.* Ventral opening of the blastoporic passage. *yk.* Yolk. *yk. c.* Yolk-corpuscles. *yk. p.* Yolk-plug. *z.* Shelf-like extension into the archenteron.

All the figures, excepting Figs. 5, 6, 14*a*, 16, and 35, have been drawn by C. Ishikawa. Figs. 1--6 and 35 have been re-drawn by M. Indō.

Figs. 1—4 have been drawn with Zeiss's A A, × 2; Figs. 7—17 with Zeiss's C C, × 2; Figs. 18—34 with Zeiss's D D, × 2; Fig. 35 with Zeiss's B B, × 2; Fig. 5 not drawn to scale; Fig. 6 is diagram.

Fig. 1*a*.—Dorsal view of the embryonic shield from an egg just deposited.

Fig. 1*b*.—Ventral view of the same.

Fig. 2*a*.—Dorsal view of the embryonic shield from an egg laid forty-eight hours.

Fig. 2*b*.—Ventral view of another embryonic shield of the same age from the same deposit.

Fig. 3.—Dorsal view of the embryonic shield from an egg laid thirty-six hours.

Fig. 4*a*.—Dorsal view of an embryo from an egg laid three days.

Fig. 4*b*.—Ventral view of the same.

Fig. 5.—Ventral view of an embryo from an egg laid five days.

Fig. 6.—Diagram of the embryonic shield.

Figs. 7—15.—Series of transverse sections of the embryonic shield given in Figs. 1*a* and *b*, arranged from behind forward.

- Fig. 7. Section of the region where the three germinal layers are free from one another.
- Fig. 8. Section of the primitive streak.
- Fig. 9. Section passing directly behind the blastopore.
- Fig. 10. Section passing just in front of the dorsal lip of the blastopore.
- Fig. 11. Section through the blastoporic passage.
- Fig. 12. Section passing through the posterior part of the ventral opening of the blastoporic passage.
- Fig. 13. Section passing through the anterior part of the ventral opening of the blastoporic passage.
- Figs. 14 and 15. Sections passing in front of the ventral opening of the blastoporic passage.
- Fig. 14*a*. Section showing the shelf-like extension into the archenteron.

Fig. 16.—Median longitudinal section of an embryo closely similar to Fig. 1*a* and *b*, and from the same deposit.

Fig. 17.—Section passing through the ventral opening of the blastoporic passage of the embryonic shield, similar to that given in Fig. 2*a* and *b*, and from the same deposit.

Figs. 18—23.—Series of transverse sections of an embryonic shield, closely like that given in Fig. 3, and from the same deposit. Arranged from behind forward.

Fig. 18. Section passing through the lateral limbs of the horseshoe shaped blastopore.

Fig. 19. Section through the blastoporic passage.

Fig. 20. Section passing slightly in front of the ventral opening of the blastoporic passage.

Figs. 21 and 22. Sections passing through the middle region of the shield.

Fig. 23. Section passing through the region of the head-fold.

Fig. 24.—Median longitudinal section of an embryonic shield, closely like Fig. 3, and from the same deposit.

Figs. 25—28.—Series of transverse sections through the head region of the embryo represented in Figs. 4*a* and *b*, illustrating the mode of the formation of the notochord. Arranged from before backward.

Fig. 29.—Transverse section through the head region of another embryo, closely like Figs. 4*a* and *b*, from the same deposit.

Figs. 30—34.—Series of transverse sections from the posterior region of an embryo, very much like that given in Fig. 4*a* and *b*, illustrating the mode of the formation of the notochord in that region. Arranged from behind forward.

Fig. 35.—Series of transvers sections from the posterior region of an embryo, with five or six mesoblastic somites, showing the neurenteric canal. Arranged from before backward.

On the Caudal and Anal Fins of Gold-fishes.

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With Plates XVIII—XX.

As is well known, the fins of fishes have been put under two categories, the paired and the unpaired. The former includes the pectoral and ventral fins, and the latter, the dorsal, caudal and anal fins. On this point, however, there is a remarkable deviation in our gold-fish. Here we find not only the pectoral and ventral fins, but also the anal and caudal fins distinctly paired in many individuals. In such cases, the fish presents a series of four paired appendages, a circumstance that has no parallel in all known natural species of Vertebrates.

That our gold-fish is an abnormal form which has been cultivated is an acknowledged fact, and many of its characters are no doubt mere products of artificial selection and devoid of all morphological significance, but the paired state of the caudal and anal fins is, in my view, of great interest when considered in connection with the problem on the nature of the paired and unpaired fins.

In the present article are embodied the results of my researches into this curious condition in the caudal and anal fins of the gold-fish.

The investigation was begun in the autumn of 1885 and continued until the summer of the following year under the supervision of Prof. Mitsukuri and Prof. Ijima. To these gentlemen, I owe my sincere thanks for their constant encouragement and valuable suggestions.

I regret to say that I have not been able to obtain access to some works more or less connected with the present subject. Under this disadvantage I do not feel qualified to enter into an extensive discussion of the question bearing on the morphology of the paired and unpaired fins. I therefore confine myself in the present paper principally to the description of the structure of the caudal and anal fins of the gold-fish and to few suggestions.

Before proceeding further, it would perhaps be well to make a few remarks on the various breeds of gold-fish. We generally distinguish three distinct breeds, viz., the "*Japanese*," the "*Corean*" and the "*Loo-chooan*." These are the commonest breeds to be met with in Japan.

The Japanese breed or the "*Wakin*" has a slender body, closely resembling that of the common Carassius or carp.

The Corean breed, otherwise called the "*Maruko*" or the "*Ranchiu*," is characterized by an exceedingly short body, being in some cases almost globular in shape (Figs. 4—5, Pl. XVIII). The dorsal fin is generally entirely absent. The head is usually disfigured by rough-looking protuberances of the skin which often attain a considerable size.

The Loo-chooan breed or the "*Riukin*" (Figs. 1—3, Pl. XVIII) has a short body with a rounded abdomen. Of all the breeds, this has the most beautiful tail, which is very large and often longer than the rest of the body.

In foreign works it is frequently asserted that these animals were first reared in Japan as well as in China. That the latter

country first produced them, is quite probable; but there is sufficient reason to show that Japan was not their original home. History bears direct testimony as to the date of their first introduction into Japan, the exact locality where they were introduced, etc.

It is known that they were introduced in all probability first from China, at the beginning of the sixteenth century, and subsequently at frequent intervals from China, Loo-choo and Corea.

It is to be gathered from such imperfect descriptions as exist, that the first introduced breed was what we call at present the "Japanese breed." The name has, it is to be assumed, nothing to do with the *nativity* of the breed. Perhaps it was the priority in the date of introduction which caused it to be so designated, in order to distinguish it from the breeds brought in later from Loo-choo or Corea.

Gold-fish breeders of the present day can freely produce the "*Riukin*" or "*Maruko*" from the "*Wakin*." Hence there can be no doubt that the last named breed represents the primitive form, a fact that is also sufficiently apparent from the configuration of the body. There certainly exist various intermediate forms between the above mentioned breeds.

In all gold-fishes, irrespective of what breed they belong to, the tail-fin is above all other parts subject to the greatest variation. It is to be found in one of the following three states: 1. it has three-lobes, one median and two lateral (Fig. 5, Pl. XVIII & Fig. 7, Pl. XIX); 2. it consists of two separate halves (paired) giving rise to a four-lobed tail (Fig 1); and 3. it is vertical and normal. In the first two cases, the lobes are more or less horizontally spread.

Next to the caudal fin, the anal fin undergoes a remarkable variation. It is either median and normal or distinctly paired. In the former case, especially when the caudal fin is also normal, the


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gold-fish closely resembles a common Carassius. There are all stages of caudal and anal fins intermediate between the normal and the paired states; thus, the tail-fin with its lower portion alone in a double state (Figs. 20—22, Pl. XIX), or the anal-fin, with either its anterior or posterior portion double and the remaining parts single, is of quite a common occurrence. These different conditions of the two fins combine in various ways in different individuals, thus giving rise to manifold varieties of form, the typical ones of which are shown in the accompanying wood-cut.

In series I those forms are shown in which anal fins occur plainly in pairs. The caudal fin however occurs in four different conditions. In No. 1 it is distinctly paired, giving rise to the "*four-lobed tail*" or the "*Yotsuo-wo*" In No. 2 the two halves of the tail are connected at the dorsal edge, giving rise to the "*tri-lobed tail*" or the "*Mitsu-wo*." No. 3 represents a case in which the caudal fin is double only at its ventral edge and No. 4, a case with a perfectly normal tail-fin.

In series II forms are represented in which the anal fin is single and median in front but double posteriorly. The caudal fin behaves in four different ways as in the preceding series.

In series III the anal fin is in a reversed condition from that represented in series II; namely its anterior portion is double while the posterior portion is single and median. The caudal fin shows the same variation as before.

In series IV the anal fin is always single and median, the caudal behaving as before.

We shall now proceed to make a closer examination of the caudal fin.

The Caudal Fin. The simplest transitional state from single to double, is seen where the tail, normal in all other respects, has the

ventral edge slightly furrowed by a median groove. Fig. 8, Pl. XIX, shows a tail of this kind. The greater part of it is vertical and median, but its lower portion (*C'*) occurs double. When viewed from the ventral side, a shallow groove extends from the point of attachment down to the extremity of the tail, as represented in Fig. 9 (*B*).

In some cases, this groove extends further upward, almost as far as the middle line of the height of the tail. In other instances, the groove is still deeper and extends above for more than half the height of the tail. The furrow may reach the dorsal edge of the tail and thus divide it into two halves which then expand right and left. These halves may or may not be connected at the median line, at the dorsal edge. In the former case, the tail is represented by a more or less horizontally expanded single piece. In the latter it is distinctly paired. I shall soon return to these various forms of tails when considering their skeletal elements.

The remark may not be out of place here that it is only the ventral lobe of the caudal fin (*V. L.*, Fig. 7) that is liable to become split into lateral halves. The dorsal lobe, i. e. that part lying dorsal to the notochord (*D. L.*, Fig. 7), has as yet never been met with in a paired condition.

It will be necessary first of all to get acquainted with the bony structures of a normal tail fin. A close study has shown that the vertical and median forms of the caudal fin of gold-fishes have the bones arranged identically as in other Cyprinoides, such as a carp or common Carassius.

I shall give here a brief description of the caudal skeleton of a carp (Fig. 6), the large size of which affords a greater facility for study than that of a gold-fish.

The few hindermost vertebræ, as is well known, differ somewhat in form. Thus, the centrum of the autepenultimate vertebra

(Fig. 6, *a.p.v.c.*) is beset with two neural spines, *a.p.n.* and *a'.p'.n'.* The posterior of these processes is prolonged backward and takes a small share in carrying the fin rays of the dorsal lobe of the tail. The hæmal spine (*a.p.h.*) of the same vertebra is thicker and longer than those of preceding ones, and carries at its thickened termination a few of the fin-rays constituting the ventral lobe of the tail. The penultimate centrum (*p.v.c.*) is beset with one neural and one hæmal spine. Both of them are respectively stronger and larger than the corresponding spines of the preceding vertebra. Each carries a few of the caudal fin rays. From the last recognizable centrum (*l.v.c.*) many processes start out. The uppermost and most anterior of these is a short lancet-shaped piece (*l.*), which occupies a position similar to that of more anteriorly situated neural spines.

The next succeeding process—the urostyle (*u.*)—consists of a pair of long, styliform plates, running obliquely upward. The two pieces are arranged in the fashion of a razor-sheath and enclose between them the hinder end of the notochord, while the spinal chord runs along their dorsal edge. The urostyle is, in fact, the continuation of the vertebral column, representing a certain uumber of coalesced vertebral bodies. At its distal end is found a pair of free slender bones (*h.*), one on each side.

The third process, structurally connected with the last recognisable centrum, is a flat, dilated bone (*e.*) which running backward forms an acute angle with the urostyle. Just below the root of the third process (*e.*) and facing obliquely backward and downward, there is in the last vertebral centrum a depression, to which the fused extremities of two flat bones (*f.* and *g.*) are connected by cartilage. The centrum (*l.v.c.*) may in fact be considered as possessing one neural (*l.*) and three caudal appendages (*e, f, g.*).

In the angular space formed by the urostyle (*u.*) and the upper-

most hæmal process (*e.*) there is a series of four bones (*a*, *b*, *c*, *d*), whose anterior extremities are wedged into the space between the two pieces of the urostyle. A narrow space on the line of prolongation of the vertebral axis divides the whole series of hypural bones—as the bones under the urostyle have been called—into two groups. The posterior edges of these hypural bones are all truncated in much the same way, and present an almost even line at which the caudal fin-rays are attached. The seven flat hypural bones and the two hæmal spines from the penultimate and ante-penultimate centra support the rays of the ventral lobe of the caudal fin.

Between the urostyle and the neural spine from the penultimate centrum there is found a freely suspended piece of bone—the "*falscher Dorn*" (*f.d.*). The urostyle, the "*falscher Dorn*" and the two neural spines from the penultimate and the ante-penultimate centra, constitute bony supports for the rays of the dorsal lobe of the caudal fin.

It is thus apparent that the greater portion of the tail is to be designated as the ventral lobe while the dorsal portion occupies only a comparatively insignificant part of the whole. The original diphycercal state, in which the ventral and dorsal lobes were equally distributed on both sides of the vertebral axis, is now disturbed by the upturning of the latter near its caudal termination. The great development of the inferior appendages and the backward condition of the superior consequent on this, resulted in the expanded, fan-shaped arrangement of the caudal skeleton.

With this brief description of the bony elements in the vertical form of the tail, we proceed to the examination of those of gold-fishes in various stages of modification. We will first take up a gold-fish with the tail divided in three distinct lobes as shown in Fig. 7. In comparing the caudal skeleton of such a form with that of a vertical

form (Fig. 6) we observe a general agreement in the arrangement of the various pieces, except in one important respect, namely, that in the former case there is a double series of bones supporting the rays of the ventral caudal lobe (in Fig. 7 *a*, *b*, *c*, *d*, *e*, *f*, *g*, *p.h*, *a.p.h.* forming one series and *a'.p'.h'*, *p'.h'*, *g'*, *f'*, *e'*, &c., forming the other) corresponding to the two lateral halves, into which the tail fin is divided. In the case of Fig. 6 in which all caudal rays are arranged in one vertical plane, the bones supporting them form one series in one plane (*a*, *b*, *c*, *d*, *e*, *f*, *g*, *p.h.*, *a. p.h.*).

Here I will mention that the bones supporting the caudal fin-rays in a gold-fish, are each tipped by a cartilaginous cap at their distal ends (Fig. 7). At the lower portion of the ventral caudal lobe and close to the cartilaginous cap just mentioned there is on each side a small piece of cartilage (*K.* Fig. 7) freely intercepted between the roots of fin-rays. In our figure the left piece alone is represented.

The comparison of some individual bones in a single and a double state of the caudal fin will bring out the points of difference into a clearer light.

Fig. 10 represents a penultimate vertebra from a common Carassius in which the tail is normal. Fig. 10 *a* is the corresponding bone taken from a gold-fish with a double tail. The neural appendage and the centrum are similar in both. In the gold-fish the hæmal arches, after uniting in the median line, again separate and diverge in different directions (*h.s.*, & *h'.s'.*). In the common Carassius however, the hæmal arch sends out but one process,—the hæmal spine (*h.s*, Fig. 10).

Fig. 11 and 11*a* represent the last vertebral centrum with their appendages. The former is taken from the vertical and normal tail of Carassius and the latter from the gold-fish with paired tail. It

will be observed that the hindmost process (*e*) is single in the former and double in the latter (*e*, *e'*).

Figs. 12 and 13 are two corresponding bones, the fourth hypural bone, from a single and a double form of the tail respectively. The former is single whereas the latter consists of two diverging limbs. Its apical portion lies wedged in between the two pieces of urostyle.

In the tail fin with its ventral edge furrowed by a shallow groove, (Figs. 8 & 9), the hæmal spines from the penultimate and antipenultimate centra proceed from their origin in a distinctly paired state in order to support the paired portion of the tail. In other cases in which the groove is deeper, those skeletal supports corresponding in position to the groove occur in pairs.

Figs. 19—23 represent diagramatic cross-sections of the several forms of the tail-fin.

Fig. 19 shows the arrangement of bones in a vertical and normal tail. On comparing this figure with others it will readily be seen that the difference in the depth of the ventral groove affects the bony structures in corresponding degrees.

The paired state of hypural bones in gold-fishes is of no small interest in connection with the view that the hypural elements are homologous with the more anteriorly situated subvertebral appendages, and that the styliform urostyle is a coalesced representative of a certain number of vertebral bodies. On comparing the subvertebral appendages in various regions of the body, we observe that, in the trunk, the appendages start in pairs (transversal processes) and end in pairs; that, in the caudal region, they start likewise in pairs (hæmal arch) which however soon coalesce and terminate in a median process (hæmal spines); and that in the caudal extremity, the appendages start from the beginning in a coalesced state as median

spines. The above is the usual arrangement that is to be met with in Teleostei. In the gold-fish however, appendages in the caudal extremity may start in pairs and end as such, somewhat like hæmal appendages in the trunk region.

It now remains to consider the system of caudal fin-rays, which in fact plays a very significant part in causing the anomalous phenomena of the tail of gold-fishes. Each fin-ray consists of a pair of similar parts running along-side and closely applied to each other for the greater part of their length. At the proximal extremity the two diverge a little and firmly clasp the terminal portion of the caudal skeleton. They are smooth and simple at the base, but become distally segmented into a number of small joints and moreover longitudinally split into a number of finer rays. Crescentic marks surrounding the caudal skeletons in Figs. 19—23 show the distribution of fin rays in various forms of the tail.

It will now be easy to conceive in what light the three or four lobed tails are to be regarded in relation to an erect and normal form. Thus, when a normal tail with an emarginate outline (Fig. 14, Pl. XIX), is split into two halves in a vertical plane, each half retaining its emarginate form, the "four- lobed tail" comes into existence (Fig. 15). When the splitting is not complete and the two halves remain united at their dorsal edge, the form known as the tri-lobed tail is the result (Fig. 16).

The Anal Fin. The anal fin of gold-fishes in its vertical form consists of nine fin-rays, supported by seven inter-hæmal bones. These numbers however vary to a certain extent with the individual. The first three of the fin-rays are solid spines, of which the third is the longest and the strongest of all. The following six fin rays branch into finer rays. The inter-hæmal bones bring the anal fin into connection with the hæmal spines.

Each fin-ray consists for the greater part of its length of two similar pieces placed side by side and closely applied to each other. At the thicker end, with which they come in contact with the inter-hæmals, the two pieces diverge and present the shape of Y. In the bay formed by the parted extremities a small bony nodule is intercepted (Fig. 17).

In the double form of the anal fin (Fig. 18), bony structures similar to those of the single form are present in pairs. The observer is at once struck with the close resemblance existing between the paired inter-hæmal and the pelvic girdle. The anal fin also very closely agrees in appearance with the ventral fin and one might think that such a fish has a third pair of extremities (Fig. 3, Pl. XVIII). The double anal fin often makes a flapping motion and seems to be capable of serving the same function as the ventral fin, although on a much more limited scale. When the anal fin is partially paired, the corresponding interhæmal bones alone occur in double state, while the rest is unpaired and median.

Primordial Fin-folds of the Gold-fish Embryo.

The examination of gold-fish embryos shows that the double anal and caudal fins are already laid out in the primordial fin-folds. In a gold-fish in which these fins are unpaired, the primordial fin-fold is likewise unpaired as shown in Fig. 27 Pl. XX. Where they occur in pairs however the primordial folds are laid out as two longitudinal thickenings along the ventral side of the post-anal section of the body. Fig. 26 represents an embryo taken from an egg three days after it was laid. The ectodermal cells aggregate in two similar ridges ($f,' f'$) along the ventral side in the posterior position of the body, forming the foundation of primordial fin-folds. A similar aggregation of

ectodermal cells forming a dorsal median thickening constitutes the beginning of the dorsal fin-fold. This fold extends backward and around the caudal termination and passes over to the double primordial ridge (*f,' f'*) on the ventral side.

With the growth of the embryo, the ridges or folds grow in height. The dorsal fold extends anteriorly as far as the level of the eyes. The two ventral ridges behind the anus diverge from each other at a later period as shown in Figs. 28 and 29.

Still later, at two regions, one behind the anus and the other at the hind end of the body, the folds greatly develope and become markedly broad, specially at the latter region as represented in Fig. 33.

The portion lying between these two local thickenings remains undeveloped and becomes finally atrophied.

The thickenings (*a.'f'.*) just behind the anus are the rudiments of anal fins and the hind ones (*c.'f'.*), those of caudal fins.

Fig. 29 represents an embryo of this stage as seen from the ventral side. This state of the ventral fold gives rise to the paired anal and caudal fins. In other cases, the paired portion is restricted to the region just behind the anus, the remaining portion being unpaired (Fig. 31). Such a state would give rise to a form in which the anal fin alone is paired. In others again, it is in the caudal extremity alone that the fold is double (Fig. 30). From such an instance there arises the form with the caudal fin alone paired, while the anal remains normal.

The varying extent to which the ventral portion of the primordial fin-fold is cleft is the cause of all those intermediate fins between the paired and the unpaired state. Fig. 32 represents a case in which only a shallow groove runs along the ventral margin of the fin-fold. Such an embryo, for instance, would give rise to fins which are only partially double.

Thus, there is in the embryo as in the adult, a complete series of gradations between the paired and the unpaired form of fins. The anomalous condition observed in the anal and caudal fins of gold-fishes is apparent from the very begining of their existence.

Preanal Folds. The structure known as the preanal fold is usually well-marked in gold-fish embryos. It often occurs in a paired state extending from the anus up to the level of the spots where the ventral fins appear later (Fig. 33, *p.'a.'f.'*) The two often unite at their anterior ends into a median fold (Fig. 29, *m.*)

The latter extends further anteriorly beyond the level of the ventral fins. The preanal folds originate as ridges of ectodermal cells like the caudal or anal fins.

The existence of double preanal folds in the embryo is another interesting feature of gold-fishes. Thus there exists a pair of continuous longitudinal folds running along the ventral side of the body from the level of ventral fins down to the very end of the body, save the single local interruption on account of the existence of the anus (Figs. 24, 25, 26, 28).

To show how such an embryo gold-fish differs from the embryo of a normal teleost in this respect, I refer to Figs. 24 and 25. I have connected the roots of the fins and preanal folds with dotted lines. Fig. 25 shows the ventral aspect of a young gold-fish, half an inch in length, in which the ventral fins have just begun to bud out and the double preanal folds are in process of atrophying. The above-mentioned dotted lines are furthest apart from each other at the pectoral region and slightly converge toward the ventral fin; behind the latter, they suddenly approach each other, almost meeting in front of the anus, behind which the two again gradually diverge.

At the commencement of the caudal fin the divergence becomes suddenly marked and finally they meet with each other. In a

normal teleost the case is quite different (Fig. 24). Here the two lines suddenly converge behind the ventral fins and unite with each other, in which state it continues down to the end of the body. The preanal folds are embryonic structures; they vanish as the embryo advances in development.

General Observations.—From what has been said about the skeletal element of the paired anal and caudal fins, it is sufficiently evident that they are accompanied by somewhat profound structural deviations from their normal type. It seemed to me exceedingly doubtful if such deviations were purely accidental productions of artificial selection, designed after the breeder's fancy and devoid of any significance whatever from the standpoint of comparative morphology. The examination of embryos has clearly proved that the paired state of those fins is anticipated from a very early embryonal period, being laid out as two longitudinal folds. The idea that the law of abbreviated heredity had here been active in shifting the artificially acquired paired condition of the anal and caudal fins into the embryonal period, seems to be untenable. On the other hand, the most plausible explanation lies in assuming that under certain circumstances certain fishes have the anal and caudal fins laid in a double state and that breeders have taken advantage of this fact in producing their double-tailed forms.

The development of double folds as "*Anlage*" for the anals and the caudals, I regard as a case of reversion to the primitive state. It stands in favor of the view entertained by St. Mivart, Thacher, Dohrn, Balfour and Mayer with regard to the origin of vertebrate limbs. These authors claim, in contradistinction to the well-known gill-arch theory of Gegenbaur, that the paired fins of fishes were derived from originally continuous lateral folds and further that the anal as well as the ventral portion of the caudal fin arose through

coalescence of the same folds.

The occurrence of double preanal folds in gold-fishes coincident with the double post-anal folds, harmonizes with my view. The median preanal and post-anal folds usually met with in a normal Teleost are then both to be looked upon as the coalesced state of the two folds.

The dorsal fin has also been interpreted by some authors to have arisen through coalescence of two dorsal folds (Parapodia). As the probable cause of this coalescence, is assigned the closing of the medullary plate. In this connection it ought to be mentioned that in gold-fishes the dorsal fin and that portion of the caudal lying dorsal to the vertebral axis are never met with in a double state,—showing that the median unpaired condition of the dorsal fold is more strongly rooted than in the ventral fold.

Provided that the pectoral and the ventral fins are really specializations of the same lateral folds as the post-anal fins, all these are to be regarded as homologous. The interhæmals might then be looked upon as representatives of the pelvic or shoulder girdles. The homology of the anal fin with the more anteriorly situated limbs is most apparently borne out in the case where it is paired as was described on page 258. The paired caudal fin, accompanying the most complicated portion of the axial skeleton, has undergone such modifications that the recognition of all parts homologous with those of the more anteriorly situated appendages has become a difficult task. The analogy of position however disposes me to assume that the free cartilaginous nodule (Fig. 7. K.), intercepted between the diverged extremities of caudal fin-rays, represents either some degenerated Interhæmals or some bony nodules similar to those found in the anal fin (Figs 17 & 18, *n*, *n'*,), or perhaps the aggregate of both.

To recapitulate;

(1) In the paired anal and caudal fins of gold-fishes, the internal bony structures are also paired.

(2) The view that the hypural bones of Teleostei are homologous with the more anterior pairs of subvertebral appendages receives a certain confirmation by the actual occurrence of paired hypural bones in the gold-fish.

(3) If the pectoral and ventral fins are to be looked upon as specializations of originally continuous lateral folds in the pre-anal section of the body, the anal and caudal fins are to be regarded as specializations of the same folds in the post-anal section of the body.

(4) The four pairs of appendages may therefore be considered as homologous structures.

(5) A certain degree of similarity is still recognizable in the structure and arrangement of parts in these four pairs of appendages in the adult.

(6) In many gold-fish embryos, a pair of pre-anal and post-anal folds are developed, which may represent the once continuous lateral folds.

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EXPLANATION OF FIGURES.

PLATE XVIII.

Fig. 1. Dorsal view of a gold-fish belonging to the breed "Riukin." Natural size. P., pectoral fin; V., ventral fin; C., caudal fin; D., dorsal fin.

Fig. 2. Lateral view of the same gold-fish. Half natural size. A., anal fin.

Fig. 3. Part of the ventral view of the same gold-fish, showing the paired anal fin A. An., anus.

Fig. 4. Dorsal view of a gold-fish belonging to the breed "Maruko," otherwise called the "Corean breed." Half natural size. Dorsal fin absent; head and face with protuberances. Tail three-lobed. S., a sickle-shaped appendage on the dorsal edge of the tail.

Fig. 5. Part of the same viewed from the ventral side and showing the paired anal fin.

PLATE XIX.

Fig. 6. Caudal skeleton of Cyprinus carpio L. Natural size. *a, b, c, d, e, f, g*, hypural bones. *h.*, a slender bone freely attached to the urostyle; left piece of the pair.

f.d., "falscher Dorn"

l., lancet-shaped process.

l.v.c., the last recognizable centrum.

p.h., haemal spine of the penultimate centrum.

a p.h., the same of the anti-penultimate centrum.

p.v.c., penultimate vertebral centrum.

p.n., neural spine from the penultimate centrum.

a.p n. the anterior of the two neural spines proceeding from the antipenultimate centrum.

a.p.n., the posterior neural spine of the same.

a.p.v.c., anti-penultimate vertebral centrum.

u., urostyle.

i.h., inter-haemal spines.

A., anal fin.

Fig. 7. Caudal skeleton of a gold-fish with tri-lobed tail, much enlarged posterolateral view. The left lower lobe of the tail-fin is omitted. Most of the letters are same as in Fig. 6.

e'., f'., g'., p'.h'., a'.p'.h'., show the right halves of hypural bones, corresponding to *e., f., g., p.h., a.p.h.*, of the left. K., the free cartilaginous nodule. The cartilaginous terminal caps of hypural bones are colored blue. D.L., the

dorsal lobe and *V. L.*, the ventral lobe of the caudal fin (the latter consisting of two halves).

Fig. 8. Tail fin of a gold-fish, with a shallow groove along its ventral margin. Letters as in figs. 1 and 2. *C′*, the paired portion of the tail-fin.

Fig. 9. Ventral view of the same. *B*, the ventral groove.

Fig. 10. Penultimate vertebra of Cyprinus carpio.

n.a., neural arch; *v.c.*, vertebral centrum; *h.a.*, haemal arch.

h.s., haemal spine.

Fig. 10*a*. The same of a gold-fish with tri-lobed caudal fin. *h. s.* occurs in a pair. Enlarged.

Fig. 11. The last vecognizable vertebral centrum of Cyprinus carpio. Lateral view. *l.v.c.* centrum; *l.*, lancet-shaped spine; *u.*, urostyle; *e.*, 5th hypural bone.

Fig. 11*a*. The same of a gold-fish with trilobed caudal-fin. Enlarged. *e′* and *e′* correspond to *e* of fig. 11.

Fig. 12. The 4th. hypural bone of Cyprinus carpio, corresponding to *d.* of Fig. 6. Vertical (*a*) and lateral (*b*) views.

Fig. 13. The same of a gold-fish. Enlarged. Vertical (*a*) and lateral (*b*) views.

Fig. 14. Normal tail-fin of Cyprinidae, with two lobes *a* and *b*.

Fig. 15. Four-lobed (*a*, *a′*, *b*, and *b′*) tail-fin of a gold-fish.

Fig. 16. Three-lobed tail-fin of a gold-fish.

Fig. 17. Transverse section through the region of normal anal-fin diagramatically represented. *i.s.*, interhaemal spine; *n.* bony nodule; *f.r.*, fin ray.

Fig. 18. Transverse section through the same region of a gold-fish with distinctly paired anal-fins. Diagramatic. *i′. s′.*, interhaemal bones; *n′.* bony nodules; *f.′r′.* fin rays.

Fig. 19–23. Diagramatic figures (transverse sections) showing the topographical arrangement of the caudal skeleton in different forms of the tail. *s.c.*, spinal chord. *u.*, *u′*, The two halves of the urostyle, intercepting the notochord.

PLATE XX.

Fig. 24. A young gold-fish, 13 mm. long. An, anus. The preanal fold (*p. a. f.*), the anal fin (*A*) and the caudal fin (*C*) are in unpaired condition. Lines drawn through the roots of various appendages on both sides of the body converge behind the ventral fins and unite altogether at the beginning of the preanal fold.

Fig. 25. A young gold-fish of the same size as the preceeding. The preanal fold (*p′*, *a′*, *f′*), the anal fin (*A′*) and the candal fin (*C′*) are all in paired state. Lines connecting the roots of various appendages on both sides of the body run separate throughout the entire body-length.

Fig. 26. An embryo gold-fish, 3 mm. in length and taken out of an egg

three days after it was laid. *p.a.f.*, rudiments of the preanal fold. *f,′ f′* paired postanal folds. *d.f*, dorsal fold.

Fig. 27. A gold-fish embryo with vertical fin-folds. 5.5 mm. in lengeth. *d. f.*, dorsal fold. *c. f.*, caudal fold. *a.f.*, anal fold. *p.a.f.*, preanal fold. *p. f.*, pectoral fin. An, anus.

Fig. 28. A gold-fish embryo, 6.5 mm. in length. The preanal fold (*p.′ a.′ f.′*), the anal fold (*a.′ f′.*) are double. The latter is continuous with the double caudal fold (*c.′ f.′*). The dorsal fold (*d. f.*) alone is median and unpaired.

Fig. 29. A gold-fish embryo, 8 mm. in length. Ventral view. m., median portion of the preanal fold. *p.′ a.′ f′*, double preanal folds. *a.′ f′*. double anal folds. *c.′ f′*, double caudal folds. *p. f.*, pectoral fin. An., anus.

Fig. 30. Postanal section of an embryo, in which the ventral fold is double only at the region of the caudal fin (*c.′ f.′*).

Fig. 31. The same, in which the ventral fold is double only at the region of the anal fin (*a.′ f.′*).

Fig. 32. The same, in which the ventral fold is double only at its free edge.

Fig. 33. Posterior half of a young gold-fish, 7.5 mm. in length. This specimen belongs to the breed " Maruko " and does not possess the dorsal fin fold. *p.′ a.′ f′*, double preanal folds. *a.′ f.′*, double anal folds (anal fin). *c,′ f′*, double caudal folds (caudal fin).

Some Notes on the Giant Salamander of Japan

(Cryptobranchus Japonicus, Van der Hoeven.)

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In the summer of 1880 and 1881, I visited some provinces in the interior of our island, for the purpose of collecting specimens of the Giant Salamander.

I succeeded in capturing seventy-one specimens, varying in length from 19 cm. to 69 cm., and in weight from 41 grams to 1926 grams, and in gathering some facts respecting their habits, and mode of life.

Although my observations on the above points are of a fragmentary character, I think I may assume that they will not be wholly devoid of interest, especially as they concern an animal which is remarkable not only in itself, but also for its close relationship with that celebrated fossil discovered more than a century and a half ago in the tertiary fresh-water deposits of Oeningen, and called by its discoverer Schenchzer, "Homo diluvii, testes."

It is now generally admitted that Schenchzer's "Homo," which he regarded as "ein recht seltenes Denckmal jenes verfluchten Menschengeschlechts der ersten Welt,": belongs to the same genus as the giant salamander of Japan.

Cuvier, who first recognized the amphibian character of this fossil, which should now be called Cryptobranchus Schenchzeri,* estimated its length at three feet five inches, a size seldom, if ever, reached by its Japanese representative.

It is remarkable that C. Japonicus is now no longer found outside a very limited area of central Japan, which, according to Temminck and Schlegel,† lies between 34° and 36° North latitude. My specimens were all collected in the three provinces, Iga, Ise, and Yamato. Siebold obtained a specimen said to have come from the mountains of Suzuga-yama, somewhat farther north.

The above named provinces are traversed in various directions by mountain ranges, between which are numerous valleys, raised several thousand feet above the level of the sea. Through these valleys pass swift running brooks, fed with clear cold water from mountain springs. I visited the streams of 13 valleys, seven in Iga, two in Ise, and four in Yamato.

These streams have everywhere stony beds, are quite shallow, and seldom attain a width of more than a few metres. In these small but swiftly flowing brooks,‡ thickly shaded, for the most part, with shrubs and trees, lives the subject of this paper.

It conceals itself in dark places under rocks, along the banks or in the middle of the stream. It seems to delight in solitary life; for so far as I was able to learn, not more than a single specimen is ever found under one rock.

The animal may be easily captured with a fish-hook, baited with

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* Naturgeschichte der Lurche, p. 132.

† Temminck and Schlegel. Siebold's "Fauna Japonica." Reptilia.

‡ The temperature of the water in the middle of August was found to be between 17° and 23° C.

a fish, frog or several earth-worms and tied to a string, a few feet in length. This is thrust by the aid of a small bamboo stick into the salamander's retreat. The string is not tied to the stick, but the point of the loaded hook is forced into one end of it, far enough to keep it in place while this end of the rod is pushed under the rock. When the bait has been thus brought near the salamander, any bite will be instantly felt through the rod.

When the bite is felt, the rod is withdrawn as quietly as possible, the hook and bait being left. As soon as a jerk of the string is noticed, a pull is made, which generally ends in the capture of the unfortunate animal. If the first pull should fail, the bait is replaced as before, and a second opportunity is offered which the unwary creature accepts as readily as the first. The fisherman having obtained one bite, is sure of ultimate success, as the salamander does not learn by experience to refuse the proffered morsel.

When captured, this animal emits a peculiar slimy secretion, having an odor much like that of the leaves of the Japan pepper (Xanthoxylon peperitum.) This secretion hardens into a gelatinous mass on short exposure to the air.

Temminck and Schlegel (*l. c.*, *p.* 128) state that the act of inspiration is ordinarily performed once every 6—10 minutes. This is true for specimens kept in tubs; but my observations lead me to think that they perform this act less frequently in their native brooks.

The eyes are remarkably small (measuring only 4 mm. in diameter), and this fact is perhaps correlated with their mode of life. For the capture of their prey (fish, frogs &c.), which they do, not by pursuing, but by waiting for the near approach of it, their eyes are obviously of comparatively little importance.

Besides they keep themselves habitually in dark places, and

thus live, as regards light, under conditions similar to those of cave life.

The manner in which they secure their food is correctly stated by Temminck and Schlegel, *p.* 129. "They slowly approach their prey and, by a swift lateral movement of the head, seize it with their teeth : holding it for some time in the mouth, the next act consists in swallowing it."

The Salamander is eaten by Japanese, and the flesh, when properly cooked, is said to be delicious. It is also used for medicinal purposes by both Japanese and Chinese, being supposed to be good for " Rogai," a kind of consumption.

Young Specimens.

The smallest individual found by Siebold measured about 30 cm., and showed no trace of external gills or branchial clefts. The youngest of my specimens, measuring 19 to 20 cm., have three pairs of very short branchial processes (from 3 to 5 mm. in length), attached just inside the branchial orifice. Each process is somewhat flattened and tapering, and most of them still have branchlets.

The color differs from that of the mature individual only in being lighter.

In another specimen, 20.5 cm. in length, the gills have almost wholly disappeared, but the branchial slits are still to be seen.

Another individual, 24.5 cm. in length, shows no trace of the gills, and the branchial orifice has completely closed, its original position being marked by a light streak. In this specimen a few of those dermal protuberances, characteristic of the adult forms, are to be seen on the dorsal surface of the head. In the larger specimens these protuberances are thickly crowded on the dorsal side of the head, extending down the sides and on the lateral portions of the

same. There are four rows of these protuberances on the dorsal side of the body. Two of them run along each side of the median dorsal line, at some distance from it; while the remaining two run just above the lateral dermal folds, extending to the tip of the tail.

The following gives the weights and measurements of five specimens.

No.	Length.	Width taken at head.	Weight.
1	20.0 cm.	2.6 cm.	42 grms.
2	20.5 "	2.9 "	55 "
3	24.5 "	3.0 "	61 "
4	55.0 "	9.1 "	970 "
5	69.0 "	11.0 "	1926 "

The Eggs, Time of Deposit, &c.

The fishermen report that the eggs are laid in August and September. I succeeded in getting a few eggs deposited about the middle of August.

The eggs are not laid one by one as in the case of our common Triton, nor in smooth cylindrical strings as in the case of the Toad, but in a string that resembles in form a rosary.

Each egg floats in a clear fluid inclosed in a bead-shaped gelatinous envelope (1.62—1.35 cm); and this envelope is connected with the next by means of a comparatively small string which is about equal in length to the longer axis of the envelope. The egg has an oblate spheroidal form, measuring about 6 mm. by 4 mm., and is yellow everywhere except at the upper pole, where it is whitish.

All attempts to make Cryptobranchus breed in captivity have failed hitherto, owing no doubt to the difficulty of obtaining in the city cool water such as the animal is accustomed to in its mountain home.

A Pocket Galvanometer.

By

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Assistant to the Professor of Physics, Imperial University.

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§ I. Coilless Pocket Galvanometer.

If a permanent magnet be fixed by an axis through its magnetic axis, it will be perfectly restrained from responding to external magnetic influences. But if the magnet be fixed by an axis which is only parallel to, and not coincident with, its magnetic axis, it will still be in neutral equilibrium in a uniform magnetic field. In other words, a magnet so supported cannot be distinguished from non-magnetic bodies whatever be the strength of the field so long as this remains uniform ; induction being neglected for the time.

If one or any number of small bar magnets be vertically attached to a suspended piece of wood, or if a horse-shoe magnet be hung by a string as in *Fig. 1.*, such a system may be realized. The astatism is quite independent of the number of magnets, as each several magnet is in neutral equilibrium.

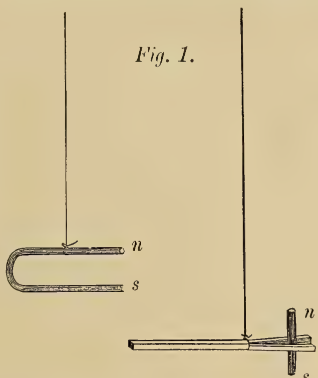


Fig. 1.

In such a state of equilibrium, if a pole of another magnet, or a wire conducting a current be brought near, the first magnet will be pulled or pushed as the case may be, so as to rotate round the fixed axis; the only necessary condition being that the forces acting on the two poles of the magnet shall be unequal, as will generally be the case in the neighbourhood of a straight conductor. The circumstances of rotation will depend upon the strengths of the magnet and of the current respectively.

Any small portion of a closed electric circuit may be looked upon as the edge of a large magnetic shell. Now, if two opposite poles of a small magnet be placed close to the edge of such a shell at equal distances from the edge, the electro-magnetic force acting on the magnet will be constant, provided the line joining the two poles always passes through the edge of the shell, no matter how the other portions of the shell may lie with regard to the magnet. If four poles in rigid connection be placed about such an edge, it will be possible to find such an arrangement of the poles that the force acting on the system of the poles will be sensibly constant when the edge is within a certain portion of space between the four poles.

Thus it becomes possible to construct an instrument which will measure the strength of such a shell without breaking its continuity, and that independently of the uniform field in which the shell may be situated. That is, an instrument which will measure the current without breaking the circuit and which can be used in any position and in any uniform field. This is the idea upon which the apparatus to be described is constructed. It is indeed simply a modified form of an astatic galvanometer.

The following is one particular form of the apparatus :—Four small bar magnets are fixed symmetrically at the four corners of a thin rectangular plate of wood, which can rotate as a whole about a

fixed central axis perpendicular to its plane and parallel to the magnets, as shown in *Fig. 2.* What we want to measure is the couple about this axis. We may accomplish this by means of two springs, one attached to the frame, the other to an index, and both springs initially stretched very much as in an ordinary aneroid barometer. The motion of the index

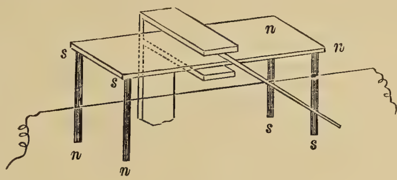
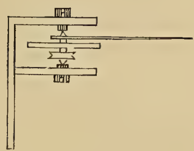


Fig. 2.



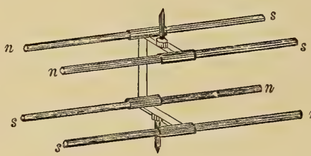
is magnified by means of a multiplying lever introduced between the index and its spring, as shown in *Fig. 4* below. As we cannot use steel springs, an alloy of gold and brass, which is used in making springs for *pince-nez* eyeglasses, answers here very well. Ordinary brass springs however are easier to make, although they are not quite so lasting.

The whole arrangement is then put in a case with a jaw-like vacant space hollowed out for receiving a conducting wire. The axis for the index projects above a graduated circular plate, and the index is fixed at right angles to the jaw, in such a way that when part of a circuit is slipped in within the jaw, the index will be moved toward that side to which the current is passing; in other words we may imagine the tip of the index to be carried by the current. The value of the indications depends upon the strengths of both the magnets and the springs, and has to be obtained by comparison with some standard galvanometer; and this calibration must be repeated occa-

sionally as with other instruments of a similar description.

There is one inconvenience however in this system, namely, that the four magnets placed round the current have all their axes perpendicular to the direction of the current and consequently they are subject to the effect of induction. The induction will be positive for the two approaching magnets (that is their moments will be increased), and negative for the receding two. The total effect on the system will be nearly nil for small currents; but with large currents, the magnets may be permanently affected and may even be reversed in magnetization. We can get rid of this inconvenience by fixing the four magnets to an axis which is perpendicular to each axis of the magnets as in *Fig. 3*. It will be seen that the system is exactly the same as the previous one, if the four magnets be of equal moments or if they satisfy the condition $\sum M = 0$ with respect to the fixed axis. The former condition we can not hope to attain in practice, but the latter can be approximated to thus :—

Fig. 3.



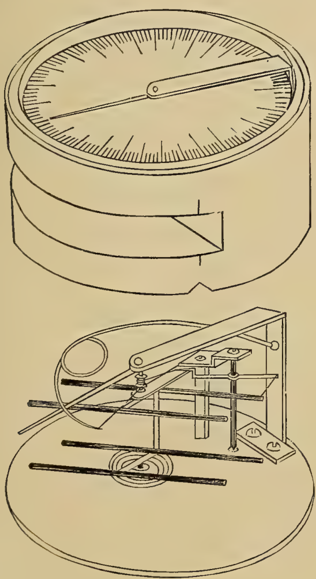
Make some 10 or 20 magnets of the same steel wire and leave them for some months or even for years, until the time-rate of the fall of moments becomes insensible. Then measure the moment of each magnet by any of the ordinary magnetometric methods, such as the deflection of a mirror magnetometer at a constant distance from each magnet, a knowledge of the relative strengths only being essential. The magnets I used in one arrangement had the following relative moments.

(*a*)	(*b*)	(*c*)	(*d*)
313.3	311.7	312.0	313.0

This gives $(a) + (b) = (c) + (d)$ within the errors of experiment, an arrangement which was accordingly adopted. We may notice here, that if the determination of any one of them were in error of 1 in the unit figure, its uncompensated effect would be $1/\sum M = 1/1250$ of the whole.

In other details of construction this form of apparatus is exactly the same as the previous one, except that the jaw is side-wise instead of vertical. This gives a greater precision to the instrument as will be seen immediately, and still more facilitates the introduction of the coils of fine wire for measuring large differences of potential and small currents in the way to be described further on. The following sketch shows one of the working instruments.

Fig. 4.



In calibrating the instrument, it is best to use a small current repeated many time instead of a single strong current which is so difficult to keep steady. For this, 200 turns of a fine insulated wire were wound round in the form of a circular ribbon having a circumference of about 1 metre. The wires were tied together with a string so that the external appearance was like one thick insulated wire of circular

section. One portion of this was put into the jaw, the remaining portions being as far removed as possible from the instrument. A current from 20 Daniell cells was passed through the ribbon ; a standard tangent galvanometer and a resistance box being in the circuit. Simultaneous readings of the standard galvanometer and the pocket instrument were taken for different strengths of current, obtained by varying the resistance. From these the value of the current corresponding to 1 division of the pocket instrument was easily calculated. The following table gives the results of the comparison thus made with the working apparatus.

Standard Galvanometer.		Pocket Galvanometer.	
Reading.	Reduced to Ampères.	Reading.	Value of 1 Division in Ampères.
12.0	.0350	12.4	.565
19.7	.0575	20.6	.558
28.9	.0844	30.5	.554
35.5	.1037	37.4	.555
		Mean	.558

As the instrument was graduated to 100 divisions we could measure with this up to 56 ampères, and, as will be seen later, by placing the wire below the instrument up to 168 ($= 3 \times 56$) ampères. In making the comparison, we must place the standard galvanometer at a considerable distance from the ribbon, which becomes a strong magnetic shell when the current is passing. In fact it is best to put the two instruments on different tables, and make two series of observations, one with the current direct and the other with it reversed. For measuring a moderate current such as 1 or 2 ampères we can advantageously repeat the circuit three or four times by simply coiling the conductor so as to make a temporary ribbon like that just described.

By altering the points of support of the springs (as in the time rating of a watch) we may, if it is required, adjust the instrument so as to make one division correspond to some simple fraction of an ampère, say .5 or .2

Field of Force and Arrangement of Magnets.

In studying the field of force due to four magnets arranged as in *Fig. 4*, and a single straight conductor parallel to their axes, we have only to consider the action of a single straight conductor upon one set of four poles which lie in a plane perpendicular to the conductor, since the other set of poles is an exact counterpart of the one considered. As proved in Maxwell, the electromagnetic force at an external point of a straight cylindrical conductor of infinite length depends only upon its distance from the center of the section, for any concentric distribution of current; and since the action between a magnet and a current is mutual, if a compound cylindrical magnet consists of concentric layers of uniform intensities, its action upon an externally placed current running parallel to the axis of the magnet, must be reducible to the action of a single equivalent pole at its centre, whatever be the law of distribution of magnetism from layer to layer. As we make our magnets of cylindrical wire, and as most conductors are cylindrical, we may safely reduce the action to the centers of the sections of the magnets and the conductor, neglecting the pole-shifting effect due to the induction of magnets on each other.

Let $2a$ be the distance between like poles, and $2b$ that between unlike ones: and put r for $\sqrt{a^2 + b^2}$.

Let x, y be the co-ordinates of any point referred to the center of the rectangle $[2a, 2b]$, measured parallel to a and b respectively.

Let ξ η be the co-ordinates of any one of the poles, referred to *any position of the current as origin*, so that, if x, y be the co-ordinates of the current,

$$\xi + x = a \text{ and } \eta + y = b$$

Then $\rho = \sqrt{\xi^2 + \eta^2}$ is the distance of any pole from the current.

The potential energy of unit current and four unit poles is

$$V = 2\sum tan^{-1}\frac{\eta}{\xi} + const$$

$$= 2\sum\theta + const \qquad (1)$$

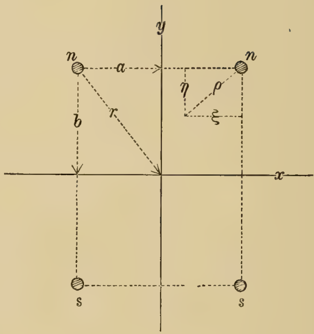


Fig. 5.

$\sum$ meaning summation for the four distinct poles, and θ being the angle between any of the polar distances and the axis of ξ. V then reduces to the algebraic sum of the two angles subtended by the two parallel lines of length $2\,b$ placed $2\,a$ apart. This is the same as the potential energy of unit pole and two parallel magnetic strips of infinite length placed $2\,a$ apart, the breadth of each being $2\,b$. The solid angles ω_1 and ω_2 of the usual notation become two spherical wedges. The two magnetic strips may be replaced by two equal pairs of parallel currents, placed along the edges of the strips, as will be evident *à priori*, since the action of a unit current upon four unit poles must be the same as that of four currents upon a unit pole so far as the dynamical aspect is concerned. This latter was the combination I used in making an experimental verification of the result.

From (1) the equation

$$\sum\theta = const.$$

gives the equipotential surfaces, and

$$\sum log\ \rho = const.$$

gives the lines of force. *Fig. 6* shows a special case of such a field.

To determine the best arrangement of the magnets so as to minimize the error due to the eccentric position of the current (which is the same thing as that due to the deflected position of the magnets) we may proceed to find the electro-magnetic forces acting upon the system of magnets, when the current is placed in any position x, y. Putting F for this force and expanding it in terms of eccentric displacement ∂x, ∂y, we have,

$$\begin{aligned} F = F_0 &+ \frac{\partial F}{\partial x}\partial x + \frac{\partial F}{\partial y}\partial y \\ &+ \frac{1}{\lfloor 2}\left\{\frac{\partial^2 F}{\partial x^2}(\partial x)^2 + 2\frac{\partial^2 F}{\partial x \partial y}(\partial x)(\partial y) + \frac{\partial^2 F}{\partial y^2}(\partial y)^2\right\} \\ &+ \frac{1}{\lfloor 3}\left\{\frac{\partial^3 F}{\partial x^3}(\partial x)^3 + 3\frac{\partial^3 F}{\partial x^2 \partial y}(\partial x)^2(\partial y) + 3\frac{\partial^3 F}{\partial x \partial y^2}(\partial x)(\partial y)^2 + \frac{\partial^3 F}{\partial y^3}(\partial y)^3\right\} \\ &+ \cdots\cdots \end{aligned}$$

where F_0 is the force at the origin. On account of the symmetry of configuration, the terms containing odd powers of either ∂x or ∂y vanish when taken for all the four magnets. The equation then reduces to

$$F = F_0 + \frac{1}{2}\left\{\frac{\partial^2 F}{\partial x^2}(\partial x)^2 + \frac{\partial^2 F}{\partial y^2}(\partial y)^2 + \frac{\partial^4 F}{\partial x^4}(\partial x)^4 + \cdots\cdots\right\} \cdots\cdots (2)$$

where $$-F = \frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\, 2\sum tan^{-1}\frac{\eta}{\xi}$$

but since

$$\left.\begin{aligned} x &= a - \xi \\ y &= b - \eta \end{aligned}\right\}$$

$$\frac{\partial^n}{\partial x^n} = (-1)^n \frac{\partial^n}{\partial \xi^n}$$

$$\frac{\partial^n}{\partial y^n} = (-1)^n \frac{\partial^n}{\partial \eta^n}$$

$$F = \frac{\partial}{\partial \xi}\sum tan^{-1}\frac{\eta}{\xi} = \sum\frac{-\eta}{\xi^2 + \eta^2}$$

also

$$\frac{\partial^n}{\partial \xi^n}\left(\frac{\eta}{\xi^2+\eta^2}\right)=\frac{(-1)^n \lfloor n \sin (n+1)\,\theta}{(\xi^2+\eta^2)^{\frac{n+1}{2}}}$$

$$\frac{\partial^n}{\partial \eta^n}\left(\frac{\eta}{\xi^2+\eta^2}\right)=-\frac{(-1)^n \lfloor n \sin (n+1)\,\theta}{(\xi^2+\eta^2)^{\frac{n+1}{2}}}$$

$$\tan\theta=\frac{\xi}{\eta} \qquad \text{as before}$$

Thus, taking only the increment of force, we get from equation (2)

$$\Delta^2 F=-\sum\left(\frac{\sin 3\,\theta}{\rho^3}\,\partial x^2-\frac{\sin 3\,\theta}{\rho^3}\,\partial y^2+\ \cdots\cdots\cdots\right)$$

but when ∂x and ∂y are each small ρ becomes very nearly r, and θ may be regarded as measured from the center of the rectangle [$2a$, $2b$]. We may then dispense with the sign $\sum$ in discussing the configuration. Thus if

$$3\theta=\pi \quad \text{or} \quad n\pi$$

or

$$\theta=\frac{\pi}{3} \quad \text{or} \quad \frac{n\pi}{3}$$

the coefficients of $(\partial x)^2$ and $(\partial y)^2$ vanish simultaneously. In other words if the magnets are arranged occupying any four opposite corners of a regular hexagon, that is if a/b be nearly 4/7,* the error due to a small displacement will be eliminated up to the third order of the displacement inclusive.

The simultaneous disappearance of the coefficients of $(\partial x)^2$ and $(\partial y)^2$ might indeed have been anticipated from the general equation $\nabla^2 V=0$ since V is here function of only x and y we have

$$\frac{\partial}{\partial x}\nabla^2 V=\frac{\partial^2}{\partial x^2}\frac{\partial V}{\partial x}+\frac{\partial^2}{\partial y^2}\frac{\partial V}{\partial x}=0$$

now $\partial V/\partial x$ is F. Hence when either of $\partial^2 F/\partial x^2$ or $\partial^2 F/\partial y^2$ vanishes the other must do so too.

* Practically it will be found better to make a/b somewhat larger than 4/7, so to obtain a greater range of uniformity in the field.

The uniformity of the field arrived at by this arrangement is shown by the following diagram of equipotential lines and lines of force. Curves representing the intensities of forces along the coordinate axes and the diagonal are also given.

FIELD OF FORCE.

$$a^2 : b^2 : r^2 = 1 : 3 : 4$$

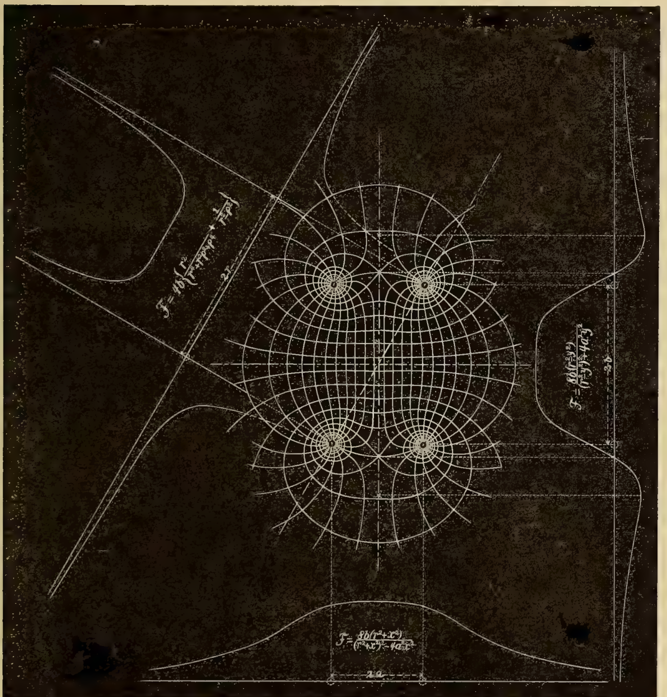


Fig. 6.

The general expression for the force is

$$-F=\frac{\partial}{\partial x}\,2\sum tan^{-1}\frac{\xi}{\eta}$$

$$=2\left\{\frac{b-y}{(b-y)^2+(a-x)^2}+\frac{b-y}{(b-y)^2+(a+x)^2}+\frac{b+y}{(b+y)^2+(a-x)^2}+\frac{b+y}{(b+y)^2+(a+x)^2}\right\}$$

If, now, we make

$$\left.\begin{aligned}&x=0\\&y=0\\&\rho=x\frac{a}{r}=y\frac{b}{r}\end{aligned}\right\}\text{ successively}$$

we obtain the expressions

$$\left.\begin{aligned}F&=\frac{8\,b\,(r^2+x^2)}{(r^2+x^2)^2-4\,a^2\,x^2}\cdots\cdots\cdots(\alpha)\quad\text{along } x\text{-axis}\\F&=\frac{8\,b\,(r^2-y^2)}{(r^2-y^2)^2+4\,a^2\,y^2}\cdots\cdots\cdots(\beta)\quad\text{,, } y\text{-axis}\\F&=\frac{4\,b\,r^2}{r^4+r^2\,\rho^2+\rho^4}+\frac{4\,b}{r^2-\rho^2}\;(\gamma)\quad\text{,, diagonal}\end{aligned}\right\}\text{ respectively}$$

The maxima and minima of these are given by

$$x=0$$
$$x=\pm\sqrt{r\,(-r\pm2\,a)}\;\cdots\cdots\cdots\cdots\cdots\cdots\;(\alpha')$$
$$y=0$$
$$y=\pm\sqrt{r\,(r\pm2\,a)}\;\cdots\cdots\cdots\cdots\cdots\cdots\;(\beta')$$
$$\rho=0$$
$$\rho=r\text{ (others being imaginary)}\cdots\cdots\;(\gamma')$$

from which we see that the pairs of maxima are superposed at the origin if $r=2a$, which is equivalent to $\theta=\frac{\pi}{3}$. The minima along the y-axis at $y=\pm\sqrt{r(r+2a)}$ are always real for any ratio of a/r. In the present case, the values become $\pm2\sqrt{\frac{2}{3}}b$. Substituting this value of y in (β) we get the value of F there $=-\frac{1}{3}F_0$. It will be

seen that at each of these points, the lines of force begin to change their curvature, and the field is sensibly constant. By constructing the instrument, so that the base lies just above one of these points, we adapt it for the measurement of very strong currents, such as 100 ampères or greater. To facilitate such a measurement a V-groove is cut out along the base in the proper position, and into this the circuit bearing the current is received. To reduce the value thus obtained to what it would have been, had it been placed in the jaw, we have only to multiply the reading by -3.

The following table giving the proportional decrement of force for given displacements of the magnets, will show to what amount of displacement we may, without sensible error, assume the uniformity of the galvanometer constant as obtained by calibration for small displacements. Computing $(F-F_0)/F_0$ we have

Along x-axis.		Along y-axis.		Along Diagonal.	
$\partial x/a$	$\partial F/F_0$	$\partial y/b$	$\partial F/F_0$	$\partial \rho/r$	$\partial F/F_0$
1/4	1/4161	1/4	1/435	1/4	1/445
1/3	1/1333	1/3	1/133	1/3	1/132
1/2	1/273	1/2	1/24	1/2	1/21
1	1/21				

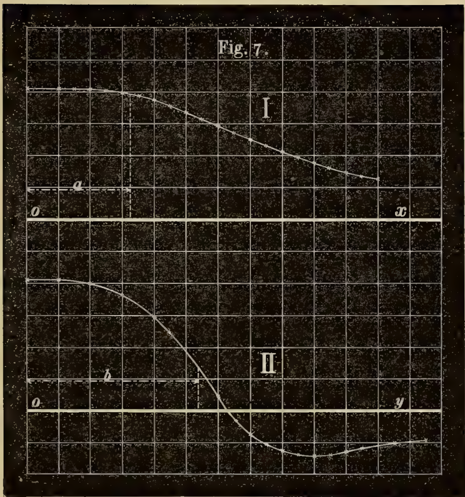
As long as the deflection is within $\frac{1}{2}a$ the error will be less than 1/2 per cent. and when it is to the extreme limit of a, the error will be about 5 per cent. It is interesting to notice that ∂F is very nearly proportional to the fourth powers of ∂x, ∂y, $\partial \rho$, even when these are $\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}r$. The above table enables us to adjust the range of the index and the jaw of the instrument so as to keep the magnitude of errors within any assigned limit. In the actual instrument constructed

the width of the jaw was $\frac{1}{2}b$ and the deflection was limited to $\frac{1}{2}a$, so that the error fell within $\dfrac{1}{273}$ for any reading even if we suppose the eccentricity in the direction of y to be as much as $\frac{1}{4}b$.

To verify the results thus far obtained the following experiment was made in the Physics Laboratory of the Science College of the Imperial University :—Six blocks of wood, each 21 cm. high and 13 cm. wide were arranged in a row upon a long laboratory table extending through a space of 3.7 metres along the magnetic meridian. These blocks served simply as guides for the stretching of two rectangular coils of insulated wire, whose distance apart bore to the height of either the ratio required (tangent 30°). Each coil consisted of six turns. Thus were obtained two parallel magnetic strips of practically infinite length.

One of Thomson's graded galvanometers with its field magnet taken away, was placed in the space between the two central blocks, its V-groove lying along the magnetic east and west line. The height of the galvanometer was so adjusted that the centre of the four small magnets, belonging to the fan-shaped compass, was always in the plane half-way between the upper and the lower lines.

A current was run round the wire and was left for about 20 minutes till its flow became steady, and then the compass was slid along the V-groove and its position and deflection simultaneously observed at several positions. From these observations the curve I in *Fig. 7* was obtained. By a slight modification of the arrangement, providing a vertical V-groove, the compass was made to move along a vertical line ; and from a similar series of observations the curve II was obtained. These should be compared with the curves of *Fig. 6*.



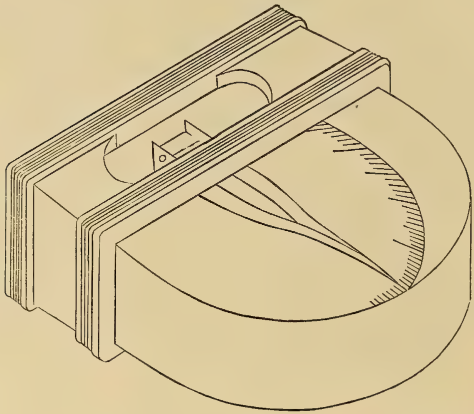
It will be seen that the system of magnets thus arranged is equivalent to Helmholtz's galvanometer of two circular coils, if we imagine the two coils to be deformed into two long rectangles of infinite length, that is, reduced to four parallel currents. For if we adjust these currents to proper positions, and replace each of them by a magnet and the central magnet by a straight current, we have the system just discussed.

§ II. Flat Coil Pocket Galvanometer.

I now pass to the description of the flat coil pocket galvanometer, which may in virtue of its compactness and simplicity of construction be found useful for some purposes, although it can not of course

take the place of circular coils when absolute determinations of electromagnetic constants are required. One such instrument is represented in *Fig. 8.* It is essentially the same as an old form of current

Fig. 8.



detectors, constructed however with due regard to the proportional dimensions which theory shows to be best.

The following calculation of the proper distance between two rectangular coils is made not so much for the sake of the flat coil galvanometer as for the sake of the internal coil mentioned above (page 279) and described in detail below (page 296).

Let $2a$ be the distance between the two coils,

,, $2b$,, ,, height of the coils

,, $2c$,, ,, length ,, ,, ,, .

Let x, y, z be the co-ordinates of any point referred to the centre of the coils as origin, axes being parallel to a, b, c.

Let ξ, η, ζ be the co-ordinates of any corner of the coils referred to any point x, y, z as origin, so that

$$\xi + x = a, \qquad \eta + y = b, \qquad \zeta + z = c.$$

Then the potential at any point due to unit current is the sum of the solid angles subtended by the coils, or

$$V = \sum sin^{-1} \frac{\eta\,\zeta}{\sqrt{\xi^2+\eta^2}\,\sqrt{\xi^2+\zeta^2}} = \sum \Omega \quad \text{say}$$

$\sum$ meaning the summation for 8 corners, there being four to each coil. Now,

$$\frac{\partial^n}{\partial x^n} = (-1)^n \frac{\partial^n}{\partial \xi^n}; \qquad \frac{\partial^n}{\partial y^n} = (-1)^n \frac{\partial^n}{\partial \eta^n}; \qquad \frac{\partial^n}{\partial z^n} = (-1)^n \frac{\partial^n}{\partial \zeta^n}.$$

Hence
$$F = -\frac{\partial V}{\partial x} = \sum \frac{-\eta\zeta}{\rho_0}\left(\frac{1}{\rho_1^2} + \frac{1}{\rho_2^2}\right)$$

where
$$\rho_0 = \sqrt{\xi^2+\eta^2+\zeta^2}; \quad \rho_1 = \sqrt{\xi^2+\eta^2}; \quad \rho_2 = \sqrt{\xi^2+\zeta^2}$$

The force at the center of the coils will be given by making $\xi, \eta, \zeta = a, b, c$ respectively and multiplying the result by n, if n be the number of turns of wire in the coil.

The increment of force at any displaced point $\partial x, \partial y, \partial z$, is given by the equation (odd terms disappearing as before),

$$\Delta^2 F = \frac{1}{\lfloor 2}\left\{\frac{\partial^2 F}{\partial x^2}(\partial x)^2 + \frac{\partial^2 F}{\partial y^2}(\partial y)^2 + \frac{\partial^2 F}{\partial z^2}(\partial z)^2\right\} + \frac{1}{\lfloor 4}\Big\{ \cdots\cdots\cdots\cdots$$

but, since

$$\frac{\partial}{\partial x}\nabla^2 V = \frac{\partial^2}{\partial x^2}\frac{\partial V}{\partial x} + \frac{\partial^2}{\partial y^2}\frac{\partial V}{\partial x} + \frac{\partial^2}{\partial z^2}\frac{\partial V}{\partial x} = 0$$

it is necessary and sufficient that any two of $\partial^2 F/\partial x^2$, $\partial^2 F/\partial y^2$, $\partial^2 F/\partial z^2$ should vanish simultaneously, in order that they all may vanish at the same time.

Thus we have

$$\left.\begin{aligned}\frac{\partial^2 F}{\partial y^2} &= \sum \frac{\eta\,\zeta}{\rho_0}\left\{\frac{3}{\rho_0^2}\left(1-\frac{\eta^2}{\rho_0^2}\right)\left(\frac{1}{\rho_1^2}+\frac{1}{\rho_2^2}\right) + \frac{6}{\rho_1^4} - \frac{4\eta^2}{\rho_0^2\rho_1^4} - \frac{8\eta^2}{\rho_1^6}\right\} = 0 \\ \frac{\partial^2 F}{\partial z^2} &= \sum \frac{\eta\,\zeta}{\rho_0}\left\{\frac{3}{\rho_0^2}\left(1-\frac{\zeta^2}{\rho_0^2}\right)\left(\frac{1}{\rho_1^2}+\frac{1}{\rho_2^2}\right) + \frac{6}{\rho_2^4} - \frac{4\zeta^2}{\rho_0^2\rho_2^4} - \frac{8\zeta^2}{\rho_2^6}\right\} = 0\end{aligned}\right\} \text{(A)}$$

which at the center of the coils (ξ, η, ζ being a, b, c) reduces to

$$\frac{\partial^2 F}{\partial y^2}=\frac{b\,c}{(a^2+b^2+c^2)^{\frac{5}{2}}(a^2+b^2)^3}\Big\{12a^6+(21b^2+15c^2)a^4$$
$$+(6[b^2+c^2]^2-2b^2c^2)a^2-b^2(b^2+c^2)(3b^2+2c^2)\Big\}=0$$

$$\frac{\partial^2 F}{\partial z^2}=\frac{b\,c}{(a^2+b^2+c^2)^{\frac{5}{2}}(a^2+c^2)^3}\Big\{12a^6+(21c^2+15b^2)a^4$$
$$+(6[b^2+c^2]^2-2b^2c^2)a^2-c^2(b^2+c^2)(3c^2+2b^2)\Big\}=0$$

Unless these two equations are simultaneously satisfied, the three partial differential coefficients will not vanish. Eliminating a between the two equations we find that the only admissible cases are when $b=c$, and either b or $c=\infty$. Thus it seems that $\Delta^2 F$ can not be made to vanish entirely except for the case of a square, and four infinite parallel straight currents. But when either b or c is great compared with the other, the partial differential coefficient with respect to the greater will be comparatively small as may be judged from the equations (A). Hence for such cases, if $\partial^2 F/\partial^2 x=0$, both the others must be small. But since $\dfrac{\partial^2 F}{\partial x^2}=-\left(\dfrac{\partial^2 F}{\partial y^2}+\dfrac{\partial^2 F}{\partial z^2}\right)$* we find from the above two equations

$$\frac{\partial^2 F}{\partial x^2}\propto 24a^{12}+72\,(b^2+c^2)\,a^{10}+[93\,(b^4+c^4)+146\,b^2c^2]\,a^8$$
$$+8\,(b^2+c^2)\,[9\,(b^4+c^4)+4b^2c^2]\,a^6$$
$$+3\,(b^2+c^2)^2[11\,(b^4+c^4)-10b^2c^2]\,a^4$$
$$+2\,(b^2+c^2)^3[3\,(b^4+c^4)-7b^2c^2]\,a^2$$
$$-b^2c^2\,(b^2+c^2)^2[2\,(b^4+c^4)+b^2c^2)\,]=0$$

from which a is to be found for any given value of b and c, the sides of the coils. Since the equation is homogeneous, we may take either b or c as unit of length and measure the other lengths in terms of it. Thus if we take b (half the height of the coils) as the unit, and express a and c in terms of it, we have all the possible cases brought out by varying c from 1 to ∞. Examining the above equation, we find that

* The equation $\partial^2F/\partial y^2+\partial^2F/\partial z^2=0$ shows that at the origin F, considered as a function of y and z, is a minimax with respect to those variables; when $b=c$, this becomes what may be called a *flat point*, and for $b=\infty$, a *flat line*.

there is only one variation of sign among the coefficients of a whatever b or c may be. Hence this equation has only one pair of real roots. Putting $a=\frac{1}{\sqrt{2}}b$, and $a=\frac{1}{2}b$ successively the expression changes sign once, so that the positive root of this equation lies between .5 and .71 whatever be the values of c/b. Dividing the equation by its last term we see that the higher powers of a rapidly converge when c increases. When $c=\infty$ the equation reduces to

$$3a^2 - b^2 = 0$$

which agrees with the previous result. When $b=c=1$ it reduces to

$$4(a^2+1)^3(6a^6+18a^4+11a^2-5)=0$$

as might be found by an independent calculation.

Since a knowledge of the solution of this equation will serve as a guide in the construction of such galvanometers, I give the following table of its roots for several values of c/b together with the values of the field and the proportional decrement of force at the point $\delta x = \frac{1}{3}a$. From these numbers we can at once judge of the uniformity of the field.

Value of c/b.	a (root).	$2\tan^{-1}\frac{a}{b}$.	$F_0=\frac{8bc}{r_0}\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}\right)$	Δ^2F/F_0 at $x=\frac{1}{3}a$.
1	.54451	57° 8′	8.144	.00085
$\sqrt{2}=1.4$	.60040	61° 58′	7.151	.00081
2	.59736	61° 42′	6.681	.00077
3	.58393	60° 34′	6.365	.00074
4	.57980	60° 13′	6.222	.00074
5	.57843	60° 6′	6.148	.00074
10	.57742	60° 0′	6.039	.00074
∞	.57735	60° 0′	6.000	.00075 *

* For Helmholtz's arrangement $\Delta^2F/F_0=$ .00077.

The values of F_0 for any actual case are to be obtained by dividing the above number by the number expressing half the height of the coils in centimetres, and multiplying by the number of turns of wire in the coil. From the table it is seen that when the length c exceeds 5 times the height b the action of the coils is not far from that of the four infinite parallel currents already treated. This justifies the experiment described at the close of the first section of the paper. Further we see that *the assumption for a large magnetic shell made in the beginning of the paper is practically correct,* if the straight part of the circuit extends more than 5 times the distance between the upper and lower magnets ($c/b = 10$) on each side of the instrument, and if the length of each magnet is not less than $5c$.

In practice however the coils are of finite section whereas the above results refer to coils of infinitesimally small section. So long as the depth and width of the sections of the coils are small fractions (say 1/10) of the height of the coils or of their distance apart, we may, without sensible error take the centers of the sections for the positions of simple equivalent coils of infinitesimal section.

If the depth of the section is small and the width finite the force at the center of the coils is got by a process of ordinary integration. Thus if the number of turns of wire per centimeter be n, we have with the same notation as before

$$F = \sum \int_{a_1}^{a_2} \frac{\partial \Omega}{\partial x} n\, dx$$

$$= \sum n \Big[\Omega \Big]_{a_1}^{a_2}$$

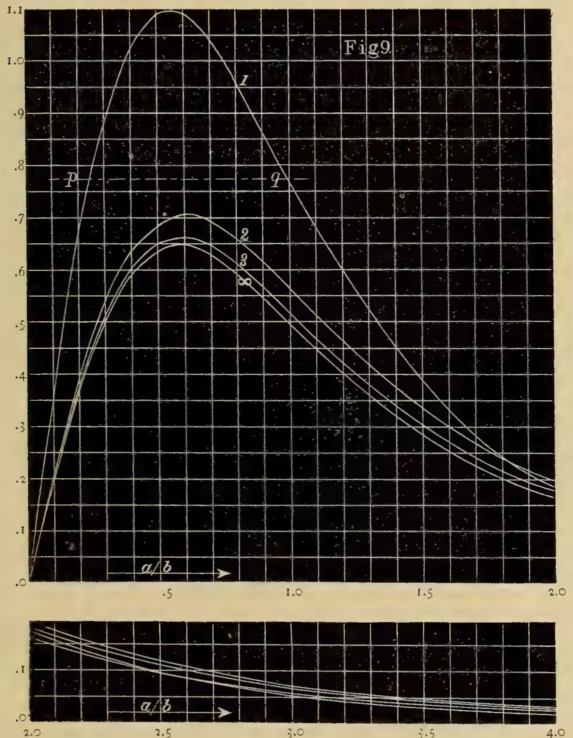
where a_1 and a_2 are the distances of the internal and external faces of either coil measured from the point half way between the two coils. Hence in order that the effect of eccentric displacement may be small we have

$$\frac{\partial^2 F}{\partial x^2} = \sum \left\{ \left(\frac{\partial^2 \Omega}{\partial x^2}\right)_{a_1} - \left(\frac{\partial^2 \Omega}{\partial x^2}\right)_{a_2} \right\} = 0$$

$$\frac{\partial^2 \Omega}{\partial x^2} = \frac{\partial^2}{\partial \xi^2} \sin^{-1} \frac{\eta \zeta}{\sqrt{\xi^2 + \eta^2}\sqrt{\xi^2 + \zeta^2}}$$

$$= \frac{\xi \eta \zeta}{\rho_0} \left\{ \frac{1}{\rho_0^2}\left(\frac{1}{\rho_1^2} + \frac{1}{\rho_2^2}\right) + 2\left(\frac{1}{\rho_1^4} + \frac{1}{\rho_2^4}\right) \right\} \qquad \text{(B)}$$

but we have seen that $\frac{\partial}{\partial x} \frac{\partial^2 \Omega}{\partial x^2}$ vanishes for some value of a/b between



.5 and .71 so that $\frac{\partial^2\Omega}{\partial x^2}$ has a maximum between $a/b = .5$ and .71 for any ratio of c/b. Hence $\partial^2 F/\partial x^2$ can always be made to vanish by taking a_1 and a_2 on both sides of the maximum. The values of $\frac{\partial^2\Omega}{\partial x^2}$ are graphically represented in *Fig. 9* for $c/b = 1, 2, 3, \infty$.

At first these curves are in the order 1, 2, 3 &c, counting from above, but afterwards when they become distinctly asymptotic their order becomes reversed. This is indeed apparent from the equation.

An indefinite number of proper values for a_1 and a_2 may be got by the following simple construction. Draw any horizontal line cutting any particular curve in the points p, q; the x-coordinates of these points at once give a special pair of suitable values a_1 and a_2. In the case of simple coils p and q coincide at the top of the curve, which is the case already discussed.

§ III. Internal Coils for Measuring Large Differences of Potential and Small Currents.

We are now in a position to consider the dimensions which ought to be given to the internal coils spoken of in page 279 as necessary for the measurement of large differences of potential and small currents. For examining the above curves we see that $\partial^2\Omega/\partial x^2$ becomes very small when a/b is more than 3. Also putting (B) in the form

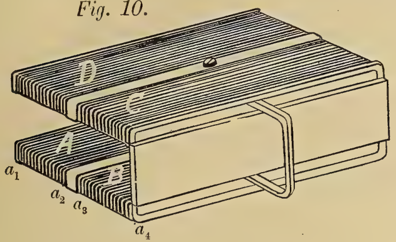
$$\frac{\partial^2\Omega}{\partial x^2} = \frac{\xi\eta\zeta}{\rho_0\rho_1^2}\left(\frac{1}{\rho_0^2}+\frac{2}{\rho_1^2}\right)+\frac{\xi\eta\zeta}{\rho_0\rho_2^2}\left(\frac{1}{\rho_0^2}+\frac{1}{\rho_2^2}\right)$$

and considering η and ζ as variables we may regard the action as due to two equivalent "Electro-magnetic strips."* The coils are shown in the

* See my paper on the 'Electro-magnetic Declinometer' published in the Proceedings of the Royal Society of Edinburgh (Vol. XII. P. 544 1883—4) or Rigakukyōkwai Zassi (Vol. II. P. 84). in which curves are shown very similar to those just given.

diagram below. There are four, one around each magnet. They are necessarily made of small height, in fact just enough to allow the wire magnets to move freely inside and yet to leave a good space for the jaw. For convenience of reference call these coils A, B, C, D, as indicated in the figure. From the symmetry of configuration

Fig. 10.



we may take any one of the eight poles and consider the action of four coils upon it. Take one of the poles inside A and call it P_A for the sake of definiteness. The variations in the actions of C and D upon P_A due to possible motions of the same may be regarded as the differentials of the actions of electromagnetic strips placed along the edges of C and D. These we may safely neglect in comparison with the variations in the actions of A and B. Thus we have only to consider the effect of the four faces of A and B. But since each coil extends a good way over the poles of the magnet, we may regard these coils as drawn out indefinitely in the direction of the magnets' axes without committing any sensible error as was shown in the last section. Hence, calling the distances of the faces of A and B from P_A, a_1, a_2, a_3, a_4, as in the diagram, we have

$$\Delta^2 F = \left(\frac{\partial^2\Omega}{\partial x^2}\right)_{a_1} + \left(\frac{\partial^2\Omega}{\partial x^2}\right)_{a_2} - \left(\frac{\partial^2\Omega}{\partial x^2}\right)_{a_3} + \left(\frac{\partial^2\Omega}{\partial x^2}\right)_{a_4} \qquad \text{(C)}$$

where $a_4 = a_1 + a_2 + a_3$ from symmetry of arrangement. But from the curves given above we see that when a/b is more than 4, the values of $\frac{\partial^2\Omega}{\partial x^2}$ become insensible. Therefore by simply extending the ends of the coils so as to have the least value of a/b more than

4, we make Δ^2F practically vanish.

Theoretically there is an indefinite number of solutions of the equation (C) obtained by selecting sets of values of a_1, a_2, a_3, a_4 such that

$$\left(\frac{\partial^2\Omega}{\partial x^2}\right)_{a_1}+\left(\frac{\partial^2\Omega}{\partial x^2}\right)_{a_2}+\left(\frac{\partial^2\Omega}{\partial x^2}\right)_{a_4}=\left(\frac{\partial^2\Omega}{\partial x^2}\right)_{a_3}$$

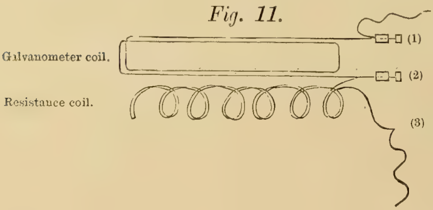
which is always possible since the curve has a maximum. But to apply such a mode of solution practically is dangerous on account of the steep rise of the curve toward the maximum. Any small errors in the values of a_1, a_2 &c. will give rise to an ultimate error greater than that caused by the neglect of the converging terms when a/b is made great. Hence in practice it is advisable to make $a/b > 4$, and limit the range of deflection so that the least value of ξ/b should not fall within 2 or 3. If the instrument were constructed on a large scale with all the dimensions determined accurately, the graphical solution just indicated might be applicable.

Fig. 10. above shows the actual coil belonging to the instrument represented in *Fig. 4.* which was purposely drawn without the coil so as to lay open its internal construction. Both these figures are drawn very nearly to full size. The central ridges which divide the upper and the lower coils into two parts serve for holding the pivots of the frame of the magnets.

The fittings of the levers and springs remain, of course, exactly the same as in *Fig. 4.*

The coil is wound with a thin copper wire in four layers and has a resistance of 40 ohms. An extra resistance of 5000 ohms is added by introducing a small bobbin of fine german silver wire not shown in *Fig. 1.*

Fig. 11.



into the back corner of the case of the instrument. The connection of the terminals is diagramatically shown in *Fig. 11.*

For the measurement of potential differences, the terminals (1) and (3) are used and for the measurement of small currents the terminals (1) and (2). To avoid the possibility of confusion as to which pair of terminals is to be used, a pair of binding screws are provided at (1) and (2) for the case of small currents, and a pair of wire ropes at (1) and (3) for the case of potential differences. An alternative construction would be to omit the pair of binding screws and provide a plug hole between (2) and (3) to shunt off the resistance when the instrument is to be used for small currents. For the measurement of very large potential differences or of moderate currents a system of shunts may be employed in the usual way. Various other like devices may be multiplied almost endlessly.

A separate calibration is required for this coil, but this is nothing more than the ordinary galvanometer gauging. The following table gives a comparison of the readings of an actual instrument with those of a standard galvanometer.

Standard Galvanometer.		Pocket Galvanometer.	
Reading.	Reduced to Ampères.	Reading.	Value of 1 Division in Ampères.
7.8	.02278	25.8	.000883
13.6	.03971	45.1	.000880
22.1	.06453	73.2	.000882
		Mean	.000882

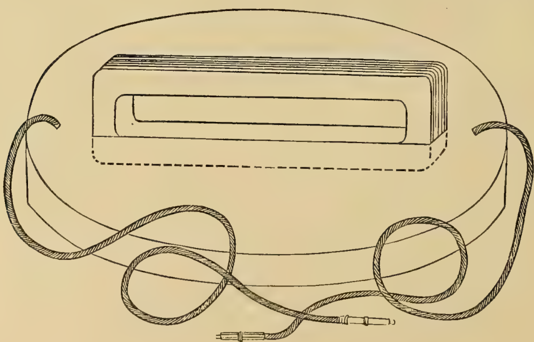
From the above result we easily find, for the given extra resistance of 5000 ohms inserted, the value of the potential difference corresponding to 1 division of the scale. It is 4.4 volts $(= .000882 \times 5040)$. If a thinner wire be wound with a greater

number of layers, it will be easy to make the above value 1 volt or less. Indeed this would have been done in the instrument now being described had it not been at the time impossible to obtain in this country a fine enough insulated copper wire. No doubt, the value required could be arrived at by simply reducing the resistance to little more than 1000 ohms. This however would be too small for the purpose; if possible 10,000* ohms or more would be desirable.

§ IV. External Coil.

A single rectangular coil consisting of several hundred turns of a fine copper wire (see *Fig. 12.*) may take the place of this internal coil just described, if one part of the coil be temporarily slipped in between the jaw in the same way as the so called ribbon was slipped in when the instrument was being gauged. In simplicity of construction, this plan is perhaps superior to that of introducing the internal

Fig. 12.



coil especially when the instrument is to be made on a small scale. But it is inferior inasmuch as it needs an extra piece of apparatus.

* Thomson's graded potential galvanometer has about 7000 ohms.

Of course, we cannot regard this coil as equivalent to a single magnetic shell of an infinite extent. But referring to the diagram of the field of force given in *Fig. 6*, and the table of page 293 we see that the variation of the action of the coil due to deflections of the magnet will be insensible, if the height of the coil be such that the part outside the jaw is just below the base of the instrument, and if the length of the coil be such that the bends are distant from the nearest poles of the magnets by more than five times the distance between the magnets.

Further we see that the part of the coil outside the jaw will add to the effect of the part inside by about 33 per cent the current necessarily being in opposite directions in these two parts.

The coil shown in *Fig. 12.* has 600 turns of copper wire and a resistance of 100 ohms.

Comparison with a standard galvanometer gave the following results.

Standard Galvanometer.		Pocket Galvanometer.	
Reading.	Reduced to Ampères.	Reading.	Value of 1 division in Ampères.
11.8	.03446	41.2	.000836
17.4	.05081	61.0	.000833
21.2	.06190	74.5	.000831
		Mean	.000833

Hence when an extra resistance of 5000 ohms is added 1 division of the scale will correspond to 4.25 volts ($= .000833 \times 5100$).

Thus we see that the present instrument measures from .001 ampères to 168 Ampères and from 4 volts to 400 volts with a probable error of 1 per cent. The galvanometer has of course three constants, one for the jaw, one for the internal coil, and one for the

external coil if it has one. When these are once determind it is not necessary however to test all the three from time to time, since the ratios of the constants amongst themselves depend only upon the configuration of the instrument. These ratios being determined once for all, any change in the value of the constants due to a change in the moments of the magnets or the strengths of springs can be readily discovered by testing for any one of them, say, that for the internal coil.

Some Occurrences of Piedmontite in Japan.

By

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With Plate XXI.

As has been already stated in my other paper(1), the occurrence of Manganese epidote or Piedmontite is often associated with the Glaucophane-bearing rock(2) in the so-called crystalline schists-system in Japan. The rock, which contains Piedmontite as an essential component, is well-characterized in outward appearance by being of a

(1) This journal, Vol. I, No. I, p. 85.

(2) I take here the opportunity of adding a few words in way of caution on the statement of Dr. K. Oebbeke in his valuable paper, entitled: 'Ueber den Glaukophan und seine Verbreitung in Gesteinen'; Zeitschrift d. deutsch. geol. Ges. XXXVIII, p. 641 and 653. Vide also Zeitschrift für Krystallographie und Mineralogie, XII, p. 285. I state it here *verbatim*: he says, 'Durch Herrn Dr. Naumann, früheren Director der geologischen Landesaufnahme von Japan, erhielt ich eine Suite von Gesteinen, unter denen einige von der Insel Sikok (Sikoku) meine Aufmerksamkeit sofort erregten wegen ihrer Ahnlichkeit mit Glaukophan-Eklogiten. *Die Untersuchung zeigte jedoch, dass in diesen Gesteinen kein typischer Glaukophan, sondern eine intensiv blaugrün gefärbte Hornblende vorkommt.*' The most excellent glaucophane occurs, however, in the same rock-complex, from which Dr. Naumann had collected the 'Eklogitschiefer' (Garnet-amphibolite?), and other karinthine-bearing schists, although, I believe, they do not come in the same geological horizon. I have given a short description of our glaucophane from Sikoku in my paper: 'A Note on Glaucophane.' I regret very much that the work of Dr. Oebbeke had not come to hand before the publication of mine, since otherwise I should have given a more detailed discussion. I now feel it my duty to call the attention of those who read the paper of Dr. Oebbeke to the fact, that in the island of Sikoku there are both kinds of amphiboles, the one a bluish-black, highly pleochroic Hornblende, the other a true glaucophane; and the rocks in which they respectively occur, show a marked difference in the characters of the associating components, such as garnet etc.. Dr. Oebbeke is perfectly right in his description of a bluish-black hornblende, but what I believe to be a true glaucophane is also a typical one, and has a striking resemblance to those of Zermatt and the island of Groix, specimens of which were kindly given to me by my honoured professor, Geheimenbergrath Dr. Zirkel. I am of opinion, that there are many intermediate forms between a hornblende of crystalline schists, and a Glaucophane, so that *in some cases* it may be difficult to draw a sharp line between them.

dark violet colour; hence the rock is locally named the "*Murasaki*" or violet rock. And this is most typically developed in the island of Sikoku, especially in the neighbourhood of the city of Tokusima.

The very first specimen that came under my notice, was brought from Mount Ōtakisan, one mile to the south-west of the last-named city; but afterwards many localities are added to the list of places where it occurs, so that we are now able to trace out the geological horizon of the Piedmontite-bearing rock everywhere within the crystalline schists-system of that island. This rock is, however, not exclusively confined to this region; it has also a wide distribution in Musasi and Kōzuke provinces, on the main island (Honsiū).

The Piedmontite occurs together with fine Quartz-grains, and in virtue of its parallel disposition gives to the rock itself a schistose structure, a vertical section of the rock presenting a regular banded appearance formed by the fine alternation of Piedmontite and Quartz layers.

The accessory components are Sericite (hydrous Mica of Prof. Bonny)(1), greenish-yellow Garnet, Rutile (which in some cases may be easily mistaken for Piedmontite), non-striped Felspars (probably Orthoclase), blood-red Iron-glance and also opaque crystals of the same mineral; tourmaline has been nowhere found so far in my slides. This is the typical Piedmontite-schist, and the general appearance of a slide of this rock as seen under the microscope with an amplification of 90 diameters is represented in *Fig. I.* In the Glaucophane-rock(2), the Manganese-epidote makes its appearance; but it is subordinate in quantity to Glaucophane, and has its place often supplied by common, yellowish-green Epidote. We shall first of all speak of the Epidote in the Piedmontite-schist.

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(1) Min. Mag. Vol. VII, No. 32, July, 1886, p. 3.

(2) Loc. cit. p. 86.

Piedmontite.

Crystals of Piedmontite are usually much elongated, traversed by transversal irregular cracks and fissures, and sometimes broken, when the dismembered parts form chains with faces striated in the direction of the axis of symmetry (ortho-axis); and nearly all the crystals lie with the supposed orthopinacoid ($\infty P \bar{\infty}$), parallel to the plane of schistosity of the rock.

In contrast to common rock-forming Epidote in which the well-defined crystallographic forms are seldom to be observed, the crystal-individuals have here usually well-developed faces of M ($o P$), T ($\infty P \bar{\infty}$), i ($\frac{1}{2} P \grave{\infty}$), r ($P \bar{\infty}$) and n (P). (*Fig. II*).

The vicinal section of the clinopinacoid ($\infty P \grave{\infty}$) is, as a rule, of a rhomboidal outline, caused by the predominance of the traces of T and i (*Fig. III*); and, if the face M is at the same time well-developed, the section will be the six-sided. This latter case is however of less frequent occurrence. In common Epidote the face r ($P \bar{\infty}$) is said to be a predominating element, and, as a rule, it is more perfect than the face T ($\infty P \bar{\infty}$).(1) In our Piedmontite, the face r is very poorly developed, and is commonly not visible even in the clinopinacoidal section (*Fig. III*). Outlines of the clinopinacoidal section are never regular, owing to the fact that there are an infinite number of prominences and indentations. They are sometimes even knee-shaped, just like twins of Rutile.

All these facts are due to the parallel growth and intergrowth ("laterale Juxtaposition und Umwachsung") of two or more individuals of different size; and the striations commonly observed on the faces parallel to the orthoaxis arise mainly from these causes, and comparatively few stripes are assignable to the formation of twins.

(1) Rosenbusch, 'Mikroskopische Physiographie,' I Band, 2te Auflage, p. 496.

Extinction of the light occurs simultaneously in all crystals that have taken part in the formation of the complex-individuals, and this should not occur in the case of twins. Some of these remarkable forms are given in the annexed plate, *Figs. III, IV, V.*

Twins are comparatively rare, and if they are present, they are of a common type whose plane of twinning and composition are T ($\infty P \overset{+}{\infty}$), and the extinction-direction of one individual makes an angle of 6° with that of another (*Fig. VI*). The trace of cleavage upon M of both individuals meets at an angle of about 130°, just as in the case of common Epidote given by E. Becke(1) and H.H. Reusch.(2) The crystal-individuals of twins differ considerably in their size, the one bearing parasitic relations to the other. The colour and the behaviour of pleochroism of twins are exactly similar in most cases so that the presence of twinning formation can only be recognized by a slight difference in the shade of colour of both crystals under crossed Nicols, and also in the direction of the traces of the cleavage upon M in both individuals. Cleavages upon the base ($o P$), and orthopinacoid ($\infty P \overset{+}{\infty}$) are sometimes observed as in the *Fig. III;* but in minute crystals they are, as a rule, less distinctly developed than in the larger individuals; for in the majority of cases the smaller ones are perfectly free from such traces of cleavage.

The angle of oblique extinction: $\mathfrak{c} : \mathfrak{a} = 3°$. The axial colour: $\mathfrak{a}$ = deep reddish-violet; $\mathfrak{c}$ = brownish-red; $\mathfrak{b}$ = light violet. The degree of absorption: $\mathfrak{a} > \mathfrak{c} > \mathfrak{b}$; while in common Epidote it may be expressed by the following scheme(3): $\mathfrak{c} > \mathfrak{b} > \mathfrak{a}$. Hence the clinopinacoidal sections of our mineral show most intense colours, while those parallel to the orthoaxis display a lighter tinge.

When a slide is made in the plane of schistosity of the rock, we

(1) Tschermak, Min. u. petr. Mitth. 1879, p. 837.
(2) N. Jahrb. f. Min. u. Geol. II, 1883, p. 87.
(3) Rosenbusch, 'Mikroskopische Physiographie.' I Band, IIte Auflage, p. 497.

usually obtain sections approximately parallel to the *b*-axis; but there is a marked difference in the colours of various sections so as to lead observers to think of entirely other minerals, the one being a deep violet, the other a brownish-yellow. As there are great differences in the axial colours already stated, it may be naturally expected that a section parallel to the basal pinacoid *M* is of a brownish-yellow (the facial colour of $\mathfrak{c} + \mathfrak{b}$); and that which is taken nearly parallel to the orthopinacoid *T* of a deep violet (the facial colour of $\mathfrak{a}+\mathfrak{b}$). The clinopinacoidal section shows the deepest shade of colour, the facial colour being of a combination of $\mathfrak{c}$ and $\mathfrak{a}$.

The extinction-direction is, of course, parallel and at right angles to the longer sides of sections in the zone of *M* and *T*, and the intensity of colours also depends upon the section in this zone. The polarization colours are magnificent, varying from an intense violet to an indigo-blue tinge, which become more pronounced, if we insert a Quartz-plate in the tube of the microscope.

The Piedmontite is ideally pure; neither liquid- or gas-inclosures nor any microlithic interpositions are discernible. The mineral was isolated from the other constituents of the schist obtained from Ōtaki-san, Awa province, by means of the Thoulet solution; and the chemical analysis was kindly undertaken by Mr. J. Takayama, of the Geological Survey of Japan, with the following result:—

$Si\,O_2$	36,16
$Al_2\,O_3$	22,52
$Fe_2\,O_3$	9,33
$Mn_2\,O_3$	6,43
$Ca\,O$	22,05
$Mg\,O$	0,40
$K_2\,O$	trace
$Na_2\,O$	,44
$H_2\,O$	3,20
	100,53

H : Ca :: 1 : 2.2 Ca : $\mathfrak{R}$: Si :: 1.25 : 1 : 1.92

Comparison with other Specimens of Piedmontite.

A comparison of the result just stated with analyses of the Swedish and Alpine Epidotes(1), shows our mineral to be in some particulars markedly different from both of them, although there is a general resemblance throughout. The Japanese Piedmontite indeed forms just the link between those of Jacobsberg, in Sweden, and of St. Marcel, in Piedmont. Mr. Takayama informs the writer that he has not yet been able to decide whether in our specimen the Manganese exists as the sesquioxide or monoxide or (thirdly) both together.

As is well known, Igelström suggests that the Swedish mineral contains manganese as the monoxide, while others are of opinion that in the Alpine Epidote there exists only the sesquioxide. Some mineralogists, therefore, hesitate whether they should be put together as the same variety.(2) The writer is unfortunately not able to express himself more decisively on this point, and awaits a more extended research.

Being of a beautiful rosy-red colour and of a highly pleochroic character, and having a needle-shape, the Piedmontite is usually confounded with a Tourmaline, and as such was formerly regarded by us. Dr. E. Naumann(3) says there are two interesting rocks among the crystalline schists of Japan; the one is "ein echter durch charakteristische rothe Färbung kenntlicher *Turmalinschiefer*, der unter dem Mikroskop schöngefärbte starke dichroitische langgestreckte Krystalle zeigt." The original specimens from which E. Naumann had drawn the above-quoted conclusion were kindly placed at my disposal by the Geological Survey of Japan. An inspection of the

(1) Rammelsberg, 'Mineralchemie,' 2te Auflage, p. 595.

(2) Naumann-Zirkel, 'Elemente der Mineralogie,' 12te Auflage, p. 577.

(3) Ueber den Bau und die Entstehung der japanischen Inseln, Berlin, 1885, p. 10.

various slides convinced the writer that the mineral was true Piedmontite, and not a Tourmaline, and the analysis given above has fully confirmed the writer's view.

Geographical Distribution of Piedmontite.

The mineral Piedmontite is not of common occurrence. Treatises on mineralogy recognise up to the present only two typical localities; the one in St. Marcel near Aoste in Piedmont, Italy, where it occurs as a rare mineral together with other manganese ores, and the other in Jacobsberg, in Wermland, Sweden, where it is found localized within a limestone. In both cases, as it seems to me, Piedmontite comes as a rare mineral, and by no means abundant enough to form an independent rock.

The occurrence of it in Japan is something remarkable, and finds scarcely its equal in other parts of the world. The Manganepidote forms with Quartz the Piedmontite-schist, and is an accessory component in the Glaucophane-schist.(1) Geologically speaking, its occurrence is confined to the same horizon as the Glaucophane-rock, i. e. the lower part of Chlorite-Sericite-Gneiss. This unique Piedmontite-bearing rock is unexpectedly of a wide distribution, constituting indeed an essential member in the archaean complex of Japan. The subjoined are some out of many of the typical localities of the Manganese epidote in our country:—

1. Ōtakisan, near the city of Tokusima, Awa province.

2. Bessi mine, in Uma Gōri,(2) Sanuki province.

3. Chihara copper mine, in Siūfu Gōri; Kitanada, in Kami-ukina Gōri; Uchinoko, and Kaya, in Kita Gōri, Iyo province.

(1) Journ. Sci. Imperial University, Tōkyō, Vol. I, Part I. B. Kotō, 'A Note on Glaucophane,' p. 85 et seq.

(2) Gōri or Kōri is synonymous with "Kreis" or a township.

4. Minano, Simo-tano and Yorii, in Chichibu Gōri; Ogawa, in Hiki Gōri, Musasi province.

5. Miyanosawa and Samba-gawa in Kanra Gōri, Kōzuke province.

6. Misaka, in Iwamae Gōri, Iwaki province.

7. Okino-Sima, Kii province, etc..

A peculiar Epidote.

There are still others which may be conveniently described on the present occasion. In speaking of the Glaucophane-schist in the other paper,(1) the writer has already given a brief notice of the presence of the remarkable Piedmontite. There we find, besides others, a peculiar Epidote in the form of long irregular plates (1/2 – 1 cm.), of a slight yellowish-green colour, and variously traversed by transversal cracks and longitudinal striæ. The morphological habitus differs from an ordinary Epidote by its more flattened tabular condition.

It possesses sometimes a faint rosy tint, and its pleochroism is weak, but distinct, being more intense when the short diagonal of the lower Nicol is at right angles to the longer sides of the Epidote. In other instances, the red pigment is localized in the centre (*Fig. VII.*), so as to form a distinct zone; but the reversed case, i. e. a red margin with the yellow centre, has never happened to be observed so far within the reach of the writer's knowledge.

The rosy pigment, which gives a peculiar feature to our Epidote, is due most certainly to the presence of a manganese oxide, and forms an intermediate stage between common Epidote and Piedmontite. One thing should not be passed unnoticed, namely, the abundant enclosures of clumps of opaque Iron glance and blood-red hexagonal scales of the same mineral, the typical Piedmontite being

(1) loc. cit. p. 85.

entirely free from any such, a fact which shows that the latter (Piedmontite) has crystallized out before the yellowish-green Epidote.

Garnet.

In the Glaucophane-schist from Ōtakisan, in the Island of Sikoku, we find a large number of rhombic dodecahedra (the size of peas) of a greenish-yellow Garnet. Under the microscope, a slide of it appears as made up of different minerals as shown in the figure (*Fig. VIII*). This crystal is, indeed, a small mineral-cabinet of all that are found in this rock except Glaucophane. The violet Piedmontite-needles, clumps of dark Iron-glance, hexagonal scales of Iron-glance, the knee-shaped twins of Rutile-needles and, lastly, highly vitreous grains of Quartz, are all thrown together within the crystal, assuming more or less a curved structure. These admixtures are supplemented and completed by a Garnet substance. The colour of the Garnet itself is deep yellow, and its crystal shows an optical anomaly, its behaviour being just like an anisotropic mineral, caused probably by the strain resulting from the interposition of the other minerals. Prof. Bonny[1] has also discovered Garnet in a Glaucophane-bearing rock near Berrioz in the Val d'Aoste, in theAlps. Here the Garnet sometimes contains Glaucophane and dark dust, which he suggests to be possible in certain cases of subsequent infiltration. Our Garnet is entirely free from the interposition of Glaucophane, although the rock itself is a Glaucophane-schist; and the above-mentioned interpositions, i. e. Piedmontite etc., seem to be formed prior to, or cotemporaneous with the formation of Garnet. It is a very remarkable fact that the brownish-red Garnet seems to be absent in the Piedmontite- and the typical Glaucophane-schist; while it is common in the Amphibolite-zone.

(1) 'On a Glaucophane-eclogite from the Val d' Aoste.' Min. Mag. Vol. VII, No. 32, p. 2. 1886.

I am rather inclined to think that the schists containing the yellowish-green Garnet in Sikoku may represent a metamorphic facies of some other schists.

EXPLANATION OF PLATE XXI.

FIG. 1. A slide of Piedmontite-schist taken parallel to the plane of schistosity of the rock, showing purple Piedmontite needles, hexagonal plates of Iron-mica (lighter shade of a blackish colour in the figure), dark Iron-glance, fibrous Sericite-lamellæ, Rutile needles together with round crystals of Garnet. The matrix of the rock left blank in the figure consists of an admixture of Quartz and Felspars. Magnified 90 diameters.

FIG. 2. A usual form of Piedmontite crystals with predominating faces of M, T, i, together with the hemi-pyramid. Such perfect crystallographic forms are not uncommon in the rcck.

FIG. 3. A clinopinacoidal section with predominating faces of T and i usually met with in rock-sections, showing at the same time few cleavage-traces parallel to M and T.

FIG. 4, FIG. 5, and FIG. 3. represent some of the remarkable forms resulting from the intergrowth and supergrowth of crystals of various sizes, whereby the outlines of sections become very irregular.

FIG. 6. Twins of a common typo with general forms of crystal-individuals, showing the scheme of optical orientation.

FIG. 7. A light yellowish-green Epidote, having a rosy colour in the centre, caused by an accumulation of the pigment of a manganese-oxide. It represents the transitional stage of Piedmontite and common Epidote.

FIG. 8. A section of the rhombic-dodecahedron of Garnet with admixtures of violet Piedmontite needles, clumps of dark Iron-glance, hexagonal plates of Iron-mica, knee-shaped twins of Rutile needles and highly vitreous grains of Quartz within the crystal of Garnet.

The Severe Japan Earthquake

of

The 15th of January, 1887.

By

S. Sekiya.

Professor of Seismology, Imperial University.

With Plates XXII–XXIV.

Soon after the occurrence of the earthquake of the 15th of January last, which caused considerable damage to property in and near Yokohama, the authorities of the Imperial University directed the writer to visit the places which had been affected by the shock, and to make a full report of all the circumstances. The results thus arrived at form the subject of the present paper. Before proceeding with this, however, it seems desirable to give some particulars respecting the principal shocks which have been felt in the Empire since 1879.

The Earthquake of February 22nd, 1880*, is the severest that has been experienced in the Plain of Musashi during the last ten years. The damage done to buildings was very much greater than on the recent occasion. Its origin was in the Bay of Tōkyō, and the boundary of the disturbed area is shown on Plate XXII. Recent severe shocks.

On the 25th October, 1881, Nemuro in Yezo, was visited by a somewhat destructive shock. Fissures were opened in the ground, and the damage to property was not inconsiderable.

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* See *The Earthquake of February 22nd*, 1880, by John Milne, Transactions of the Seismological Society of Japan, Vol. 1, Part II.

The well-known Atami Spa and its neighbourhood were convulsed on the morning of September 29th, 1882, by a sudden and severe movement, which damaged embankments, destroyed an historical monument, and did sundry other mischiefs.

The earthquake of October 15th, 1884, also marked on Plate XXII, originated in the Bay of Tōkyō, and affected the Plain of Musashi. It overturned a considerable number of chimneys, cracked walls, and broke articles in museums and elsewhere. In Tōkyō, the greatest horizontal movement, in a soft ground, was 42 mm., or double the amount observed on the 15th of January last. However, the total damage, taking the whole affected area into account, was smaller.

The seismic waves in the disturbance of October 30th, 1885, extended over the whole of Northern Japan and part of Yezo, shaking a land area of 34,738 square miles. But, though of great extent, they fortunately did little harm.

On July 23rd, 1886, quite a destructive earthquake visited Shinano and the neighbouring provinces, overthrowing several houses, and forming fissures in roads and hill-sides. The shock also stopped the flow of a hot spring at Nozawa. The part most severely shaken was a mountainous district some 2000 feet above the sea, including the famous active volcano of Asama, and many extinct craters. This case was an unusual one, as most of the larger earthquakes in Japan extend along the sea-shore.

Next in the list comes the severe shock of last January.

It thus appears that this empire is visited by a more or less destructive earthquake *almost once a year*, and that the Plain of Musashi is affected in like manner *at intervals of a few years*.

Earthquake of January 15th, 1887.

The shock of last January was of most unusual violence. It originated near the coast, about 35 miles southwest of Tōkyō, and

the seismic waves propagated nearly 200 miles to the west and north-east along the Pacific seaboard. On the northwest they approached but do not quite reach the shore of the Japan Sea. They shook, in all, about 32,000 square miles of land area. The limit of the disturbance is indicated on Plate XXII.

Instrumental observations.

In Tōkyō the disturbance began at $6^h 51^m 59^s$ P.M., with slight tremors. After thirty seconds from the commencement, the greatest horizontal motion (21 mm.) was recorded. The time taken to complete one to-and-fro motion of the ground was 2.5 seconds. The maximum vertical motion was only 1.8 mm., being, as usual, very small compared with the horizontal movement. The principal motion continued for more than two minutes, during which time no less than *sixty distinct shocks* occurred. The maximum velocity and maximum acceleration, which measure the overthrowing and shattering power of earthquakes, have been calculated from the above numbers, and found to be respectively 26 mm. and 66 mm. per second. These numbers, considering the range of motion, are small; or, in other words, the oscillations of the ground were comparatively gentle and slow, which serves to explain the fact that but little harm was done to property in the Capital. In Yokohama Hipp's Seismograph registered a horizontal motion of 35 mm.

The Cause of the shock.

The origin of the shock was in a narrow band of country running from west to east in the province of Sagami, parallel to the coast, at a distance from it of about seven miles. The red shading on Plate XXII indicates this band. It emanates from the western or mountainous parts of the province, passes through the southern foot of Ōyama (4125 feet above the sea-level), and reaches the Bay of Yokohama in a total distance of about thirty miles. I believe the most probable cause of the shock to have been faulting or dislocation of the earth's crust along the band above named. This

inference is supported by the fact that the parts of the country through which the western half of the band passes consists of rocks of different geological formations, interwoven in such a way that their junctions present lines of weakness favourable to earth-snaps. The topographical features of the district—high mountains on the north and comparative law plateau and sea-shore, on the south—also lend strength to this conclusion. Unequal distribution of loads on the earth's surface tends to facilitate bending and folding of the rocks.

Effects on Land and Water. It is along the above-named axis or band that the effects were most striking. They were mainly confined, however, to a small breadth on either side of it, so that places as little as two or three miles to the north or south experienced a well-marked diminution of seismic energy. This is not the first instance in the history of the severer shocks in which the destructive effects have been practically limited to a small area near the origin.

More especially on the hilly or western portion of the origin, land-slips and cracks were numerous. The cracks mostly took place in banks, hill-sides, or other situations favourable for their formation. The writer counted no fewer than seventy-two in a distance of seven miles, the largest measuring a foot wide and five hundred feet long, and all of them running parallel to the axis of origin, which is also parallel to the general contour of the country. Several wells became turbid. In some of artesian character the water permanently decreased; in others it increased. There is a ferry across the large river Banyū where it is crossed by the axial band; but the water was so agitated by the shock that for some times afterwards the boat could not be used. The water in one of the rivulets on the west became muddy. The shock was severely felt on board of vessels in Yokohama harbour, the people in many of them rushing on deck under the impression that they had been run into. The effects upon these

vessels were doubtless caused partly by motion communicated through the cables, and partly by agitation of the water due to movements of the sea-bottom. The earthquake was preceded by the usual warning roar or rumbling, as of distant cannon, emanating apparently from the western part of the origin-band. In that district, too, the after-shocks on the same night were five in number, while in Tōkyō there was only one. There were four tremors near the origin during the night of the 16th.

Effects on Buildings.

Dwelling houses in country-towns and villages are always built of wood. Their frame-work is of timbers from four to seven inches square, crossing one another at right angles. The uprights are placed about three feet apart, and stand on rows of squared stones or boulders, the intervening spaces being filled with bamboo-laths, on which is laid the mud-plaster that forms the walls. Tiles and straw are principally used for the roof-covering. In the district near the origin these wooden houses shook with great violence. Several of them were more or less twisted, cracked or unroofed. Sliding doors, covered with paper or of wood, which serve as shutters, partitions and windows in Japanese houses, broke and were shot out of their grooves. The joints between the frames were in some cases badly loosened. Fig. 2 on Plate XXIII shows how one of these joints suffered, together with the paper covering of a sliding door which was rent by the vibrations. Although there are thousands of wrecked houses, in the district of origin, on the verge of falling down, and looking as if a strong breeze would be enough to blow them over the buildings of this class nevertheless withstood the violence of the earth movements so far as to escape actual demolition. The writer saw only two small rotten hovels which had been thrown down. This circumstance shows the tenacity of wooden framed structures. Prof. T. Mendenhall, in a report* on

* The *Monthly Weather Review*, U. S. Signal Service, August, 1886.

the recent catastrophe at Charlestown, says—"As was to be expected, buildings constructed of wood suffered much less than those of brick. The interior of wooden buildings, however, would often exhibit a scene of total destruction, furniture, book-cases, etc., having evidently moved with great violence."

Fire-proof stores, or *Kura*, suffered severely as to their walls. These buildings have wooden frames, strongly joined by horizontal and vertical pieces, and closely covered with laths, the whole making up a compact box-like structure. The roof is tiled, and carefully plastered with a mud which has a slight cementing property, to the thickness of from three to nine inches. This plaster is put on in several layers, each layer being added after the preceding one has dried. The whole process is an expensive one. The walls, on account of their great thickness and the poor tenacity of the mud, are easily cracked or stripped. As many as sixty or seventy per cent. of the *Kura* suffered from the recent shock. Fig. 1, Plate XXIII shows one of them with its walls badly damaged, and shows also the method of framing the timbers. It is evident that these thick-walled structures should be replaced by brick buildings, which are equally fire-proof and much stronger.

It may be mentioned, however, that the framework of *Kura*, after having been entirely stripped, have withstood the most violent earthquake on record.

In Yokohama houses are built of different types and with a variety of materials, so that they afford a fair field for the comparison of seismic effects. It is very fortunate that, judging from the effects wrought by the recent earthquake on both land and buildings, the seismic intensity in this town was less than one-third of that in the western or hilly parts of the origin-band. But for this, the results would have been highly disastrous.

The houses which suffered most were the composite structures of wood and stone. They are built of wooden frames encased with stone blocks, each of the latter measuring 2 ft. 9 in. long, 9 in. wide and 6 in. thick, and being clamped to the wooden planks inside by three iron-nails. Plate XXIV shows one of these houses that was affected by the recent shocks, together with the details of the stone attachments. The nail, called *Kasugai*, is 5 in. long and $\frac{7}{10}$ in square, and bent at right angles at its two ends. The stone is soft and brittle, being volcanic rock of the worst quality. In time the iron-nails get rusty, and the stones are so acted on by rain and frost as to be easily cracked, or detached from the wooden frames, even by moderate shakings. These buildings, erroneously called European houses, already exist in abundance, and unfortunately increase each year in number. They are generally constructed with bad materials and on faulty principles; the object of the builders being to attain fair protection from fire, along with the appearance of a stone building, at the least practicable cost.

Two brick structures received serious damage (Plate XXIII, Fig. 3), cracks having been formed, as usual, at the corners of the buildings and over the windows. The seismic vibrations, however, left no traces on the Town Hall, the Custom House, Prefectural Office, and other well-built structures of brick or stone.

In Yokohama, wooden houses sustained no damage worth mentioning. Joints were more or less loosened and tiles occasionally fell down from the roofs. The tiles that are fastened to the framework of wooden houses, to form walls, were in some case detached in large quantities. There are decidedly many improvements which might be made in the present wooden buildings, both of Japanese and so-called European styles, especially in the arrangements of their joints, the scientific distribution of materials, etc. If these and other defects

were properly remedied, such dwellings might be made pretty safe as against earthquakes. In sites little liable to danger from fire, one may find, in this country, wooden houses built three and even four centuries ago. Wood, no doubt, will continue for a long time to be the chief building material in this country

In Japan, however, fire is a more constant and even more dread enemy than earthquakes, while terrible conflagrations are often brought about by destructive shocks. Hence, brick and stone should, and probably will in time come to be largely employed for building, especially in towns. The question, then, is to select certain types of brick or stone houses which are best calculated to resist earthquake shocks. Sheet and bar iron houses, as used in Australia, would make very efficient earthquake-proof buildings, although they are not free from several objections.

After the terrible catastrophe of 1883 in the island of Ischia, the Italian Government appointed a Commission * to consider the reconstruction of the buildings in that island. The Commission, after investigating the different modes of constructions most suitable for earthquake countries, submitted models of houses in wood and in combinations of wood and masonry, which were adopted. The commission recommended that buildings should be chiefly constructed with an iron or wooden framework, carefully joined together by diagonal ties, horizontally and vertically, the spaces between the framework being filled in with masonry of a light character. Not more than two stories above ground were to be allowed, etc., etc.

In Italy brick houses are joined by iron tie-rods; and similar devices are now, to a certain extent, used in this country. Concerning the erection of brick or stone houses in Japan, much valuable information is to be obtained from the Italians, who, like ourselves,

* Proceedings of the Institution of Civil Engineers, Vol. LXXXiii, Session 1885-86—Part i.

have lived for centuries amidst terrible shakings, and who, no doubt, have gained much experience in the constructive arts suitable to the conditions of our existence here.

Chimneys.

A prominent feature in the effects of the recent earthquake was the overthrowing of brick chimneys in Yokohama, especially on the Bluff. Soon after the shock, circulars were sent round to the principal residents, asking for information as to the effects of the shock on the buildings occupied by them. More than fifty answers were received, and the facts embodied in them have been of great value in preparing this paper. The writer takes this opportunity of expressing his warmest thanks for the kind assistance thus rendered to him. From these answers, from the Police Reports, and from actual observations fifty-three chimneys appear to have been destroyed. In one instance a heavily-coped chimney fell in a large mass through the roof, and severing a strong beam of 1 ft. by 8 in. on the second story, penetrated to the ground floor.

About one-half of the chimneys thrown down during the shock were cut in two at their junction with the roof; while some dislodged the tiling and did sundry other damage to the buildings at their points of contact. Evidently the chimneys and the houses moved with unequal range and with different vibrational periods. Professor Milne has more than once recommended that chimneys should be built thick and squat, without heavy ornamental mouldings or copings; and be, if possible, disconnected from the roofs. Those houses in which his suggestions had been adopted suffered no damage on the 15th of January.

Nature of rocks and topographical features.

Generally, the relations of the seismic effects to the geological, topographical and other features of the various localities were found to corroborate previous experience. That the seismic vibrations in hard ground are very much less than in soft soil was well illustrated

on the recent occasion. At the University, where the ground is hard and firm, the seismograph recorded only 8 mm. horizontal motion, as compared with 21 mm. registered by a similar instrument placed on soft soil a mile distant. Totsuka is a small town, with a single long street running along the foot of a hill; one side of the street, however, is built on made-up ground. Most serious damage was done on that side, while the opposite houses suffered very much less, though not more than twenty feet distant. Houses built on cliffs and hill-brows received more damage than those situated at the base or on the flat summits of the same hill. To observe the effects of marginal vibration, the writer recently placed one seismograph at the steep edge of a loamy hill 38 feet in height, and another similar instrument at its foot. The motions, thus far measured, at those two levels are found to be in the ratio of 2 to 1. A third instrument will shortly be set up on the flat summit of the same hill. Observations of a similar nature, on different rocks and at various heights, will form the subject of a further paper. It is probably owing to marginal vibration that houses on the Bluff of Yokohama are always heavy sufferers from earthquakes.

Observations on Marginal Vibration.

Kerosene Lamp.

The extensive and rapidly increasing use of kerosene lamps in Japan constitutes a grave danger in severe shocks. The lamps now in common use are of very brittle materials, contain the most combustible of oils, and are usually poised on ill-balanced stands. In the great earthquake of 1855, at a time whem kerosene was unknown in this country, fire broke out in Yedo at more than thirty points, setting a very large part of the city in a blaze. In the event of another such shock, the mischief which would be produced from this cause alone is awful to contemplate. Great credit will be due to any one who can invent a convenient earthquake safety-lamp, which, it is to be observed, will also constitute a valuable safeguard in ordinary

daily life. It is true, so-called safety-lamps are sold in Tōkyō, but they are very ineffective and miserable affairs. The use of metallic oil-holders would doubtless greatly lessen the danger.

During his inquiry the writer was shown sixteen lamps that had been broken in the recent earthquake. In one instance the kerosene caught fire, and it was with great difficulty that the residents extinguished it by the aid of wet mats.

NOTE:—Since writing the above paper, the writer has observed in "*Iron*" of February 25th, 1887, an account of Mr. Phillips' Shaftesbury Petroleum Lamp, which apparently fulfils all the requirements of an Earthquake Safety-Lamp. "The invention consists of a sliding rod passing through the body of the lamp, one end being attached to an extinguishing cap, whilst the other, which rests on the table, is weighted. Directly the lamp is put out of the perpendicular, the rod, by means of very simple gearing, slides through the tube and brings the cap over the wick, instantly extinguishing the flame. It is real protection to life and property, for if knocked over or dropped, it goes out instantly." It is stated in that journal that the lamp stood all the tests well. One complete set can be purchased at the low price of two shillings and nine pence (nearly 90 *sen*) at No. 1, Holburn Viaduct, London.

Electrical Resistance of Nickel at High Temperatures.

By

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This note is an abstract of a paper which is being published in the Transactions of the Royal Society of Edinburgh. The experiments were made in the Laboratory of the Imperial University of Japan, and carried out in part by Messrs. Hirayama and Saneyoshi, IVth year students of Physical Science. The object was to find if nickel was similar to iron in having any peculiarity as regards resistance change at or about the temperature at which the very striking thermoelectric change occurs. As shown first by Tait,* iron and nickel are exceptional amongst metals in having what is technically called an abrupt bend in their thermoelectric lines. In fact it is possible to combine iron or nickel with other wires so as to obtain two or more successive neutral points as the temperature of one of the junctions is raised indefinitely; such a phenomenon cannot be obtained with other metals substituted for the iron or nickel. According to Tait's theory, this bend in the thermoelectric line means an abrupt change in the sign of the Thomson Effect. Some twelve years ago it was discovered by Mr. (now Professor) C. Michie Smith

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* Transactions Royal Society of Edinburgh (1874.)

and myself* that at a dull red heat, which is just the temperature at which the thermoelectric change occurs, a sudden change in the rate of increase of resistance of iron as compared with the corresponding quantity for platinum seems to take place. Von Waltenhofen has also shown that for steel wires the rate of change of resistance above a red heat tends to fall off. If, then, there should be any connection between the two phenomena, something similar ought to exist in the case of nickel. Also, since the thermoelectric peculiarity for nickel occurs at a considerably lower temperature, somewhere between 200° and 320° C., the comparison obviously may be more easily effected. A preliminary experiment in which the wire was heated up in oil to a temperature of 300° C. gave a promising result. The method of experiment finally adopted was as follows. The wires to be tested were raised to a red heat inside a porcelain vessel set in a charcoal furnace. Two wires were tested simultaneously, or rather in rapid succession, as the furnace slowly cooled. One of the wires was always a certain piece of platinum wire which served as a thermometer. The other wire was nickel, palladium, or iron, the last two being studied so as to make sure that the peculiarity shown by the nickel was a real property of the metal. In the graphical representation of the results, giving the resistances of the various metals at different temperatures in terms of the corresponding platinum resistance, palladium showed as a straight line, iron as a curve of constantly increasing steepness, that is concave upwards, and nickel as a curve with a point of inflexion, concave upwards at temperatures below 300° C. and concave downwards at higher temperatures. At 320° a very sudden decrease occurred in the rate of increase of temperature. The following table, abstracted from the complete paper, brings out the differences between

* Proceedings Royal Society of Edinburgh (1875).

the various metals. The accompanying temperature scale is of course only approximate.

Temperature (Centigrade.)	Resistances (with Differences).							
	Platinum.		Palladium.		Iron.		Nickel.	
0°	10	3	10	5	10	8	10	5.2
140	13	3	15	5.1	18	10.4	15.2	8.3
280	16	3	20.1	4.8	28.4	14.1	23.5	6 2
420	19	3	24.9	4	42.5	20.8	29.7	4.1
560	22		28.9		63.3		33.8	

Further experiments with the nickel were made in which resistance changes and thermoelectric changes were measured simultaneously ; and these completely established the fact that the two peculiarities occur at the same temperature. The final conclusions are these. (1) At about 200° C. the rate of resistance growth for nickel increases markedly, and continues practically steady, till about 320° C., when a sudden decrease occurs, and thereafter the resistance steadily increases at this diminished rate. In other words, between the limits of temperature specified, the slope of the resistance curve is much steeper than anywhere else. The same peculiarity is probably possessed by iron between the temperatures of a dull red and a bright red heat. (2) The peculiarity occurs (in each case) between the limits of temperature within which the striking thermoelectric peculiarity discovered by Tait also occurs. This peculiarity, which is most briefly described as an abrupt change in the sign of the Thomson Effect, is not known to be possessed by any other metal. (3) There is thus a strong presumption that the Thomson Effect in metals has a close connection with the mutual relations of resistance and temperature—at any rate in metals in which the Thomson Effect is proportional to the absolute temperature (according to Tait's theory)

the "logarithmic rate" of change of resistance seems to be very approximately inversely as the absolute temperature. In nickel and iron, in which the law of the Thomson effect is peculiar, such a simple relation between resistance and temperature does not hold.

Electrical Properties of Hydrogenised Palladium.

By

Cargill G. Knott D. Sc. (Edin.) F. R. S. E.
Professor of Physics, Imperial University.

This note is also an abstract of a paper, read before the Royal Society of Edinburgh in July 1886, and now appearing in the Transactions. It consists of two parts, the first being a study of the electrical resistance of palladium at various stages of hydrogenisation, and the second a study of the thermoelectric properties.

It has been known for some time that the resistance of a palladium wire charged with hydrogen at ordinary atmospheric temperatures increases at a rate almost strictly proportional to the amount of charge. When fully saturated the resistance of the wire is about 1.7 times its resistance in the pure state. The interesting question as to the effect of change of temperature on the resistance of hydrogenised palladium seems, however, never to have been approached by former experimenters. If a pretty strongly charged palladium wire be taken and slowly heated, the resistance will be found to grow steadily up to a temperature of 130° C. Above this temperature a somewhat more rapid increase of resistance is generally observable until 200° C. is

reached. Thereafter, in virtue of the escape of the hydrogen from the palladium, the resistance ceases to increase; and as the heating is continued past 250° C. a very rapid *diminution* of resistance sets in, until, as the temperature approaches 300° C. all the hydrogen is driven out, so that the wire becomes pure palladium and behaves accordingly.

An unexpectedly simple relation was found to hold very approximately between the temperature coefficients for different amounts of charge. The relation may be expressed in two ways, thus:—the *resistance* of a given wire in different states of charge increases by approximately the same amount for a given rise of temperature; or the *total* increase of resistance of a palladium wire when charged to a certain amount is the same at all temperatures below 150°. This requires that the temperature coefficient is smaller for the higher charges. Thus, if R_0 is the resistance of pure palladium at 0° centigrade and a the temperature coefficient so that the resistance at $t°$ is

$$R_t = R_0 (1 + at)$$

and if r_0 be the resistance at 0° C. of the same wire with a given charge of hydrogen, then its resistance at $t°$ is

$$r_t = r_0 + R_0 at$$

$$= r_0 \left(1 + \frac{R_0}{r_0} at\right)$$

Hence the temperature-coefficient for a given specimen of wire charged with hydrogen is inversely as the resistance of the wire. In the following table the first column gives the resistances of the same wire at successive saturations; the second column contains the corresponding temperature-coefficients; and the third gives the product, which in accordance with the above statement, should be the same for all.

Resistance.	Temperature-Coefficient	Product.
1	.00293	.00293
1.07	.00319	.00341
1.13	.00299	.00338
1.27	.00273	.00347
1.41	.00257	.00362
1.51	.00214	.00323
1.63	.00198	.00323

When the peculiar conditions of the experiment are borne in mind, more especially the uncertainty of the charge being uniformly distributed in the wire, it will probably be granted that the products are sufficiently alike to justify the conclusion drawn. At any rate the approximation to constancy among these products is very remarkable when taken in connection with the rapid increase of resistance due to charging as shown in the first column.

The thermoelectric current obtained with a palladium hydrogenium pair is one of surprising magnitude, being greater than for a palladium copper circuit. When the heated junction is at or near a temperature of 200° C., peculiar irregularities appear which depend upon whether the temperature is for the moment rising or falling. This no doubt is due to the hydrogen being driven out of the highly heated parts near the junction, and to its partial return from colder contiguous portions of the wire as the junction is being cooled down again. After experiencing such a high temperature, the charged wire at the junction does not, however, wholly regain its original condition—the electromotive force in the circuit for a given difference of temperature being appreciably, sometimes very markedly, smaller than at first. So long as the temperature is kept below 150° C., the charged wire is as constant in its thermoelectric properties as the pure

wire. In all cases, the thermoelectric current is from the pure palladium to the charged palladium through the hot junction, is for any given pair very nearly proportional to the difference of temperature of the junctions, and is greater for the greater charge of hydrogen in the one wire. Thermoelectrically, fully saturated hydrogenium lies between iron and copper at ordinary atmospheric temperatures. On the thermoelectric diagram the hydrogenised palladiums of different charge are represented (up to a temperature of 150° C.) by a series of straight lines approximately parallel to palladium, an appreciable deviation from parallelism occurring only for the highly charged specimens. The thermoelectric powers at 0° C. expressed in C. G. S. units range roughly from -600 (pure palladium) to $+ 1400$ (saturated palladium), the thermoelectric power for lead being zero.* In other words, the electromotive force in a circuit of palladium and saturated hydrogenium, when the temperature of the junctions are 0° C. and 100° C., is about 20×10^4 C. G. S. units or .002 volts.

The deviation from parallelism between the lines for pure palladium and saturated hydrogenium, referred to above, is of such a nature as to produce intersection of the lines at a point corresponding to $-350°$ C. This of course is an unattainable temperature ; but the existence of such a 'neutral point' (as it were) must have some significance. The data, upon which the estimation of this point was made, are somewhat doubtful, the range of temperature employed in the experiments being limited. With the object of testing this supposed convergence of the palladium and hydrogenium lines, Messrs. Saneyoshi and Hirayama undertook a series of experiments, in which

* Compare Everett's "Units and physical constants" page 151, where, however, the signs are reversed. It seems preferable to draw the diagram as in Tait's classical memoir or as described in Maxwell's *Elementary Treatise on Electricity* and in Tait's *Heat*. The lines which have *positive* inclination according to the usual convention are then the lines of metals, in which the Thomson Effect is *positive*. [I notice that Professor Everett has, in the 2nd edition lately published, altered the signs so as to agree with this convention.]

the hydrogenium line was traced out by its neutral points with conveniently situated imaginary lines formed by combination of iron and nickel wires with adjustable resistance in the two branches. They obtained for the point of convergence the value $-260°$ C. The conclusion seems to be that the hydrogen line, if it could be found, would pass through this same point—*i. e.* have a neutral point with palladium at temperature of about $-300°$ C.

The thermoelectric peculiarities of hydrogenium may be prettily shown by the following experiment. Let a palladium wire, by immersion to half its length in the electrolytic cell, be hydrogenised throughout that half length. Attach the ends of this seeming single uniform wire to the terminals of a galvanometer, and let a flame be allowed to play gently at the central point of the wire. A large current is at once obtained, which grows to a maximum, and then diminishes to zero as the temperature rises to a red heat. There is no such current during cooling. This spurious neutral point during heating is due to the hydrogen being driven out of the heated portion, partly, no doubt, into the contiguous colder portions. By following up with the flame the ever-shifting point of separation between the charged and uncharged portions, we may repeat the experiment indefinitely until the hydrogen is all driven out of the wire or until the distribution of the charge has become fairly uniform.

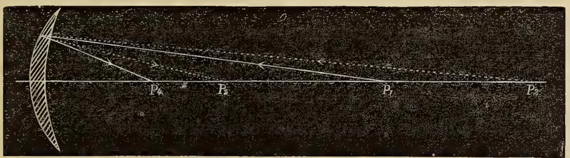
The Constants of a Lens.

By

Aikitu Tanakadate.

Assistant Professor of Physics, Imperial University.

The curvatures of the faces and the index of refraction of the substance of a lens, may be found by the following simple optical method.



In the diagram, let P_1 be the source of light and P_2 the focus due to one refraction, and p_1 p_2 their respective distances from the lens. Also let r_1 and r_2 be the radii of curvature of the front and back faces of the lens respectively. Then

$$\frac{1}{p_1}-\frac{\mu}{p_2}=-\frac{\mu-1}{r_1} \quad \text{......................} \quad (1)$$

Considering P_2 as a new source of light we have, for the determination of P_3 the focus due to reflection at the internal back surface of the lens, the equation

$$\frac{1}{p_2}+\frac{1}{p_3}=\frac{2}{r_2} \quad \text{..............................} \quad (2)$$

neglecting the thickness of the lens.

This converging pencil of rays is refracted again at the front face, and the real image is formed at P_4: its focal distance p_4 being given by the equation

$$\frac{1}{p_4}-\frac{\mu}{p_3}=-\frac{\mu-1}{r_1} \quad \text{..............................} \quad (3)$$

multiplying (2) by μ, and adding (1) (2) (3) we get

$$\frac{1}{p_1}+\frac{1}{p_4}=-\frac{2(\mu-1)}{r_1}+\frac{2\mu}{r_2}$$

This shows that the lens is equivalent to a mirror whose curvature is equivalent to $-(\mu-1)/r_1+\mu/r_2$, and is concave or convex according as $(\mu-1)/r_1 <$ or $> \mu/r_2$. This equivalent mirror becomes a plane when $r_1/r_2=(\mu-1)/\mu$.

Let $\dfrac{1}{\rho_1}$ be the curvature of this equivalent mirror, or as we shall call it for brevity, the "equivalent curvature," then

$$\frac{1}{\rho_1}=-\frac{\mu-1}{r_1}+\frac{\mu}{r^2} \quad\text{..............................}\quad (4)$$

Now reverse the lens, that is interchange the reflecting and refracting faces, and let the equivalent curvature be $\dfrac{1}{\rho_2}$ then

$$\frac{1}{\rho_2}=\frac{\mu-1}{r_2}-\frac{\mu}{r_1} \quad\text{..............................}\quad (5)$$

But the principal focal length of the lens is given by

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad\text{..............................}\quad (6)$$

From (4) (5) (6) we get at once *

$$\left.\begin{aligned}
-\frac{1}{r_1}&=\frac{1}{\rho_2}+\frac{1}{f}\\
\frac{1}{r_2}&=\frac{1}{\rho_1}+\frac{1}{f}\\
\mu&=1-\frac{\dfrac{1}{f}}{\dfrac{1}{\rho_1}+\dfrac{1}{\rho_2}+\dfrac{2}{f}}
\end{aligned}\right\} \quad\text{..............................}\quad (7)$$

* The signs of r_1 and r_2 are considered with regard to the first position of the lens.

When the thickness t of the lens is taken into account, equation (2) becomes

$$\frac{1}{p_2+t}+\frac{1}{p_3+t}=\frac{2}{r_2}$$

This combined with (1) and (3) gives the relation

$$\frac{1}{\mu\Big/\left(\frac{1}{p_1}+\frac{\mu-1}{r_1}\right)+t}+\frac{1}{\mu\Big/\left(\frac{1}{p_4}+\frac{\mu-1}{r_1}\right)+t}=\frac{2}{r_2} \quad \text{.........(8)}$$

We have a similar equation for the reversed position of the lens, and also, for direct refraction through the lens, the equation (corresponding to (6) above)

$$1\Big/\left(\frac{1}{q_1}+\frac{\mu-1}{r_1}\right)-1\Big/\left(\frac{1}{q_2}+\frac{\mu-1}{r_2}\right)=\frac{t}{\mu} \quad \text{...............(9)}$$

q_1 and q_2 being the respective distances of the light and its image from the front and back faces of the lens. These equations are strictly rigorous and can be worked out to any desired degree of accuracy.

The most favorable values of p_1 and p_4 for minimizing the errors of experiment, are when $p_1=p_4$ i.e. when the light is placed at the center of curvature of the equivalent mirror. In this case, any ray of the pencil is reflected normally at the back surface of the lens, and returns along the same track. Further this gives at once $p_1=p_4=\rho_1$, and (8) reduces to

$$\begin{aligned}\frac{1}{\rho_1}&=-\frac{\mu-1}{r_1}+\frac{\mu}{r_2-t}\\ &=-\frac{\mu-1}{r_1}+\frac{\mu}{r_2}+\frac{\mu t}{r_2^2}+\frac{\mu t^2}{r_2^3}+\ \ldots\ldots\ldots\ldots\end{aligned}$$

Thus the results in (7) are to be corrected to the first order of small quantities, by diminishing $1/\rho_1$ by $\mu t/r_2^2$ and similarly $1/\rho_2$ by $\mu t/r_1^2$

As for $\frac{1}{f}$, the correction is given by

$$\frac{1}{q_2}-\frac{1}{q_1}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)-\frac{t}{\mu}\left(\frac{1}{q_1}+\frac{\mu-1}{r_1}\right)^2+\ \ldots\ldots\ldots$$

The terms depending upon the thickness t absolutely vanish, when $q_1 = -r_1/(\mu - 1)$ and consequently $q_2 = r_2/(\mu - 1)$,—*i. e.*, when the distances of the object and image from their nearest faces of the lens are proportional to the radii of curvature of those faces. In this case, the pencil of light becomes cylindrical within the substance of the lens. This condition may be experimentally realized by using the approximate values of μ and r as given in (7). There are two cases however in which this correction vanishes without using approximate values:—(1.) When the lens is plano-convex, and the parallel rays of light pass in at the plane surface; (2.) when the lens is double convex with equal curvatures, and the object and image are equally distant from their respective nearest faces of the lens.

The above method fails with meniscus lenses in the case when the "equivalent mirror" becomes convex: but in such instances, the curvature of one face can always be found by direct reflection, and when the lens is reversed the equivalent mirror becomes concave. Thus in general, the curvatures of the faces and index of refraction of the substance of a *convex* lens can be found by a method which is essentially the same as that for finding the focal length of a concave mirror.

The method was tested experimentally and gave for curvatures and index of refraction of a particular lens

$$r_1 = 103.8 \text{ cm.} \quad r_2 = 105.3 \text{ cm.} \quad \mu = 1.515$$

Measured with a spherometer these quantities were found to be

$$r_1 = 104.5 \text{ cm.} \quad r_2 = 105.0 \text{ cm.} \quad \mu = 1.514$$

A clear aperture of 1 cm. diameter is sufficient for applying the method.

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Prof. **S. Sekiya.**

ERRATA.

Page 332, 7th line, insert "a" after "at."

Seite 337, Z. 6 lese man für "jetzt in thun kann," "jetzt in dieser Mittheilung thun kann."

Page 359, foot-note, for "Part III" read "page 313."

Page 360, foot-note, for "Part I" read "page 61."

Page 378, 8th line, for "foumed" read "found."

Über einige Tricladen Europa's.

von

Dr. phil. **Isao Ijima.**

Mit Tafel XXV.

Mit dieser kurzen Mittheilung[1]) beabsichtige ich nur einen Nachtrag zu meiner in dem 40. Bande der Zeitschrift für wissenschaftliche Zoologie veröffentlichten Abhandlung über den Bau und die Entwicklung der Süsswasser-Tricladen zu liefern. Die ursprünglich gehegte Absicht, noch mehr Arten der uns noch wenig bekannten Tricladen in Betracht zu ziehen, als ich jetzt in thun kann, habe ich, wegen anderseitiger Beschäftigung und der Rückkehr in meine Heimath, aufgeben müssen.

Die Folgenden sind nun die beobachteten Arten :

Planaria torva M. Schultze (= *Pl. Schultzei* Dies.). In meiner oben citirten Abhandlung (p. 362) musste ich unbestimmt sein lassen, ob diese Art auch in den Gewässern Leipzigs vorkäme. Dort habe ich erwähnt, dass ich drei jungen Planarien begegnete, welche in der Gestalt und in der Lage ihrer Augen der *Pl. torva* ähnelten. Später fand ich zwei geschlechtsreife Exemplare, welche mir Aufschluss

1) Diese war schon vor einigen Jahren während meines Aufenthaltes in Deutschland geschrieben. Dieselbe sollte, wie ich ursprünglich beabsichtigte, erst in Zusammenhang mit einer anderen Arbeit über die Würmer veröffentlicht werden. Da nun aber die letztere, wegen der inzwischen herangetretenen Hindernisse, in kurzer Zeit nicht beendigt werden kann, sah ich mich veranlasst diesen den europäischen Tricladen bezüglichen Abschnitt gesondert für sich zu publiciren.

gaben, dass jene jungen Thiere in der That *Pl. torva* waren. In Bezug auf ihr äusseres Aussehen, weiss ich den Ausführungen von O. Schmidt [1] und den diesen beigefügten Abbildungen nichts hinzuzufügen. Gefunden wird diese Art, also sehr selten, in stehendem oder mässig fliessendem Wasser, zusammen mit *Pl. polychroa*. Ein einziges Exemplar, unter mehreren *Pl. polychroa* habe ich auch aus Nicolausrieth in Thüringen erhalten.

Planaria gonocephala Dugès. Diese zuerst aus Frankreich beschriebene Art [2], wurde durch O. Schmidt [3] in der Mur wiedergefunden. Leydig bildet den kopftheil dieses Thieres in seinen "Tafeln zur vergleichenden Anatomie" ab, aber die von ihm in Müller's Archiv (1854) aufgeführte Art aus Genua ist sicherlich eine andere, die meines Wissens noch nicht mit Namen belegt worden ist. Im Jahre 1883, fand ich *Pl. gonocephala* ziemlich zahlreich in einem klaren Bächlein, welches durch eine vom Marienthal im Thüringerwald sich abzweigende Schlucht rieselte. Dugès fand diese Art auch in klaren Wasserströmen. Ebenso giebt O. Schmidt an, dass sie nur in schnellfliessendem Wasser vorkomme. [4]

Die von Dugès gegebene Figur genügt vollständig um die Art zu erkennen. Die allgemeine Färbung ist gerade so, wie bei Pl. polychroa und ähnlichen Variationen ausgesetzt. Zwei seitliche, mehr oder minder deutlich hervortretende, dunkle Bänder verlaufen vom Halse bis zum Schwanzende. Dugès und O. Schmidt erwähnen nichts von diesen Bändern ; der letzte spricht aber bei *Pl. sagitta.* von

---

1) O. Schmidt. "Über Planaria torva." Z. f. w. Z. Bd. XI.

2) Dugès, "Aperçu de quelques Observations nouvelles sur le Planaires et plusieurs genres voisins." Ann. Sci. nat. Tome XXI. 1830.

3) O. Schmidt. "Die dendrocoelen Strudelwürmer aus der Umgebung von Graz." Z. f. w. Z. Bd. X.

4) Übrigens scheint *Pl. gonocephara* eine weit über der Erde verbreitete Art zu sein, denn ich finde sie auch in Japan,

ähnlichen Streifen.

Das Eierlegen geschah noch im Anfang Oktober. Der Cocon hat dasselbe Aussehen, wie der von Pl. polychroa bis auf den verhältnissmässig kurzen Stiel. Einmal entnahm ich vierzehn junge Thiere aus einem Cocon.

Es sei noch bemerkt, dass *Pl. gonocephala* und *Pl. polychroa* in allen wichtigen anatomisehen Punkten unter sich übereinstimmen, wie wir später sehen werden. Die beiden bilden eine Gruppe, die sich vor Allem durch den Bau des Gehirns von den übrigen Tricladen auszeichnet. *Pl. torva* ist, trotz ihres Aussehens, dem *Dendrocoelum tacteum* eher verwandt als jenen beiden.

Planaria abscissa nov. sp. Fig. 1—5, Taf. XXV. Diese schlanke, ziemlich abgeplattete Planarie characterisirt sich durch den Besitz zweier kopflappen und die von dem vorderen Rande weit entfernte Lage der Augen. Auf den ersten Blick sieht der vordere Rand wie abgeschnitten aus. Mit schwachen Vergrösserung erkennt man in der Mitte eine ganz schwache Convexität, deren seitlicher Theil jederseits zu einer ebenfalls schwachen Concavität allmälig übergeht. Die letztere kommt dadurch zu Stande, dass die Kopflappen nicht ganz nach den Seiten, sondern auch ein wenig nach vorn hingerichtet sind. Diese Kopflappen sind zwar bei weitem nicht so augenfällig, wie die von *Polycelis cornuta* (O. Schmidt, *l. c.*), doch mehr länglich gestaltet wie die von *Dendrocoelum lacteum*. Am Ende sind sie abgerundet. Während des Kriechens, trägt das Thier dieselben etwas nach oben gerichtet und macht Bewegungen, gleichsam, als ob es damit betasten wollte, gerade wie die anderen, mit solchen Lappen versehenen Arten.

Hinter den Kopflappen und vor den Augen, hat das Thier eine schwache halsähnliche Einschnürung. Allmälig nimmt der Körper wieder an Breite zu und bleibt sich bis zum Schwanzende ziemlich

gleich. Das letztere ist mehr abgerundet als zugespitzt.

Das Auge ist sehr klein. Der Abstand der Augen von einander ist geringer, als die Entfernung eines Auges von dem seitlichen Körperrande. Die letzte Distanz verdoppelt, ist ungefähr gleich dem Abstande der Augen von dem vorderen Kopfrande.

Unsere Art erreicht die Länge von 12 mm und die Breite von $1\frac{1}{2}$ mm. Die meisten Exemplare sind jedoch kleiner.

Die Grundfarbe des Thieres ist hell-grau, mit, oder ohne schwach-gelblichen Ton. Junge Exemplare haben diese Farbe, aber die grösseren haben ein dunkles oder schwarzes Aussehen, wegen der Entwicklung des Pigments, welches oft im Form unregelmässiger Flecken oder verästelter Streifen auftritt. Der vordere Kopfrand und die beiden Lappen resp. die Seitenränder des Halses entbehren immer des schwarzen Pigments. Der vielfach quergerunzelte Pharynx und die Lage der äusseren Genitalien schimmern undeutlich auf der Rückenfläche durch. Wie bei anderen Arten, bleibt die Entwicklung des schwarzen Pigments an der Ventralfläche bedeutend zurück, so dass hier eine asch-grau Farbe herrscht. Die Ovarien und Hoden sind als dunkle Flecken auf dem Bauche sichtbar.

Gefunden wurde diese Art, sehr zahlreich unter Steinen, zusammen mit *Pl. gonocephala* im Thüringerwald. Das klare rieselnde Wasser des Bächleins, worin diese zwei Arten wohnhaft sind, sammelt sich bald zu einem kleinen Teich. In dem letzteren ist keine der beiden Arten zu finden, wohl aber *Polycelis tenuis* und umgekehrt, kommt diese letzte Art gar nicht in dem Bächlein vor. Der Fundort unserer *Pl. abscissa*, ist nicht der Thüringerwald allein. Im Sommer vorigen Jahres hat mir Herr Dr. O. Zacharias, meiner Aufrage gütigst nachkommend, eine Anzahl einer Riesengebirgischen Planarie zugeschickt, welche sich als vollkommen identisch mit *Pl. abscissa*

herausgestellt hat.[1] Aus seiner brieflichen Mittheilung, gebe ich die zwei von ihm constatirt wordenen Fundorte des Riesengebirges an: 1) findet sich diese Planarie in einem schnellfliessenden Graben mit sehr kaltem Wasser (+ 4° R., auch im Hochsommer) in 4360 Fuss Höhe über dem Meeresspiegel, nahe der Wiesenbaude (auf der sog. "Weissen Wiese"); und 2) kommt sie weniger zahlreich in dem kleineren der beiden Hochgebirgsseen, im sog. "Kleinen Teich" vor, welcher 3600 Fuss hoch liegt. Das Wasser dieses kleinen Sees hat im Hochsommer eine Temperatur von etwa + 11° R.

Gunda Ulvae *mihi.* Fig. 6–13, Taf. XXV. Die von Oersted[2] an dänischen Küsten aufgefundene und als *Planaria Ulvae* beschriebene Art (= *Fovia littoralis* Stimpson), muss ich in die Gattung *Gunda* einreihen. Ulianin[3] führt auch *Pl. Ulvae* Oerst. unter den Turbellarien der Bucht von Sebastopol auf. Seine Abbildung der Genitalien stimmt schwerlich mit denen der *Gunda Ulvae* überein; auch aus seiner Abbildung der ganzen Thieres lässt sich nicht ersehen, ob er wirklich dieselbe Art vor sich gehabt hat.

Meine Exemplare habe ich an dem grobsteinigen Ufer von Klanpenborg, unweit von Kopenhagen, gesammelt. Dort fand ich sie sehr zahlreich unter Steinen. Ein wenig nordwärts, in Skodsborg, habe ich vergebens nach ihnen gesucht. Nach Oersted soll diese Art sehr allgemein, überall im Sunde, in der nähe der Küsten, vornehmlich auf den Ulven, gefunden werden.

Die Zeichnung Oersted's von dieser Planarie ist gut. Offenbar hat er sie in nicht ganz ausgestrecktem Zustande gezeichnet. Das Verhältniss der Breite zur Länge bei einem kriechenden Individuum

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1) Inzwischen hat Herr Dr. Zacharias von seinem Befunde Erwähnung gethan; "Studien über die Fauna des grossen und kleinen Teiches im Riesengebirge." Z. f. w. Z. Bd. 41. S. 498.

2) Oersted "Entwurf einer systematischen Eintheilung u. speciellen Beschreibung der Plattwürmer etc." 1844.

3) Ulianin, im Berichte des Vereins der Freunde der Naturwissenschaft zu Moskau. 1870.

ist etwa 1 : 7. In Länge erreicht das Thier 7 mm oder noch etwas mehr. Der Körper (Fig. 6) ist zum grösseren Theile ungefähr gleich breit. Die halsähnliche Einschnürung ist deutlich. Die seitlichen Kopflappen, welcher der *Gunda plebeia* Lang fehlen, sind mehr vorwärts, als nach den Seiten gerichtet. *Gunda segmentata* Lg. soll noch kleinere Lappen besitzen. Der vordere Kopfrand zwischen den Lappen ist schwach convex. In dieser Hinsicht weicht unsere Art von *Gunda lobata* O.S. ab. Hinten endigt der Körper nur zuweilen abgerundet; gewöhnlich ist das äussere Schwanzend ein wenig eingeschlitzt, wie in meiner Figur dargestellt ist. Die Augen stehen zweimal so weit von einander ab, wie eins der beiden von dem Seitenrande entfernt ist. Sie liegen von dem vorderen Kopfrande etwa anderthalbmal so weit entfernt, wie der Abstand zwischen ihnen gross ist.

Betreffs der Färbung sind auch ähnliche Variationen vorhanden, wie wir sie bei allen dunkelgefärbten Tricladen vorfinden. Kleinere Exemplare sind blass-grau, die grösseren sind dunkler wegen des schwarzen Pigments. Das letztere sieht man auf dem Rücken, unter schwacher Vergrösserung, als unregelmässige Flecke oder Streifen, die ein schmutziges Aussehen bewirken. Zwei seitliche schwarze Linien, die hie und da Zweige abgeben, verlaufen unregelmässig von hinter den Augen, bis zum Schwanztheile. Diese Linien waren jedoch nicht auf allen Thieren bemerkbar. Vorn hat das pigmentirte Feld des Rückens, eine ganz charakteristische Anordnung. Gerade hinter den Augen spaltet es sich in zwei seitliche schmale Theile und ein mittleres Band. Die ersteren gehen nach vorn immer schmäler werdend, den Körperrändern entlang und verlieren sich schon hinter der Halseinschnürung. Das mittlere Band zieht sich zwischen den Augen hin und dann theilt es sich in drei oder vier Bänder, die nach dem vorderen Kopfrande zu allmälig verschwinden. Jedes Auge liegt

daher am hinteren Ende einer pigmentfreien Strecke, die sich vorn bis in die Kopflappen hinein erstreckt.

Ich habe selten Gelegenheit gehabt, die gleitende Kriechbewegung bei dieser Art zu beobachten. Bei weitem häufiger sieht man das Thier spannende Locomotionsart ausführen, etwa wie die Hirudineen.

Anatomisches von den obigen Arten. Alle Individuen von *Planaria abscissa* liessen Cilien am ganzen Körper deutlich nachweisen. Dieselben sind nach hinten umgeschlagen und befinden sich häufiger in Ruhe, als in Bewegung. Vereinzelte oder pinselartig gruppirte, starre Borsten von drei bis vierfacher Länge wie die allgemeinen Cilien, kommen überall in unregelmässigem Abstande vor. Wohl mit Recht, schreibt Lang [1] solchen Borsten der Polycladen Empfindlichkeit zu. Querschnitte zeigen eine auf dem Epithel anliegende, körnige Kruste, welche noch zuweilen, besonders auf der Ventralfläche, als Cilien zn erkennen sind.

Die seitlichen, mit starken Cilien besetzten Tastorgane [2] sind bei allen Arten vorhanden. Bei *Pl. polychroa*, sind deren mehr dorsalwärts gelegenen Theile schon auf dem lebenden Thiere als helle Striche sichtbar. Die "vordere Randrinne" der Polycladen halte ich für dasselbe Organ.

Wahrscheinlich sind die von Lang [3] zuerst beschriebenen Klebzellen auch bei *Gunda Ulvae* vorhanden. Hierauf bezügliche Beobachtungen habe ich versäumt, bei lebenden Thieren anzustellen.

Die Haut–und Körpermuskulatur von *Pl. gonocephala* stimmen mit denen von *Pl. polychroa* [4] überein. Bei den drei übrigen Arten fehlt das System von Quermuskelfasern, welche sich unter dem

1) Lang. "Monographie der Polycladen." 1884.

2) Vide p. 436 meiner früheren Arbeit über Tricladen.

3) Lang. "Der Bau von Gunda segmentata." Mittheil. a. d. Zool. Stat. Bd. III.

4) Vide meine frühere Arbeit, p. 378.

Darme in querer Richtung hinziehen sollen. Von der Hautmuskulatur, habe ich bei *Pl. torva*, trotz der Angabe Lang's, die zweite Schicht (äussere Längsfasern) und bei *Pl. abscissa*, ebenfalls die zweite und dritte (Schrägfasern) Schichten, nicht constatiren können. Die Hautmuskulatur von *Gunda segmentata* besteht nach Lang aus äusseren Ringfasern und inneren, stark entwickelten Längsfasern. *Gunda Ulvae* zeigt zwischen diesen beiden noch eine Schrägfaserschicht. Übrigens schien es mir, als ob die äusseren Ringfasern von *G. Ulvae* nicht ganz parallel, sondern etwas schräg verliefen, so dass sie sich oftmals kreuzten.

Die sich nach aussen öffnenden einzelligen Drüsen verhalten sich gerade so, wie ich früher beschrieb. Über das Mesenchymsbindegewebe, gab ich früher an, dass es aus verästelten Zellen bestehe, ähnlich wie Walter[1] bei verschiedenen Trematoden oder Salensky[2] bei *Amphilina* beschreibt. Ich zweifelte mit Unrecht, ob die "Bindegewebszellen" und die "Bindegewebsbalken" Graff's als verschiedene Elemente betrachtet werden dürften. Das Mesenchym von *Polystomum integerrimum* oder *Calicotyle Kroyeri* wovon es an einer anderen Mittheilung die Rede sein wird, bietet in jeder Hinsicht dieselben Verhältnisse dar, wie das nämliche Gewebe der Tricladen. Ovale oder rundliche Zellen, welche Minot erwähnt, finde ich bei *Pl. torva* und *gonocephala* zahlreich in der Peripherie des Körpers, zwischen den Bündeln der Längsmuskelfasern. Die Zellen, welche ich bei *Dendrocoelum lacteum* in unmittelbarer Umgebung des Ovidukts gefunden habe,[3] sind nichts anderes als Bindegewebszellen. Ebenso auch diejenigen, welche sich in der Nachbarschaft der muskulösen Wand der Penisscheide von *Pl. abscissa* befinden (bd., Fig. 3 u. 4,

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1) Walter, "Beiträge zur Anat. u. Histol. der Trematoden." Arch. f. Naturgesch. 1858.
2) Salensky, "Üb. den Bau u. die Entwickl-gesch. der Amphilina." Z. f. w. Z. 1874.
3) Vide p. 414 meiner früheren Arbeit.

Taf. XXV).

Die Verdauungsorgane übergehe ich mit Schweigen. Die Verästelungsweise derselben bei *Planaria abscissa* und *Gunda Ulvae* wird von Fig. 2 und 6, Taf. XXV, ersichtlich sein.

Von dem Excretionssystem habe ich nur eine fragmentäre Beobachtung an *Pl. abscissa* machen können. Bei deiser Art treten die Kanäle am vorderen Ende des Körpers, wo die geringe Entwicklung des Pigments und die Abwesenheit anderer undurchsichtiger Organe das Studium begünsten, sehr deutlich hervor. Die seitlichen, vielfach gewundenen Hauptkanäle sind oberhalb des Darmes gelegen und verlaufen nach vorn ausserhalb der Augen, um sich dann bald mit einander zu vereinigen, ähnlich wie bei *Dendrocoelum lacteum*. Von ihnen gehen zahlreiche Äste nach vorn und den Seiten, weniger aber nach dem mittleren Theile des Körpers. Diese verzweigen sich wiedrum und anastomosiren sich häufig mit benachbarten Zweigen. Eine grosse Anzahl der länglich gestalteten Wimpertrichter sitzen solitär auf den feineren Verzweigungen. Einmal glaubte ich, auf einem gauz jungen Thiere, eine continuirliche Flimmerströmung im Lumen eines feinen Gefässes wahrgenommen zu haben. Ebenso auch ein paar dorsale Öffnungen der Hauptgefässe in der Gegend der Augen.

Was die Geschlechtsorgane anbetrifft, so mache ich in dem folgenden auf die, jeder Art eigenthümliche Beschaffenheit aufmerksam.

Bei *Gunda Ulvae* weisen alle Theile der Genitalien, bis auf die Anordnung der Hoden und Dotterstöcke, eine vollständige Übereinstimmung mit *G. segmentata* (Lang, *loc. cit*) auf, so dass ich von denselben sehr wenig zu sagen brauche.

Das einfache Geschlechtsantrum von *Pl. torva* besteht aus einem median gelegenen, den Penis enthaltenden Theil und einem engeren,

zur Seite des letzteren liegenden Abschnitt, in welchen, das von M. Schultze zuerst beschriebene birnförmige Organ (vide p. 422 meiner früheren Abhandlung) hineinragt. Gerade da, wo die beiden Theile in einander übergehen, liegt die Genitalöffnung. Diese Eintheilung darf indessen mit der anderer Arten in Penisscheide und Vorraum, nicht verwechselt werden.

Das Geschlechtsantrum von *Pl. gonocephala* bietet einen Fall dar, welcher einen Übergang zwischen dem einfachen und dem in zwei auf einander folgenden Kammern getheilten Antrum darstellt. Es ist namentlich durch eine niedrige, von der Wand sich erhebende Ringfalte gewissermassen in zwei Theile getheilt, welche beide aber im innern von dem voluminösen Penis eingenommen werden.

Ähnlich verhält es sich bei *Pl. abscissa*, indem der Penis auch hier aus der Penisscheide mehr oder minder hervorragt (Fig. 3. Taf. XXV). Die Penisscheide (*ps.*) ist hier indessen sehr scharf markirt wegen ihrer ausserordentlich muskulösen Wandung. Unmittelbar unter dem Epithel, liegt eine mächtige Schicht von dicht geflochtenen, feinen Ringmuskelfasern (*rm.*, Fig. 3 u. 4). An der Aussenseite dieser Schicht liegt ein Stratum von stärkeren Meridionalfasern, deren Verlaufsweise aus den Figuren ersichtlich sein wird. Auf Querschnitten (Fig. 4, *mm*) nimmt man wahr, dass die Fasern in radialen Gruppen angeordnet sind, welche durch die, aus dem benachbarten Bindegewebe (*bd.*) entstammenden Faserzüge von einander getrennt werden. Die ganze Wand der Penisscheide kann man herauspräpariren; sie sieht dann wie ein rundlicher oder ellipsoider Körper aus.

Den Penis von *Pl. torva* hat O. Schmidt richtig abgebildet. Allein die beiden Samenleiter öffnen sich nicht direkt an jener erweiterten Stelle des Penisganges, sondern sie vereinigen sich erst zu einem kurzen Gang, der dann in die Erweiterung übergeht. Die

Vereinigung beider Samenleiter geschieht inmitten des Muskelgeflechtes, welches den basalen Theil des Penis bildet.

Der Penis von *Pl. gonocephala* stimmt, in Bau und Gestalt, im Wesentlichen mit dem von *Pl. polychroa* überein (vgl. p. 408 meiner früheren Arbeit). Die Schmidt'sche Figur ist nicht befriedigend. Das von ihm abgebildete "penisartige Organ," welches die Samenleiter aufnimmt, entspricht dem basalen Theil des Penis von *Pl. polychiroa* und soll nicht als ein besonderes Organ betrachtet werden.

Bei *Pl. abscissa* ist der Penis (*pe.*, Fig. 3 u 4) sehr schlank und dessen Ende, wie gesagt, ragt stets durch die Öffnung der Penisscheide in den Vorraum heraus. Er ist auffallend weniger voluminös wie bei anderen Arten. Der enge Penisgang bildet in dessen Verlauf nirgends eine Anschwellung. Ringmuskelfasern sind unter dem äusseren und dem den inneren Gang auskleidenden Epithel vorhanden. Zwischen diesen beiden Schichten verlaufen die Längsfasern und die an beiden Enden verästelnden Fasern, welche sich räderartig zwischen entgegengesetzten Punkten der Peripherie anspannen (Fig. 4). Die Muskeln zeigen aber im Ganzen keine bedeutende Entwicklung. Übrigens fehlt dem Penis ein aus geknäuelten Fasern bestehender knolliger Theil, welcher in Ejaculation der Sperma von grosser Bedeutung sein sollte. Als Ersatz für diesen Mangel könnte man die ausserordentliche Entwicklung der Muskeln an der Wand der Penisscheide in Anspruch nehmen.

Der zapfenförmige Penis von *Gunda Ulvae* ist von oben nach unten gerichtet, während derselbe bei anderen Arten stets eine von vorn nach der Bauchseite gerichtete, schiefe Lage hat.

Penisdrüsen habe ich diesmal bei allen Arten nicht mit Sicherheit nachweisen können.

Die zahlreichen Hoden sind bei *Pl. torva* in zwei Schichten, ober-und unterhalb der Darmverzweigungen, angeordnet, gerade wie

ich sie bei *Dendrocoelum lacteum* beschrieben habe.

Bei den übrigen Arten, liegen die Hoden in einer Lage, entweder an der dorsalen oder an der ventralen Seite des Körpers. Die erstere Anordnungsweise zeigen nämlich *Pl. gonocephala* und *Gunda Ulvae*, welche beiden somit der *Pl. polychroa* ähneln. Übrigens sind die Hoden bei diesen Arten hinter den Ovarien bis zum Schwanzende vertheilt, wogegen dieselben bei *Pl. abscissa* nur auf beiden Seiten des vorderen unpaaren Hauptdarmes, innerhalb der Ovidukte, beschränkt sind (Fig. 2). Hand in Hand mit dieser beschränkten Verbreitung der Hoden, tritt die Thatsache auf, dass dieselbe hier nur an dem ventralen Körpertheile vorkommen. Es sind dieselben Verhältnisse, die wir bei *Polycelis tenuis* kennen gelernt haben.

Betreffs der Samenleiter, sowohl wie der Ovarien und Dotterstöcke, zeigen alle Arten die nämlichen Verhältnisse, wie ich sie bei *Pl. polychroa* und *Dendrocoelum lacteum* geschildert habe. Nur sei erwähnt, dass die Ovarien von *Gunda Ulvae*, gleich wie die von *G. segmentata*, ausserhalb der Längsnervenstämme gelegen sind. Dagegen sind aber die Dotterstöcke in den Septen und unterhalb des Darmes strangartig angeordnet.

Was die Ovidukt anbetrifft, so verdient die Art und Weise ihrer Öffnung in die Geschlechtscloaka eine kurze Erwähnung. Bei *Pl. torva* und *Pl. abscissa*, vereinigen sich die beiden Ovidukte oberhalb der Penisscheide zu einem unpaaren Gang, der, nach kurzen Verlaufe von vorn schräg nach hinten und unten, bei ersterer Art gerade über der Spitze des Penis und bei letzterer (Fig. 3) kurz innerhalb der Öffnung der Penisscheide, ausmündet. Bei *Pl. gonocephala* öffnen sich die Ovidukte, wie bei *Pl. polychroa*, jede für sich in dem Endtheile des Uterusganges ein, eine Thatsache, die wir schon durch O. Schmidt kennen gelernt haben. Ebenso mündet auch der, durch Zusammentreffen beider Ovidukte gebildete unpaare Gang von *G.*

Ulvae in den Uterusgang ein, gerade wie es sich bei *G. segmentata* verhält.

Zahlreiche Eiweissdrüsen sind bei allen Arten sehr leicht nachweisbar.

Es bleibt nun noch übrig den Anhangsorganen wenige Worte zu widmen. Die drei beobachteten Arten von *Planaria* haben einen sog. Uterus, der eine ihm characteristische Lage und Bau besitzt. Der muskulöse Uterusgang ist bei *Pl. gonocephala* und *Pl. torva* am mächtigsten entwickelt. Bei der ersteren Art, öffnet sich derselbe gerade wie bei *Pl. polychroa*, bei der letzteren über der Spitze des kolbigen Anhangsorgans wie bei *Dendrocoelum lacteum*. Der Uterusgang von *Pl. abscissa* hat einen sehr bescheidenen Umfang und mündet in der Penisscheide unmittelbar hinter der Öffnung des Ovidukte aus (Fig. 3). Der Uterus von *Gunda Ulvae* stimmt in jeder Hinsicht mit dem von *G. segmentata* überein (Fig. 6).

Ich gehe jetzt zur Schilderung des Nervensystems über. Zunächst mache ich darauf aufmerksam, dass es auch auf dorsaler Seite des Körpers zwei seitliche Längsnerven giebt, welche allerdings den ventralen Längsnervenstämmen gegenüber sehr an Stärke zurückbleiben. Diese Beobachtung habe ich auf *G. Ulvae* und *Pl. abscissa* gemacht, bei welchen man die genannten Nerven schon an gequetschten Exemplaren wahrnehmen kann. Dieselben entstehen eine Strecke vor den Augen, ein Umstand, woraus man entnehmen kann, dass sie nicht direkt aus dem Gehirn entstammen, denn dieses ist hinter den Augen gelegen. Es wäre möglich, dass sie sich als direkte, nach der Dorsalseite umgeschlagene Fortsetzung der sog. vorderen Längsnerven herausstellen würden. Sie verlaufen zwischen den Augen und sind bis hinter den Pharynx zu verfolgen (*Pl. abscissa*). Ähnlich wie die ventralen Längsnerven, sind diejenigen der Dorsalseite nicht nur unter sich durch feinere Nerven verbunden,

sondern sie schicken auch nach den Seiten hin plexusbildende Nerven aus, welche wahrscheinlich am Körperrande mit den ventralen Seitennerven in Verbindung stehen und somit einen Nervenschlauch vervollständigen, wie von Gaffron[1] bei *Distomum isostomum* beschrieben wurde. Den von Lang[2] gefundenen sog. Randnerv (äusserer Längsnerv), in welchen die ventralen Seitennerven einmünden, habe ich auch bei *Pl. abscissa* und *G. Ulvae* wahrgenommen. Bei *Pl. torva* und *gonocephala* lösen sich die Seitennerven, gegen den Körperrand zu, in Plexusnetze auf, gerade wie bei *Pl. polychroa* und *D. lacteum*.

Die strickleiterbildenden Quercommissuren des peripherischen Nervensystems machen bei *Pl. gonocephala* so viele Verästelungen und Kreuzungen, dass es kaum möglich war, deren Zahl zu bestimmen. Weniger zahlreich kommen sie bei *Pl. torva* und *abscissa* vor, bei welchen derer doch über vierzig vorhanden sind. Die Zahl derselben bei *G. Ulvae* beträgt etwa 20–25.

Hinsichtlich des Centralnervensystems von *Pl. gonocephala*, ist zuerst zu bemerken, dass sich dasselbe von dem der *Pl. polychroa* gar nicht unterscheidet. Erwähnt sei nur, dass die Zahl der Sinnesnerven jederseits etwa 20 beträgt. Dagegen sind die Gehirne von *Pl. torva* und *abscissa* von demselben Typus, wie die von *D. lacteum* und *Polycelis tenuis* und zeigen unter sich vollkommene Übereinstimmung, so dass ich die beiden in dieser Hinsicht zusammen betrachten kann.

Fig. 5, Taf. XXV, stellt halbschematisch das Gehirn von *Pl. abscissa* dar, wie man es von der Ventralseite betrachtet. Die beiden Längsnervenstämme (*hln*), die sich nach vorn über die Lage des Gehirns hinaus als vordere Längsnerven (*vln*) fortsetzen, schicken

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1) Gaffron. Schneider's "Zoologische Beiträge." Bd. I. Heft 2. p. 112.

2) Lang. "Vergleichende Anat. des Nervensystems der Plathelminthen." Mitth. a.d. zool. Stat. Bd. III.

nach den Seiten Seitennerven (*stn*) aus und sind unter sich durch Quercommissuren (*co*) verbunden, unter (oben, in der Fig.) dem Gehirn sowohl, wie im hinteren Körpertheile. Diese Commissuren und Seitennerven, die sich gerade unter dem eigentlichen Gehirn befinden, sind in jeder Hinsicht denen des hinteren Körpertheils gleich und müssen dem peripherischen Nervensystem zugerechnet werden. Indessen habe ich jene Commissuren in meiner früheren Arbeit als "motorische Commissuren des Gehirns" bezeichnet und meinte dabei, sie derjenigen zur Seite zu stellen, die von Lang bei *G. segmentata* beschrieben wurde. Nachdem ich *G. Ulvae* näher studirte, wurde ich aber gewahr, dass jene Homologisirung eine unberechtigte war. Die "motorische Commissur" Lang's ist in dem von mir als "Gehirncommissur" (*gco*) bezeichneten Theile des eigentlichen Gehirns zu suchen, während jener Theil, den ich mit demselben Namen belegte Lang entgangen zu sein scheint.

Die Gehirnlappen von *Pl. torva* und *abscissa* unterscheiden sich von denen des *Dendrocoelum* oder *Polycelis* dadurch, dass jeder von ihnen durch eine ansehnliche Säule (*si*) von Ganglienzellen und Muskelzügen in dorsoventraler Richtung durchbohrt ist. Diese Säule, oder Substanzinsel, wenn man sie lieber so nennen will, mag wohl auch Bindegewebe in geringer Menge enthalten. Man könnte sie für eine Einrichtung gelten lassen, durch welche eine Anzahl Ganglienzellen mit dem inneren Theile der seitlichen Gehirnpartien in nähere Beziehung gebracht würden. Die Insel liegt stets von dem vorderen Rand des Gehirnlappens mehr entfernt als von dem hinteren Rand desselben. Übrigens zeichnet sich der Gehirnlappen unserer Planarien auch dadurch aus, dass er sich am Seitenrande, wovon mehrere Sinnesnerven ausgehen, nach hinten und den Seiten ausdehnt. Dabei wurden die Ausgangspunkte der neben einander verlaufenden Sinnesnerven um ein Weniges vermehrt. Der Augennerv stammt

aus dem Lappen, vor der oben beschriebenen Substanzinsel. Hinter der Gehirncommissur (*gco*), sind einige mit Ganglienzellen belegte Querfasern vorhanden (*qu*), welche die hinteren Theile der Gehirnlappen mit einander verbinden. Dieselben betrachte ich als dem eigentlichen Gehirn angehörig, zumal als sie nicht in demselben Niveau der die Längsnerven verbindenden Quercommissuren gelegen sind. Regelmässig traf ich ein paar dorsalwärts aufsteigender Nerven, welche von den Längsnerven gerade hinter dem Gehirn ausgingen.

Das Gehirn von *Gunda Ulvae* zeigt, unter wesentlicher Übereinstimmung, einige bemerkenswerthe Unterschiede von dem oben beschriebenen. Besonders nahe schliesst sich dasselbe natürlich an das Gehirn von *G. segmentata* an. Um sich einigermassen über das allgemeine Habitus desselben zu orientiren, betrachte man zunächst die Figuren 7 und 13, Taf. XXV. Während es bei *G. segmentata* jederseits vier Sinnesnerven giebt (Lang, loc. cit), hat *G. Ulvae* deren nur drei. Das erste Paar (I, Fig. 7 u. 13) ist am stärksten entwickelt. Das zweite Paar (II, Fig. 7 u. 13) nimmt, alsbald nach dessen Ursprung, einen etwas höher gelegenen Verlauf als das erste. Diese beiden Paare verästeln sich bevor sie schliesslich unter der Basalmembran am Kopfrande endigen. Das dritte Paar (III, Fig. 7) ist der Augennerv, welcher, schräg nach Vorn und Seiten, dorsalwärts aufsteigt. Dieser ist, im Gegensatz zu dem der *G. segmentata*, der dünnste aller Sinnesnerven.

Auf einem senkrechten Längsschnitte durch den Gehirnlappen (Fig. 13) sieht man, dass der hintere Längsnerv (*hln*) allmälig nach Oben steigt und wo derselbe den höchsten Punkt erreicht, der dickste Theil (0.09 mm.) des Gehirns ist. Die Ausdehnung dieses höchsten Theiles habe ich auf Fig. 7 mit einer punktirten Linie eingezeichnet. In dem medianen Theile entspricht diese Linie mit dem obersten

Rande der Gehirncommissur. Seitlich biegt sie sich nach Hinten und geht, sich hinter den Substanzinseln bogenförmig umwendend, schliesslich in jene Theile des Gehirns über, aus denen die Augennerven stammen. Von dieser Linie aus nach Vorn senkt sich das Gehirn nieder.

Die dem Gehirn aufliegenden Ganglienzellen sind, so weit wie sie deutlich hervortreten, meistens unipolar. Die von ihnen entstammenden Fasern sind so zart, dass sich unmöglich nach ihrem Verlauf in allen Fällen forschen lässt. Ich habe jedoch zwei ansehnliche Faserzüge jederseits in Gehirn näher verfolgen können. Der eine stammt aus einer Gruppe von Ganglienzellen, die gerade hinter der Substanzinsel, vornehmlich auf dem vorderen Abhang des Gehirns gelegen sind (*v.*, Fig. 9 u. 13). Die Fasern verlaufen nach Unten und Hinten und schliessen sich zuletzt denen des unteren Theiles der hinteren Längsnervenstämme an (Faserzug *a*, Fig. 9 u. 13). Auf Horizontalschnitten wird dieser Zug quer getroffen (*a*, Fig. 7) und erscheint als rundliche Stelle, worin die Körner *d. h.* die quergescnnittenen Fasern sich lockerer angeordnet befinden, als in der umgebenden Punktsubstanz. Der zweite Faserzug geht von einer Ganglienzellengruppe aus, welche sich hauptsächlich auf dem hinteren Abhang des Gehirns, innen nach den Ganglienzellen des ersten Zugs zu befindet (*u*, Fig. 10, 12, u. 13). Die Fasern verlaufen nach Vorn in das erste und zweite Sinnesnervenpaar hinein (b, Fig. 7, 12 u. 13). Auf Querschnitten des Gehirns, ist dieser Zug auch quer getroffen (*b*, Fig. 8 u. 9). Wohl mit Bestimmtheit, könnte man diese sich in die Sinnesnerven fortsetzenden Fasern als sensoriell in Anspruch nehmen ; den anderen nach Hinten verlaufenden fehlen alle Entscheidungspunpte, ob sie als sensoriell oder als motorisch betrachtet werden dürfen. Lang spricht den Längsnerven sensorielle Elemente ab. Freilich ist es schwer für diese Behauptung den Nach-

weis zu liefern, denn es sind im peripherischen Nervensystem centripetal ebensowohl wie centrifugal leitende Elemente zu erwarten, vorausgesetzt natürlich, dass solche Differenzirung auch beim Nerven unserer Thiere streng ausgeführt wäre.

Die Gehirncommissur hat hier einen sehr beschränkten Umfang. Auf einem in der Medianlinie geführten Längsschnitte, zeigt sie eine ovale Gestalt, deren Längsachse eine von Oben nach Unten und Hinten gerichtete schräge Lage hat. Erwähnungswerth ist ein dünner, mediangelegener Bündel der Muskelfasern, welche stets die Gehirncommissur in ihrem Mittelpunkt durchbohren (*y*, Fig. 7—10). Nach Lang besteht die Gehirncommissur aus lauter querverlaufenden Fasern, von denen die mehr dorsal gelegenen als sensorielle und die sich noch ventralwärts befindlichen als motorische Commissuren aufgefasst worden sind. Indessen weisen die Ergebnisse meiner Untersuchung auf einen weit complicirteren Bau. Der grösste Theil der Gehirncommissur besteht aus sog. Punktsubstanz, welche mit derjenigen der seitlichen Gehirnpartien continuirlich ist. Inmitten dieser Punktsubstanz habe ich einen ganz dünnen Faserzug festgestellt (*c*, Fig. 8, Taf. I). Derselbe verläuft in einen nach Oben concaven Bogen, unmittelbar unter dem schon erwähnten Muskelbündel *y*, und verliert sich nach den seitlichen Substanzinseln zu. Einen anderen, ebenfalls ganz dünnen Faserzug glaube ich constatirt zu haben, gerade oberhalb des eben genannten Muskelbündels. Die übrigen Querfaserzüge beschränken sich auf die Peripherie und zwar hauptsächlich auf den ventralen, untersten Theil und auf die hintere Oberfläche der Gehirncommissur. Auf der vorderen Seite vermochte ich keine deutliche Fasern zur Anschauung bringen können.

Von den peripherischen Querfasern der Gehirncommissur, schienen mir die mehr oben gelegenen (*e*, Fig. 9) den Eindruck hervorzubringen, als ob sie die ganz an den Seiten des Gehirns, hinter

den Ausgangspunkten der Augennerven gelegenen Ganglienzellen (*w*, Fig. 9) mit einander verbänden. Noch etwas weiter unten auf der hinteren Oberfläche der Gehirncommissur, dringen die Querfasern (*d*, Fig. 7 u. 10) in die seitlichen Gehirnpartien hinein, um sich über oder hinter jenen Faserzügen (*a*) der Gehirnlappen, welche in die hinteren Längsnerven führen, unserem Auge zu entziehen. Die zu unterst gelegenen Querfasern erscheinen, auf Querschnitten, auch an den Seiten der Gehirnbasis sich fortzusetzen, so dass das Gehirn hier durch sie von Unten umfasst wird (*f*, Fig. 8—10). Auf einem durch den untersten Theil des Gehirns geführten Horizontalschnitte, sieht man jedoch, dass dieselben Fasern in die beiden Längsstämme hinein gehen, so dass ich keine sichere Auskunft über ihr Verhalten geben kann. Wir wissen noch zu wenig über den Verlauf dieser oder jener Faserzüge, um sie als sensorielle oder motorische Commissuren angeben zu können. Gerade hinter der eigentlichen Gehirncommissur sind noch einige Quernerven (*qu*, Fig. 7) zu finden, die von Ganglienzellen umgeben sind. Ähnliche Nerven haben wir auch bei *Pl. torva* und *abscissa* beschrieben (*qu*, Fig. 5).

Die paarigen Substanzinseln (*si*, Fig. 7 u. 8), die entschieden mit denen der *Pl. abscissa*, *Pl. torva* und *G. segmentata* identisch sind, ziehen sich durch die Lappen, schräg von Oben nach Hinten und Unten. Auf Fig. 8, die einen gerade innerhalb der Insel getroffenen Längsschnitt darstellt, sieht man noch einige schräg verlaufende Muskelfasern (*si*), welche eigentlich der Insel angehören. Nach Aussen wird die Insel durch einen schmalen Streifen (9, Fig. 7 u. 8) des Gehirnlappens umfasst. Die Fasern dieses Streifen gehen direkt in den Augennerv hinein. Wäre nun derselbe wirklich als motorisch-sensorielle Commissur anzusehen, wie Lang behauptet, so hätten wir die merkwürdige Thatsache vor uns, dass ein so specieller Nerv wie der Augennerv von einer nicht rein sensoriellen Partie des Gehirns

ausginge.

Zwei Bündel der Muskelfasern habe ich stets durch das Gehirn in querer Richtung sich hinziehen sehen. Der eine geht durch den Lappen, hinter den Wurzeln des ersten und zweiten Sinnesnervenpaares (x, Fig. 7). Das zweite Bündel ist da gelegen, wo die seitlichen Partien des Gehirns in die Längsnerven übergehen (z, Fig. 7 u. 11).

Zu dem peripherischen Nervensystem von *G. Ulvae* ist noch Folgendes zu bemerken. Der grössere Theil der Längsnerven trägt zur Bildung des Gehirns bei; ein kleinerer, unten liegender Theil davon setzt sich weiter nach Vorn (t, Fig. 10, 12 u. 13), getrennt von der Gehirnbasis durch einen mit Ganglienzellen besetzten Raum fort, um jenen Nerv zu bilden, den wir als vorderen Längsnerv zu bezeichnen pflegen (*vln*, Fig. 13). Einige Quercommissuren, die von der Gehirncommissur durch die Ausführungsgänge der Schleimdrüsen getrennt liegen, verbinden die beiden Fortsetzungen (t), welche auch einige Seitennerven abgeben. An vereinzelten Stellen habe ich die Verschmelzung der vorderen Längsnerven mit darauf liegenden Gehirnlappen oder Sinnesnerven beobachtet. Aus dem oben Gesagten geht hervor, dass das Nervensysten von Gunda ganz dieselben Verhältnisse darbietet, wie wir sie, besonders bei Süsswassertricladen mit zweilappigen Gehirn gesehen haben. Ein bemerkenswerther Unterschied liegt nur darin, dass bei den letzteren jene vordere Fortsetzung des hintern Längsnervenstammes (t, Fig. 12 u. 13) mit der Gehirnbasis vollständig verschmolzen ist.

Erklärung der Abbildungen.

Tafel XXV.

Fig. 1. *Planaria abscissa* auct. Etwa 8-mal vergr., in kriechendem Zustande.

Fig. 2. Schematisirte Darstellung der Anatomie derselben. *da*[1] Hauptdarmkanal. *da*[2], Darmzweige. *dtg.*, Dottergang. *ge.*, Gehirn. *go.*, Genitalöffnung. *ho.*, Hoden. *ov.*, Ovidukt. *pe.*, Penis. *ph.*, Pharynx. *ut.*, Uterus. *vd.*, Vas deferens.

Fig. 3. Genitalien (vergr.) von *Pl. abscissa*. Combinirt aus Horizontalschnitten. *bd.*, Bindegewebszellen. *go.*, Genitalöffnung. *mm.*, Meridionalmuskelfasern. *ovd.*, Ovidukt. *pe.*, Penis. *ps.*, Raum der Penisscheide. *rm.*, Ringmuskelfasern. *ut.*, Uterus. *utg.*, Uterusgang. *vd.*, Vas deferens. *vor.*, Vorraum.

Fig. 4. Theil eines Querschnittes durch den Penis und die Wand der Penisscheide (*Pl. abscissa*). 200 ×. *pl.*, Lumen des Penis. Andere Bezeichnungen wie in Fig. 3.

Fig. 5. Halbschematisirte Darstellung des Gehirns von *Pl. abscissa*. Von Unten gesehen. *co.*, motorische Quercommissur. *hln.*, hintere Längsnerven. *gco.*, Gehirncommissur. *qu.*, Die der letzteren folgenden Quercommissuren des Gehirns. *si.*, Substanzinseln. *vln.*, Vordere Längsnerven.

Fig. 6. Anatomie von *Gunda Ulvae* auct. 10 ×. Die Körpergestalt ist die eines völlig ausgestreckten Individuum. *mu.*, Mundöffnung. Sonstige Bezeichnungen wie in Fig. 2.

Fig. 7. Gehirn von *G. Ulvae*, combinirt aus Horizontalschnitten. ca. 120 ×. I, II, III, Sinnesnerven. *a.*, ein von oben nach hinten verlaufender Faserzug im Querschnitt. *b.*, nach Vorn in den Sinnesnerven verlaufende Faserzüge. *d.*, Fasern der hinteren Fläche der Gehirncommissur. *g.*, eine Partie des Gehirnlappens, welche die Substanzinsel nach Innen umfasst. *hln.*, *qu.*, und *si.*, wie in Fig. 5. *x*, *y*, *z*. Muskelbündel.

Fig. 8. Querschnitt des Gehirns (*G. Ulvae*), durch den beiden Substanzinseln. 120 ×. *f*, *c*, Zwei Faserbündel der Gehirncommissur. Andere Bezeichnungen wie in Fig. 7.

Fig. 9. Querschnitt des Gehirns (*G. Ulvae*) hinter den Substanzinseln. 120 ×. *e*, die die Ganglienzellen *ww* verbindenden Fasern. *v*, wie in Fig. 13.

Fig. 10 u. 11. Zwei Queschnitte der hinteren Partie des Gehirns. 120 ×. *t.*, Fortsetzungen der hinteren Längsnerven.

Fig. 12 u. 13. Längsschnitte des Gehirnlappens. 120 ×. Fig. 13 trifft gerade die innere Grenze der Substanzinsel. Fig. 12 trifft noch ein wenig medianwärts. *u*, Ganglienzellen der in die Sinnesnerven verlaufenden Fasern (*b*, Figs. 7—9). *v*, Ganglienzellen der in die hinteren Längsnerven verlaufenden Fasern *a*. Andere Bezeichnungen wie in vorhergehenden Figuren.

A Model showing the Motion of an Earth-particle during an Earthquake.

By

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With Plates XXVI – XXVII.

During an earthquake, seismographs, such as are now used in this country, record the three rectangular components of the successive motions of the ground. These components may then be again compounded to form resultants, which, when joined together, denote the real path pursued by an earth-particle during a prolonged shaking.

The model represented on Plate XXVII was constructed on the above principle to delineate, on a magnified scale, the earthquake motion. It was copied from the diagram (Plate XXVI) of the earthquake * of January 15th of the present year, obtained by Ewing's Horizontal Pendulum and Vertical-motion Seismographs. The waves of the two inner circles in that diagram denote the horizontal components of the earthquake magnified five times while those on the outermost of the three circles record vertical motion enlarged eight times. The plate took two minutes and eight seconds to complete one revolution and the radial lines on it mark the successive

* Japan Earthquake of January 15th, 1887, by S. Sekiya. Vol. I., Part III, of this Journal.

seconds of time from the commencement of the shock up to the seventy-second second.*

In the model the motion-path is represented by means of a copper wire, the course of which was determined by successively compounding the three rectangular components recorded on the earthquake diagram ; numbers marked on small metal plates attached to the wire at different intervals correspond to the number on the radial lines on the diagram. In the actual model the motion is magnified fifty times. To avoid confusion the model was made in three parts each showing the motion for twenty seconds or so; thus, Fig. 1, Plate XXVII indicates the motion from the beginning of the shock to the end of the twentieth second, Fig. 2 from the latter instant to the end of the fortieth second, and Fig. 3 thence to the end of the seventy-second second. They are not traced beyond that point as the vertical motion practically ceases to exist and the movements may be simply represented on a horizontal plane. Each model is firmly mounted on a stand. The figures show the northern aspect of the model, but looking slightly from above.

During these seventy-two seconds, quantitative relations of horizontal and vertical displacements and constantly varying changes in their directions and oscillating periods—in fact everything concerning the movement of the ground—can be studied with ease.

Looking at the model one will observe the complexities and irregularities of the earthquake motion ; at certain moments, the ground moves nearly in straight lines, while in others, it describes somewhat circular or elliptical paths. Similarly the vertical component varies in its range and period and consequently the angles which the motion-path makes with the horizontal plane are constantly changing.

* For more detailed description of these diagrams, see Comparison of Earthquake Diagrams, etc., by S. SEKIYA. Vol. I., Part I, of this Journal.

Before going further it ought to be stated that the origin of this earthquake was in the S.W. in a narrow band of country, seventeen to forty miles from the spot where the recording instrument was placed.

The earthquake begins, as usual, with short-period tremors. During the third second there appears for the first time a vigorous horizontal motion, N.W. and S.E., (that is, at right angles to the line joining the origin of the disturbance and the instrument), accompanied by a considerable vertical displacement. Both horizontal and vertical motions then continue with great activity; at the ninth second (Fig. I), the upward displacement of 1.3 mm. or $\frac{1}{20}''$ is recorded, which is the largest vertical motion during this shock. Synchronously with it, there occurs the horizontal motion of 5 mm. or $\frac{1}{5}''$ N.W. and S.E., the complete period of oscillation in both being 1.5 seconds. Equally large vertical motion also appears at the tenth second. Vertical motions are most marked during the first part of the disturbance and give to Fig. 1 more striking features than to the other two.

The maximum horizontal displacement of 7.3 mm., or nearly $\frac{1}{4}''$, occurs later on from the thirty-third to thirty-fourth second with the complete period of two seconds. Its direction is then nearly W. S.W. and E.N.E. or nearly in a line with the origin of the shock. There is, however, no prominent vertical motion simultaneous with it.

The directions of the principal horizontal motions in Fig. 3 are S.E. and N.W., or transverse to the direction of the origin of the disturbance, but it would be premature to draw thence any conclusion as to the relations between the directions of the local movements of the ground and the position of the origin, as the seismic waves are influenced in their passage by a great many circumstances.

In the portions of the disturbance exhibited in Fig. 2 only few important up-and-down oscillations occur, as is shown by its

flatter appearance. In Fig. 3, however, several vertical outbursts of considerable amplitude are observed, with inert intervals between. They practically subside beyond the seventy-first second and the disturbances are then entirely confined to the horizontal plane where they continue with great force for more than one minute. Disappearance of the vertical motion long before the horizontal is the usual phenomenon. Also when large vertical motion occurs there is usually to be found simultaneous large horizontal displacement, but the latter often may be recorded without the former.

On Aluminium in the Ashes of Flowering Plants.

By

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It has long been known that lycopodium contains aluminium (as acetate) dissolved in its juices, and quite recently the presence of much alumina in the ash of another cryptogamous plant has been announced, though the evidence does not seem quite satisfactory. But the occurrence of this element in the ashes of phanerogamous plants has hitherto been attributed to particles of soil adhering to the plants submitted to calcination. Accordingly, in ash analyses, even when elaborate, no mention is found of its presence as a constituent of the real ash, and A. H. Allen's statement, in his *Commercial Organic Analysis*, Vol. I. page 38, that "flowering plants do not contain aluminium as a normal constituent," appears to express the current opinion of chemists.

There have been published, however, at least two exceptions to the supposed non-occurrence of aluminium in phanerogamous plants, one by myself (*Journ. Chem. Soc.* **43**, 481), and the other quite recently by M. L'Hôte (*Comptes rendus* **104**, 853). In the aqueous part of the latex of *Rhus vernicifera*, the lacquer tree of Japan, aluminium is present in solution, apparently as arabate, for on treating the latex with absolute alcohol, boiling the precipitate thus formed with water, filtering and evaporating the aqueous extract, a residue

of gum was obtained, which when burnt left 5 per cent. ash, containing 7.8 per cent of its weight of alumina.*

L'Hôte's recognition of aluminium as a normal component of a flowering plant was made in the juice of the grape, all samples of the fresh fruit and of wine, except one, examined by him, having yielded alumina in small quantity.

There being little or no evidence, except my own, as to whether aluminium is or is not to be ranked among the mineral constituents of flowering plants, I undertook, at the suggestion of my respected colleague, Dr. Divers, to make a few careful examinations of some beans and grains, in order, if possible, to decide the matter. These were completed two or three months ago, and before the notice in the *Chemical News* of L'Hôte's interesting observations, or the *Comptes rendus* had come to hand.

The fact that the soil of the plain of Musashi, in which Tōkyō is situated, and which is of volcanic origin, is remarkable for the large proportion of alumina in it soluble in hydrochloric acid, gave promise that, if anywhere, then certainly here would aluminium be found in flowering plants.

The samples examined, for which I have to thank Dr. O. Kellner, had been grown on the farm lands of the Imperial College of Agriculture, at Komaba, near Tōkyō. Each one was carefully picked over, and all imperfect and visibly soiled grains rejected.

The peas and beans were soaked and well washed in water. Some of these were rubbed with cloths, and left to dry in the air and sun. The rest were, in their soddened state, broken up by hand in

* As recorded by me at the time in the *Journ. Chem. Soc.* Since then, however, I have determined the phosphoric acid, which I had regarded as present in too small quantity to need estimation, and as I now find it to be more considerable than I thought, I believe that 6.3 per cent. more closely expresses the proportion of alumina than 7.8 per cent. as formerly stated. At the first opportunity I shall redetermine it.

water into hull and cotyledons, the two carefully separated and then dried on filter paper in the sun. The rice (hulled),* barley, millet, and buckwheat were also picked over and were then bruised in an iron mortar, winnowed, washed and rubbed on a sieve until the water passing through showed scarcely any turbidity, and then dried on sheets of paper in the sun.

The calcination was carried out in large platinum dishes, one hundred grams at a time, and in most cases several hundred grams of the prepared sample were burnt, and the ashes mixed and ground together in an agate mortar.

The peas, beans, and buckwheat were calcined from first to last over the lamp. Rice, wheat, and the rest, being more difficult to calcine, the samples were charred and partially burned over the lamp, and then the dish transferred to a capacious blind muffle to complete the calcination. The muffle was kept nearly closed and at as a low temperature as possible. It was of clay, but well seasoned by work, and the dishes were always removed when charcoal had to be added to the fire, or when the muffle was heating in the morning and cooling at night, and every other care and watchfulness taken that no contamination with clay or charcoal ash should occur. Generally the calcination was effected with but little fusion of the ash. The prepared ash was boiled with dilute hydrochloric acid and evaporated to full dryness, again treated with dilute hydrochloric acid and filtered through ashless paper. The residue washed and calcined was weighed.

The filtrate nearly neutralised and mixed with sodium acetate, was boiled and filtered through ashless paper and the precipitate

* Even the best hulled rice of commerce, has in this country, adhering to it some of the dust which had been employed in polishing the grains after hulling, and rendering them translucent. This dust or powder is Bôshū sand, a white tufa containing 10—12 per cent. of alumina.

washed. The latter was transferred to a platinum dish, and digested with sodium hydroxide which I had prepared myself from sodium in a platinum dish. The filtered solution was acidified with hydrochloric acid and precipitated with ammonia. The precipitate was washed, dried and ignited, and regarded as aluminium phosphate. In one or two cases, its composition was verified by determining the phosphoric acid in it. In another part of the ash, the total phosphoric acid was estimated by precipitation as ammonium phosphomolybdate and weighing as magnesium pyrophosphate.

In the following table, there are included the numbers for percentage of ash in the grains, and of phosphoric acid and silica in the ash, because they serve to show the normal character of the samples and of my analyses.

	Ash per cent of air-dried Sample.	Per cent. of Ash of.		
		Alumina.	Phosph. acid.	Silica etc.
Pea, whole. *Soja hispida.*	—	0.053	33.48	—
Pea, cotyledons.	4.22	0.000	—	0.50
Pea, hull or skin.	4.31	0.268	5.66	3.60
Red bean (Azuki), *Phaseolus radiatus.*	2.60	0.096	32.89	0.25
Rice. (Hill).	0.87	0.161	51.33	9.36
Rice. (Paddy).	0.56	0.189	52.79	10.99
Wheat.	2.62	0.106	65.55	1.31
Barley.	1.09	0.140	33.19	1.19
Millet (Awa), *Panicum italicum.*	1.68	0.272	40.43	8.91
Millet (Hiye). *P. crus corvi.*	0.94	0.185	39.87	8.62
Buckwheat.	1.72	0.113	1.94	0.81

From this table it will be seen that I have found alumina in every case except in that of the cotyledons of the pea, while in the hull or skin of the pea, one of the largest amounts of alumina occurs.

The result here recorded may at least serve to indicate the propriety of reconsidering the accuracy of the dictum that aluminium is not a constituent of flowering plants.

The Effects of Dilution and the Presence of Sodium Salts and Carbonic Acid upon the Titration of Hydroxyamine by Iodine.

By

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Although hydroxyamine is readily oxidisable in solution by many reagents, its estimation volumetrically is more or less interfered with by the fact that the products of its oxidation may vary. Meyeringh and others have shown that cupric hydroxide, permanganic acid, and other oxidising agents give irregular results. Titration with free iodine in non-acid solutions is the only method which has yet been proved to be fairly reliable. With this method I have become very familiar in connection with researches carried out, not only by myself, but by my former colleagues, Mr. Kawakita and Mr. Shimidzu, in conjunction with Dr. Divers, under whom we had studied, and I have long become aware that it is accurate only when some degree of uniformity is maintained in its practice. In order to learn what deviations in the process, as ordinarily carried out, might cause such irregularities as I have at times observed in the results, I made, last winter, a series of estimations of a known quantity of hydroxyammonium chloride, under definitely varied conditions, the results of which furnish the matter for this communication.

The volumetric estimation of hydroxyamine is performed by

adding a decinormal solution of iodine in potassium iodide to the hydroxyamine solution in presence of alkali bicarbonate, so long as it is bleached, generally using starch as an indicator. In order to get the best results, I find, firstly, that the solution for titration should not be excessively dilute, and, secondly, that it should contain very little alkali salts, and accordingly, neither contain, to begin with, much acid,—for this needs to be neutralised,—nor receive during titration more sodium bicarbonate than is necessary to take up the hydrogen iodide formed. The hydroxyammonium chloride used in all my experiments had been prepared from mercury fulminate.

Influence of the strength of the solution :—Very concentrated solutions of hydroxyammonium chloride give results too low. For example, 5 ccs. of a solution of which the true titer was 10 ccs. of decinormal iodine solution, required, after evaporation to dryness and addition of a few drops of sodium bicarbonate solution, 9.3 ccs. only of iodine ; 5 ccs. of another solution with a titer of 9.9 ccs. iodine solution, required after evaporation only 9.4 ccs., and 5 ccs. of a third solution with a titer of 9.55 ccs. iodine required after evaporation only 9.25 ccs., the differences being not due to loss during evaporation, but to irregular reaction with the iodine. Moderately concentrated solutions give good results, but when the dilution reaches about 1000 to 1 the results become too high, and more and more so with increase of the dilution. With a dilution of 3000 to 1 the results of titration are very markedly too high, as may be seen from the first of the tables which follow.

With an iodine solution approximately decinormal and containing 0.1256 gram in 10 cub. centims., a solution of hydroxyammonium chloride was used containing 0.0330 gram in 10 ccs., and requiring by calculation 9.6 ccs. of iodine solution. Of this 10 ccs. were taken for each experiment.

Dilution.	Cub. cents. of iodine solution required.			
	(1)	(2)	(3)	(4)
Undiluted	9.55	9.65	9.55	9.55
Diluted to 100 ccs. ...	10.1	10.3	10.0	10.0
„ „ 200 ccs. ...	10.15	10.25	10.2	10.2
„ „ 500 ccs. ...	10.9	10.95	10.7	10.7

A single drop of the iodine solution, added to 500 ccs. of water containing a little sodium bicarbonate, gave in the absence of hydroxyamine a strong colour-reaction with starch, so that dilution of the reagent was not the cause of the greater quantity of it required by the attenuated hydroxyammonium chloride solutions.

But by the large dilution of the hydroxyammonium chloride some dissociation of the salt appears to take place, for the iodine solution is consumed by the diluted solutions, while it is not at all attacked by the stronger ones, in the absence of sodium bicarbonate. The following results show this. The solutions used were the same as before, but no addition of sodium bicarbonate was made.

Dilution.	Ccs. of iodine solution required.
Undiluted	0.0
Diluted to 100 ccs. ...	0.4
„ „ 200 ccs. ...	0.8
„ „ 500 ccs. ...	1.4

Probably dependent upon this dissociation is the partial decomposition on standing for some hours of hydroxyamine in very

dilute solutions of its hydrochloride. Dilution of 10 ccs. of the hydroxyammonium chloride solution to 500 ccs., and of another 10 ccs. to 1000 ccs., was made, and the solutions left all night and then titrated by iodine, with the usual addition of sodium bicarbonate. The 500 ccs. required then only 10.1 ccs. of iodine solution instead of about the 10.8 ccs. which would have been wanted with that dilution if the titration had been proceeded with at once, as may be seen from the first of the preceding tabular statements. The effect of time upon the dilution to 1000 ccs. was still more marked, for this required only 8.8 ccs.

Influence of sodium chloride, sulphate, or carbonate present in the solutions :— Large excess of sodium bicarbonate gives high results, but I have not made any systematic series of measurements of its effects, since the use of unnecessary quantities can always be avoided. But in many cases, which occur during investigations, the solution of hydroxyamine to be titrated is very acid, and to neutralise this acid with the bicarbonate is to charge the solution with sodium chloride or sulphate, and carbonic acid. Now either of these salts, when present in rather large proportions, affects the titration of the hydroxyamine, while the carbonic acid is also active in this way in presence of one of these salts, although by itself it appears to be without action. Free acid should therefore be nearly all removed—hydrochloric acid by evaporation, sulphuric acid by addition of baryta,—before the sodium bicarbonate is added. The effect of sodium chloride was measured in a series of experiments which are tabulated below. The solutions employed were :—

Hydroxyammonium chloride	6.9772	grams per liter.
Iodine....	12.6238	,, ,, ,,
Sodium chloride	100.0	,, ,, ,,

The same, charged with carbonic acid.

5 ccs. of hydroxyammonium chloride, equivalent to 10.1 ccs. of iodine solution, were used in each trial, after dilution to the volume shown in the table by addition of either water, salt solution, or salt solution carbonated.

Diluent.	Vols. of the dil^d 5 ccs. hydroxyammonium chloride.					
	15	25	35	45	55	65
	Ccs. of iodine solution required.					
Water...	10.2	10.25	10.4	10.45	10.55	10.65
Salt solution ...	—	10.45	10.7	11.0	11.25	10.85
S. s. carbonated...	10.4	10.8	10.8	10.9	11.5	12.10

Other results confirming those in the above table were obtained with different solutions. Blank tests showed that the diluents of themselves consumed no iodine.

The influence of carbonic acid becomes more marked with lower degrees of dilution of the hydroxyamine solution, when instead of employing sodium-chloride solutions charged with the gas by sending a stream of it through them for an hour, solutions were prepared, in imitation of what happens when an acid solution of hydroxyamine is neutralised preliminary to titration, by mixing equivalent volumes of hydrochloric acid and sodium carbonate. The strength of the solutions employed were:—

Hydroxyammonium chloride	6.8220	grams per liter.
Iodine (as in last series)	12.6238	" " "
Sodium carbonate (anhydrous)	96.3075	" " "
Hydrochloric acid....	64.5079	" " "

The acid was of equivalent strength to the carbonate.

5 ccs. of the hydroxyammonium chloride solution, equivalent to 9.9 ccs. of the iodine solution, were used in each case, and diluted, to the extent stated in the table, by addition, after the water when this was also used, first of the acid and then of the same volume of the carbonate. A little bicarbonate was added as usual during titration.

Acid and carbonate, ccs. of each.	Total vols. of the dil^{d} 5 ccs. of hydroxy^{m} chlor.							
	15	25	35	45	55	65	75	85
	Ccs. of iodine solution required.							
0	9.95	10.1	10.2	10.3	10.25	10.45	10.5	10.6
5	10.7	10.7	10.6	—	11.2	—	11.2	—
10		11.2	—	11.4	—	11.7	—	—
15			11.0	—	11.2	—	11.7	—
20				11.7	—	12.0	—	—
25					11.35	—	12.0	—
30						12.0	12.3	—

Blank tests showed that the diluents alone decolourised no iodine.

A corresponding series of trials were made with sulphuric acid in place of hydrochloric acid. The solutions used were:—

Hydroxyammonium chloride	6.6492	grams per liter.
Iodine (as before)	12.6238	,, ,, ,,
Sodium carbonate (as befors)	96.3075	,, ,, ,,
Sulphuric acid	89.0409	,, ,, ,,

5 ccs. of the hydroxyammonium chloride, equivalent to 9.65 ccs. of iodine solution, were used in each case, and diluted on the same plan as before to the extent shown in the table.

Acid and carbonate, ccs. of each.	Total vols. of the dil^{d} 5 ccs. hydroxy^{m} chlor.						
	15	25	35	45	55	65	75
	Ccs. of iodine solution required.						
0	9.7	10.0	9.95	10.0	10.0	10.0	10.2
5	10.3	10.35	10.55	—	10.45	—	10.6
10		10.7	—	10.65	—	10.6	11.0
15			10.8	—	10.4	—	—
20				11.0	—	10.85	11.3
25					11.2	—	—

The experiments here recorded show that high dilution with water causes hydroxyamine to give a higher titer with iodine than that calculated; that this effect of dilution is increased when sodium salts are present in large amount; and that it is still more increased when carbonic acid is also present. They also show that with very high strengths of the solution the titer may be too low. It would seem, therefore, that there must be some deviations from the reaction which is expressed by the equation:—

$$2\,NH_3O + 2\,I_2 = N_2O + 4\,HI + H_2O$$

Should the action of the iodine not go so far, nitrite would be formed, and the low results obtained with very concentrated solutions be accounted for. The production of nitrite from hydroxyamine by various oxidising agents has been noticed by Bertoni, so that its formation through the action of iodine should not be improbable. On testing the point I have found that a distinct formation of nitrite does take place, and that, too, not only where concentrated solutions of hydroxyamine are acted upon, but in all cases, even in those where the iodine titer is higher than that indicated by the above

equation.* This fact makes greater the need already present of assuming that the hydroxyamine in decomposing must yield a very little deoxidised nitrogen, either in the free state or in union with iodine. Evidence of the correctness of this assumption seems to be afforded by a remarkable phenomenon to be observed in titrating hydroxyamine with iodine. A very weak solution being mixed with very little sodium bicarbonate, insufficient for the complete titration by iodine, the solution of the latter is added just beyond the point when it is no longer bleached. If now to this solution a little more sodium bicarbonate is added, a milkiness soon appears, strikingly resembling that of iodoform appearing under similar conditions when in place of hydroxyamine a trace of alcohol is present. It disappears on adding more iodine, but may be reproduced several times in succession before the hydroxyamine is exhausted. The substance causing this turbidity may perhaps be in reality an iodamine, related to nitrogen as iodoform is related to carbon.

I am much indebted to Dr. DIVERS for suggestions in carrying out this investigation.

* The solution after titration always yields a little free iodine when acidified, and this in absence of any iodate in the potassium iodide used, is one proof of the presence of nitrite. The solution also, when freed first from all iodide, and then tested with acidified ferrous sulphate, always gives the brown colour caused by a nitrite.

Notes on a Large Crystal Sphere.

By

Cargill G. Knott, D. Sc., F. R. S. E.

Professor of Physics, Imperial University.

Some short time since I made, at the request of the owner, a careful determination of the dimensions of a large Sphere of Quartz. It was nearly 6 inches in diameter, and weighed $10\frac{1}{4}$ lbs. avoirdupois. Its history is briefly as follows.

Nearly 30 years ago Mizuno Izumo-no-Kami, a prince in the service of the late Shōgun, offered on behalf of the Government a high reward for the discovery of exceptionally large crystals. The miners at Mitake Mountain, in the district of Higashi, Province of Kōshū, being roused to special exertions, eventually found two crystals of remarkable size. But before these were shaped, ground, and polished, the Prince died and the Government repudiated the bargain. The wealthy men of Kōfu kept the crystals in possession until a favourable chance occurred for disposing of them. About 1872 the larger one was bought by the Kunaishō or Department of the Imperial Household ; but the smaller remained in stock till 1883. Since then it has passed through several hands, and has recently been shipped to New York by Messrs Griffin & Co. Yokohama.

The larger of the two crystals, which is about 7 inches in diameter, is of a white snowy or milky colour and consequently lacking

in the exquisite purity of the smaller one, which forms the subject of the present note.

The exact dimensions of the latter are as follows.

Diameter, 15.08 cms. or 5.94 inches.

Volume, 1796 c. cms. or 109.6 c. inches.

These numbers are correct certainly to the third significant figure, with a probable error of ± 1 in the fourth. Within the errors of observation, the crystal was foumed to be a perfect sphere. The mass measurements are as follows:

True Mass, 4650.8 grms. or 10.250 lbs. Specific Gravity, 2.59.

It is impossible in words fitly to describe the exquisitely clear transparency of the crystal. It was absolutely colourless and free from any surface flaw or internal blemish. Shining in all lights with a peculiar lustre, it was an object fitted to charm every eye, and more especially, because of its perfect sphericity, the eye of the mathematician. The inverted image of the landscape as viewed through it appeared beautifully defined, only a slight indistinctness arising from the superposition of the two images due to double refraction. It was this optical property indeed which alone enabled me to distinguish definite directions in the crystal, the two images coinciding along one unique diameter and also along every other diameter at right angles thereto.

Several distinct experiments gave as a good mean for the mean refractive index, 1.56.

Resting the sphere on a sheet of finely divided section paper, and rolling it about until the two images of any line showed widest apart, I was able to make a fairly good estimate of the ratio of the indices of refraction of the extraordinary and ordinary rays. It is doubtful if as good a value was ever obtained by so simple a method.

Consider the case of a ray falling normally on the surface of a double-refracting sphere. Then whatever be the course of the extraordinary ray within the sphere, it must clearly emerge normally also. Hence the apparent deviation of the extraordinary ray from the original direction of the incident ray (a quantity very easily measured) will be twice the true deviation due to one refraction. The greatest possible deviation of the extraordinary ray from its course at normal incidence was, in this way, found to be 19′ 45″. This gives for the ratio of the two refractive indices 1.00576; Rudberg gives for quartz 1.006.

The possibility of obtaining such a result from optical measurements which, at the time, were made from simple curiosity with no studied attempt at scientific accuracy, is I think an indication of the remarkable purity and exquisite finish of the mineral which it fell to my good fortune to have in possession for a few days.

The Marine Biological Station of the Imperial University at Misaki.

By

K. Mitsukuri, Ph. D.
Professor of Zoology, Imperial University.

With Plates XXVIII–XXIX.

Thanks to the liberality of the Department of Education and of the authorities of the Imperial University, a marine station in connection with the biological laboratories of the Imperial University has at last become a reality, and naturalists in Japan will not hereafter have to have recourse to dark and inconvenient fishermen's huts, when they wish to study marine life. A piece of ground in the town of Misaki on the west side of the entrance to the Bay of Tōkyō was ceded some years ago by the Imperial government to the then University of Tōkyō, but owing to unavoidable causes, building of the station was not proceeded with until last autumn. The laboratory of which a view from the sea is given in Pl. XXVIII was finished last March and is now ready for use. The building is of plain wood, and one story high, except in the middle part which has a second floor. The main laboratory room which occupies the whole sea front (See the plan in Pl. XXVIII.) is 48 feet long, 12 feet wide at the two ends and 18 feet in the middle, and is able to accommodate about ten workers. A number of small aquaria for the use of investigators will be placed in this room. Of the rooms at the back of the main laboratory, one (*B*) has a cement floor and is for assorting and preserving specimens brought in from the sea. Another (*E*) is to

be used as the library-room, and a third (*C*) as the store room. The second floor over the central part of the building is able to give sleeping accommodation for a few persons. From a tank placed outside the building, fresh sea-water is carried into the main laboratory room and the assorting room, and is delivered out of many facets.

Misaki, (see Map Pl. XXIX) the town in which the station is situated, is an important fishing town on the western side of the entrance to the Bay of Tōkyō. It is the southern end of the peninsula of Miura which divides the Tōkyō Bay from the Bay of Sagami. It is easily reached from Tōkyō and Yokohama in one day. In front of the town, there is an island called Jōgashima, making between it and the main land a spacious and well-sheltered strait which forms the harbor of Misaki. The strait is perhaps over two square miles in extent and about eight fathoms in the deepest place. It is this piece of water which the laboratory fronts. Several villages besides Misaki are situated along the strait. It is an interesting fact that there is a certain amount of division of labor among these fishing villages. For instance, the fishermen of Misaki itself are engaged mainly in deep-sea fishing, while those on the island of Jōgashima confine themselves to animals found close to the shore. These towns together supply the fish-market of Tōkyō with a large amount of marine produce.

Misaki has long been a favorite collecting grounds for naturalists. The harbor itself, the tide pools on the ocean side of Jōgashima, and the neighboring inlets of Muroiso, Koajiro &c. furnish all kinds of bottom, while out in the sea off the shore there are beds which furnish the world-renowned Hyalonema. Almost every group of marine animals is represented in this region in more or less abundance. I do not think we have, by any means, become acquainted with even a small part of all the interesting animals to be found, but I might mention here some of the more important ones

already known to us. From what little examination we have made, Foraminifera are likely to furnish a great many species for investigation. On this head, I expect that we shall know more before long, as an English gentleman, Mr. A. Durrand, who spent a week in the station, sent to England bottom-washings from Misaki to be examined by specialists. Of the Radiolarians we have seen some—mostly of the Acanthometridae. Sponges are well represented. Of these, the most renowned is of course Hyalonema. We have not yet had the pleasure of bringing this beautiful sponge up by ourselves, as we do not possess at present means for dredging in deep-sea, but the fishermen of Misaki often bring in magnificent specimens during the winter months. They are brought up clinging to the fishing-line. Specimens of Hyalonema in museums of Europe and America are in reality mostly from Misaki, although they are marked as from Enoshima where they are bought and sold. Not less interesting than Hyalonema, although not as beautiful, is Tetilla Japonica, *Lampe* (Arch. f. Naturgesch. 52 Jahrg. I Band I Heft). This is found in great abundance in the harbor of Misaki during summer months. Of the Coelenterata, hydroid colonies are not very numerous, although we found one species of Aglaophenia in great abundance in December. Hydro-medusæ, Acalephæ, and Sea-anemones are fairly numerous. Corals are found living, as also Veretillum and other Pennatulids. Of the Echinoderms, there are several species of sea-urchins, star-fishes, ophiurans and holothurians, some species being found in great number. A Comatula is also found. A magnificent Pentacrinus two or three feet long is brought up in the same manner as Hyalonema. The Mollusca are exceptionally abundant. Tide pools &c. may be said to be alive with them in the spring, and their egg-masses form conspicuous objects at the same season of the year. Some of the more noticeable molluscs are Chiton, Haliotis, Aplysia,

a curious Tethys and other beautifully colored Nudibranchs, Patella, &c. Cephalopoda are caught in abundance by fishermen. Crustaceans are very largely, and worms fairly well, represented. Lingula is found here as in almost every part of Japan. The inlet of Muroiso is fairly choked with Ascidians, and their bright red egg-masses form striking objects at the breeding season. Surface collection also furnishes many interesting animals. Besides the usual number of the Crustacean larvæ etc. we have caught Doliolum, Salpa, Pteropoda, Heteropoda, (Atlanta, Pterotrachea) Actinotrocha, Tornaria, Siphonophora, Pilidium, Loven's larva etc. Physalia and Charybdea are also found. The Kuro-shio (Kuro-siwo) which passes off the coast of Japan has no doubt some influence on the surface fauna of this part.

Arrangements will be made by which students in the biological course of the University will be required to pass at least one term in this station.

Credit is due to Messrs Kojima and Yamaguchi, Architects of the Educational Department for designing the building. My colleague Prof. I. Ijima has also furnished many ideas in regard to the building and its internal fittings. Thanks are due to Dr. Anton Dohrn of Naples for kindly examining the plan of the station and for making several useful suggestions.

[For an account of a zoological excursion in this part of Japan, see Döderlein: Faunistische Studien in Japan, Arch. f. Naturgesch. 49 Jahrg. 1 Heft.]

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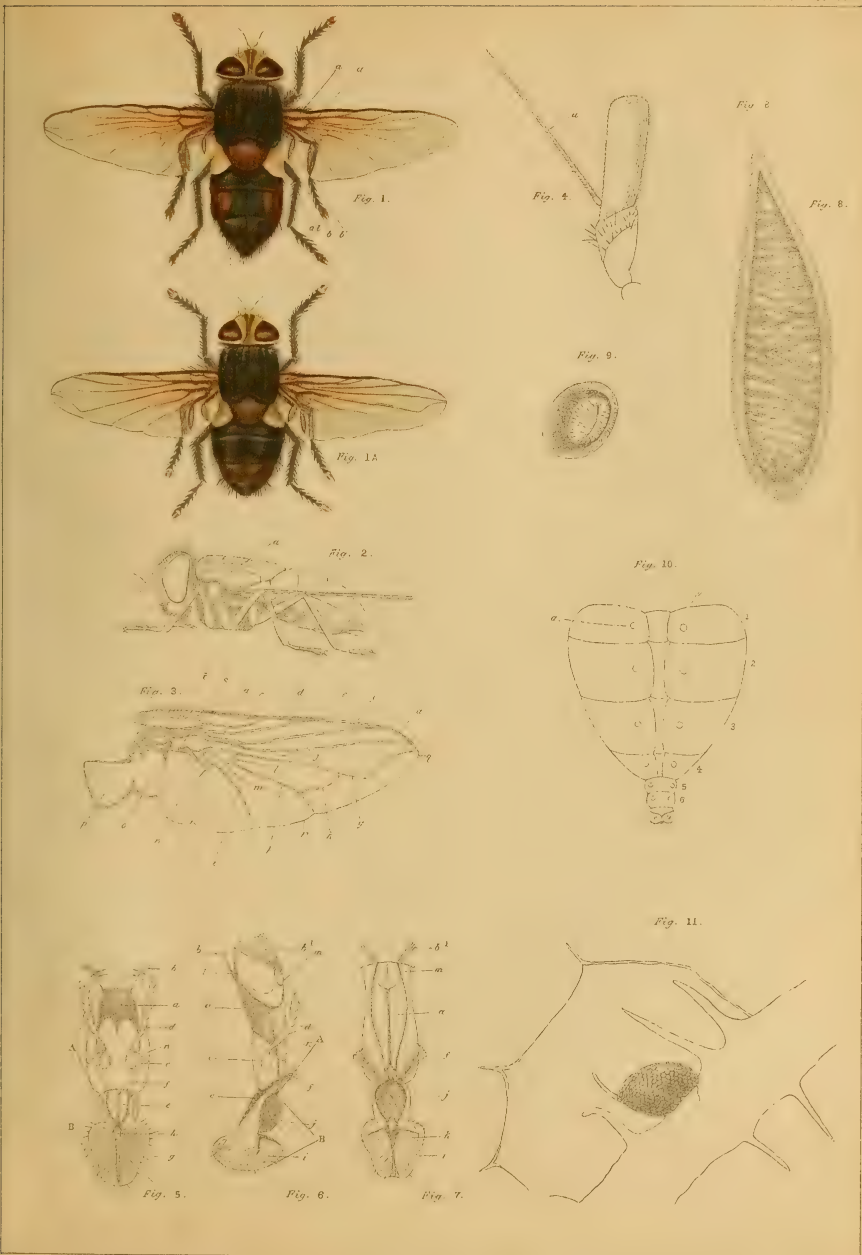
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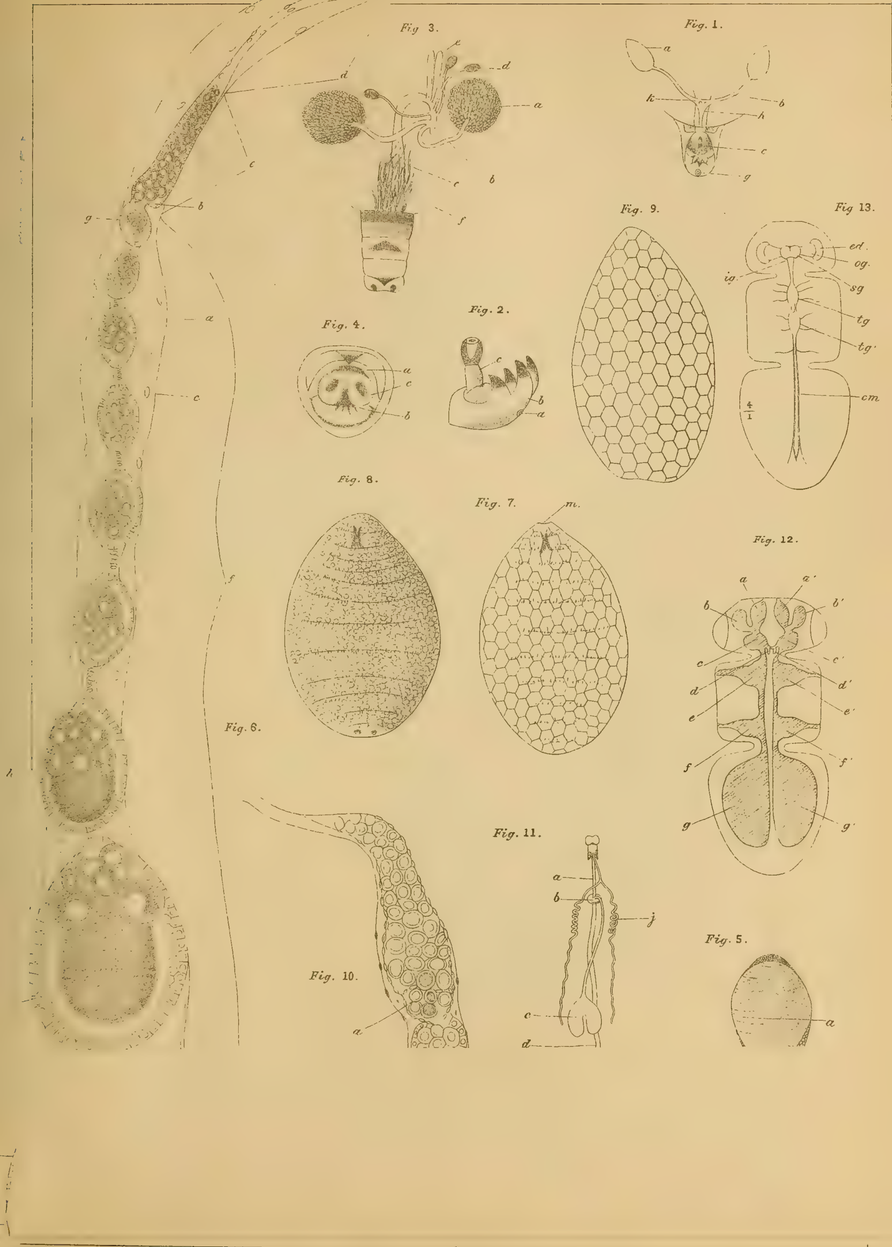
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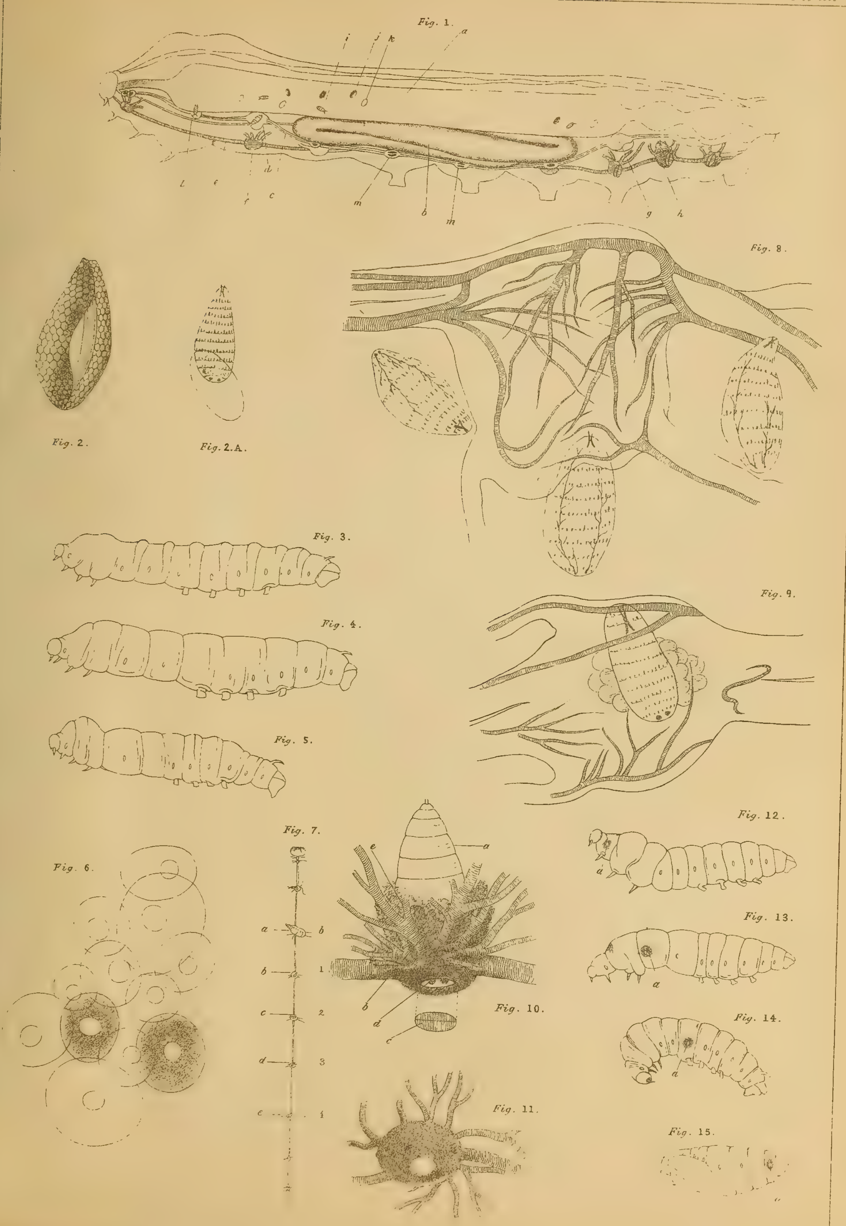
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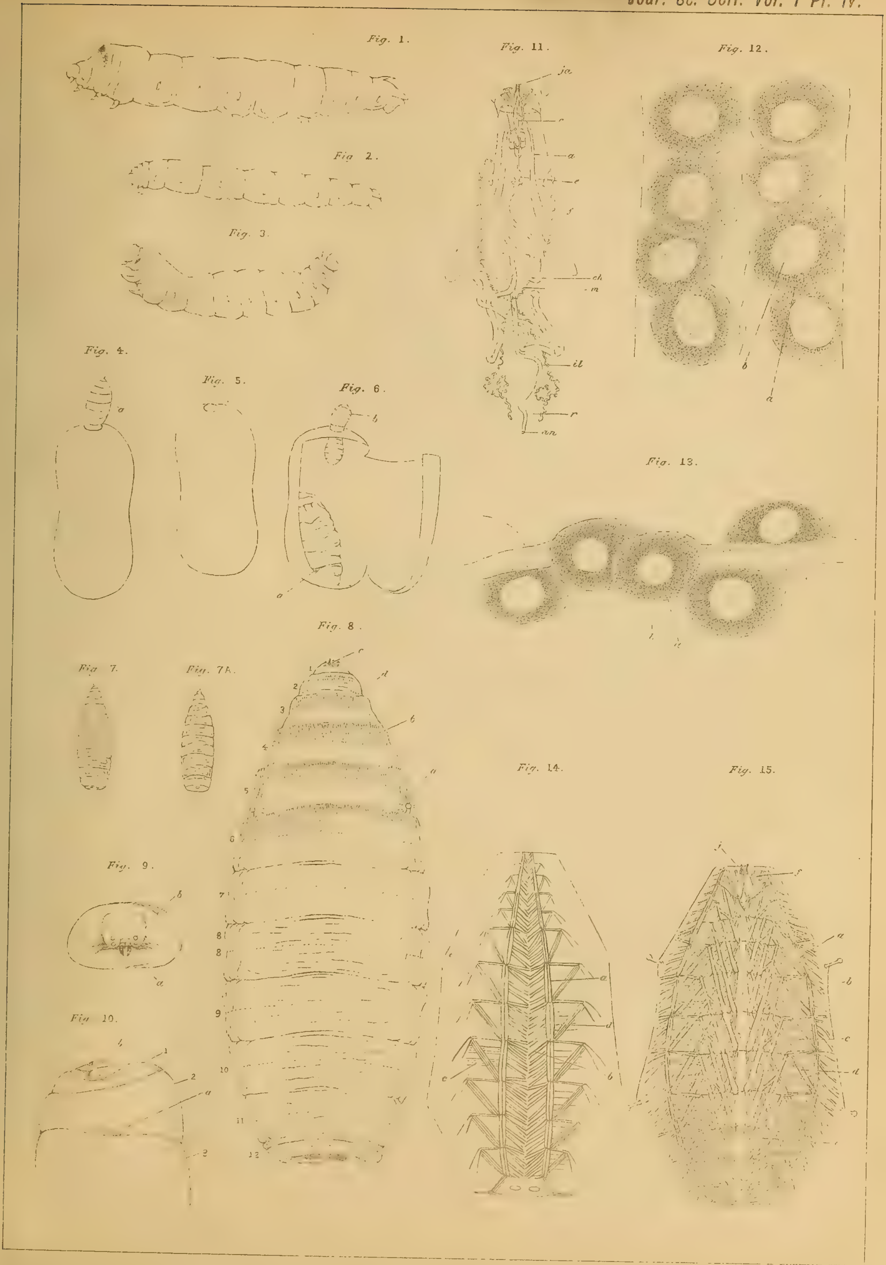
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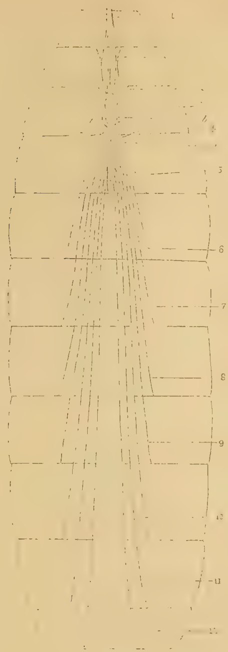

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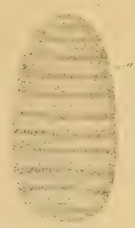

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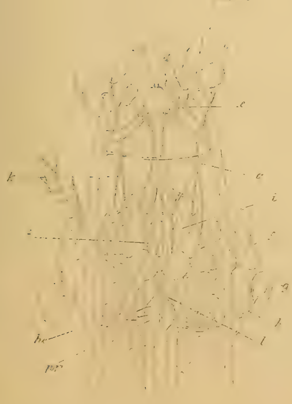

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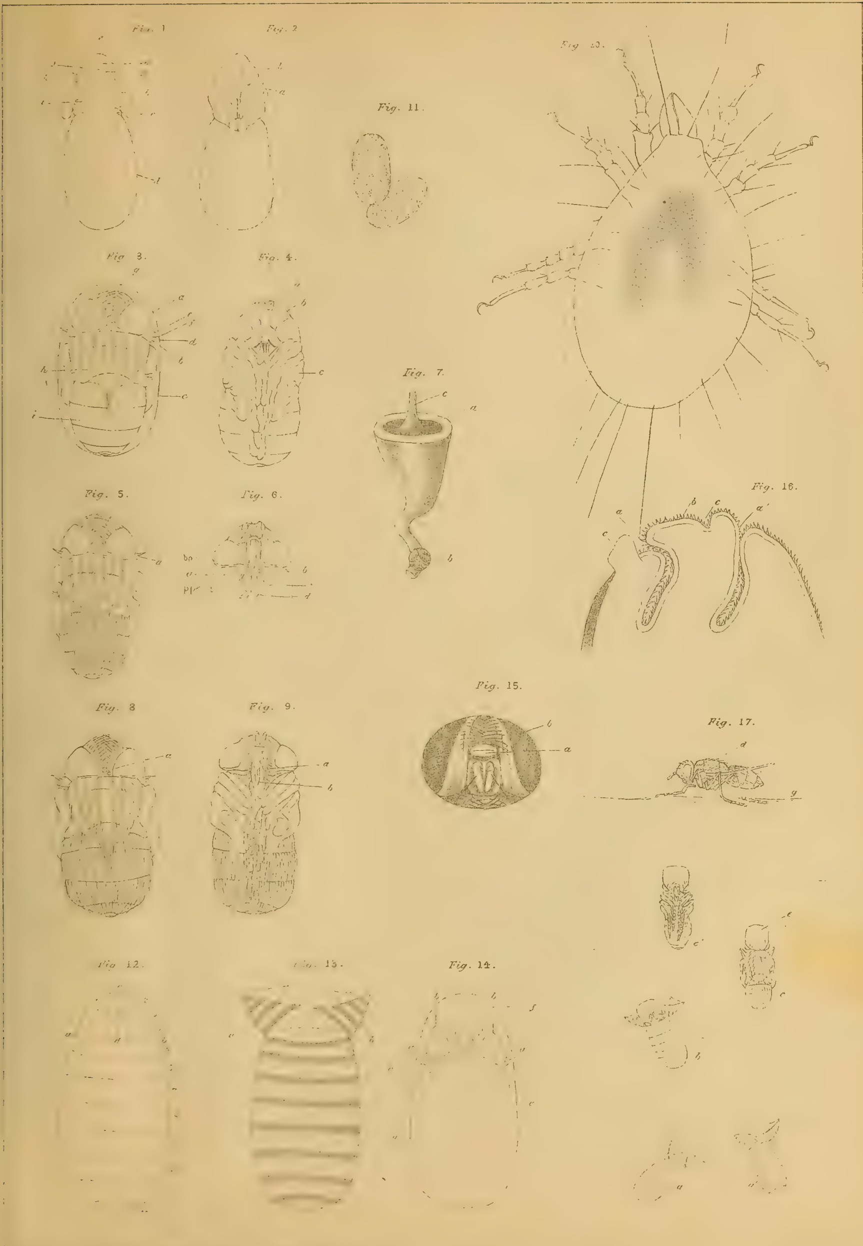
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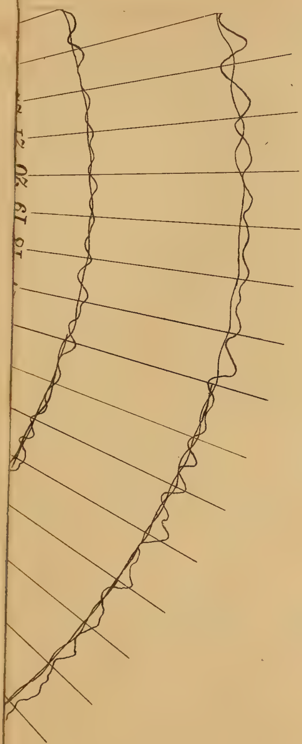
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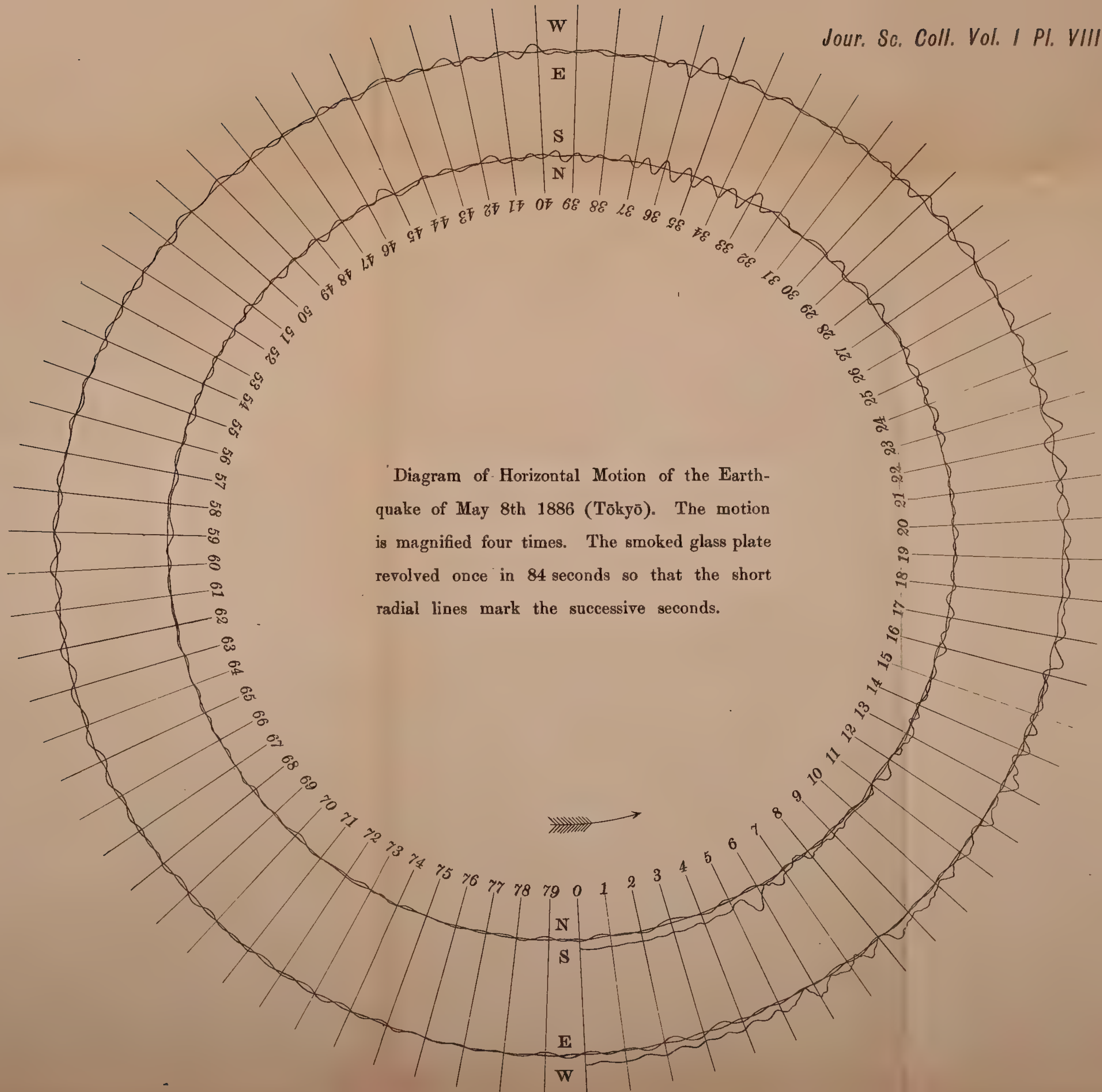

Diagram of Horizontal Motion of the Earthquake of May 8th 1886 (Tōkyō). The motion is magnified four times. The smoked glass plate revolved once in 84 seconds so that the short radial lines mark the successive seconds.

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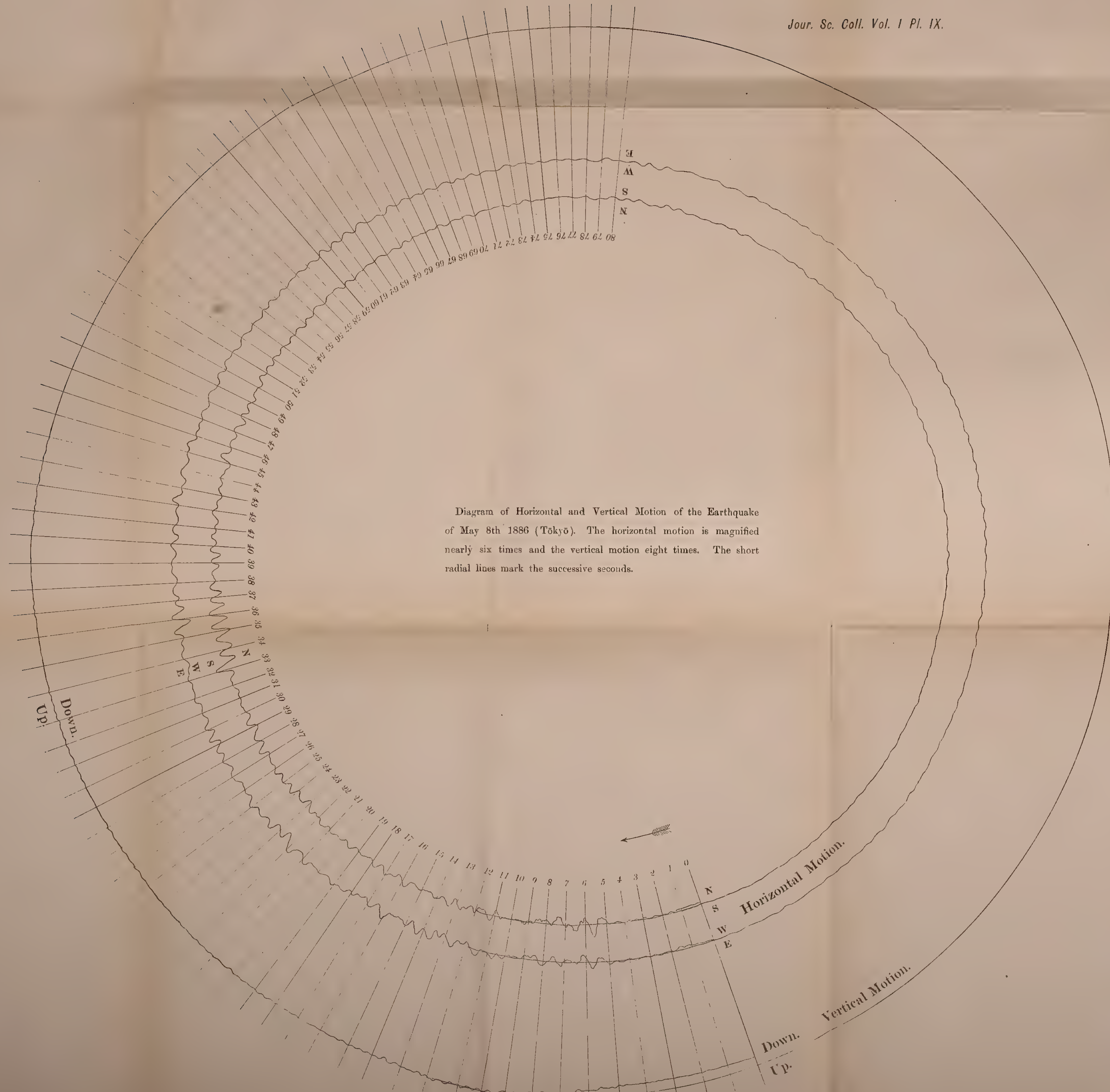

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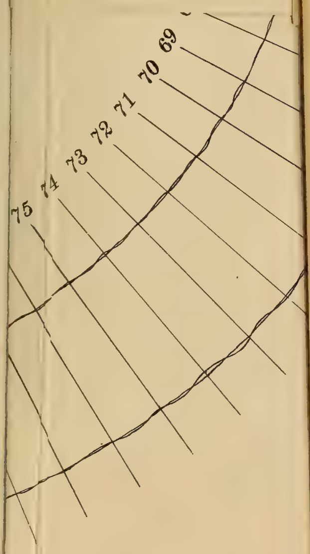
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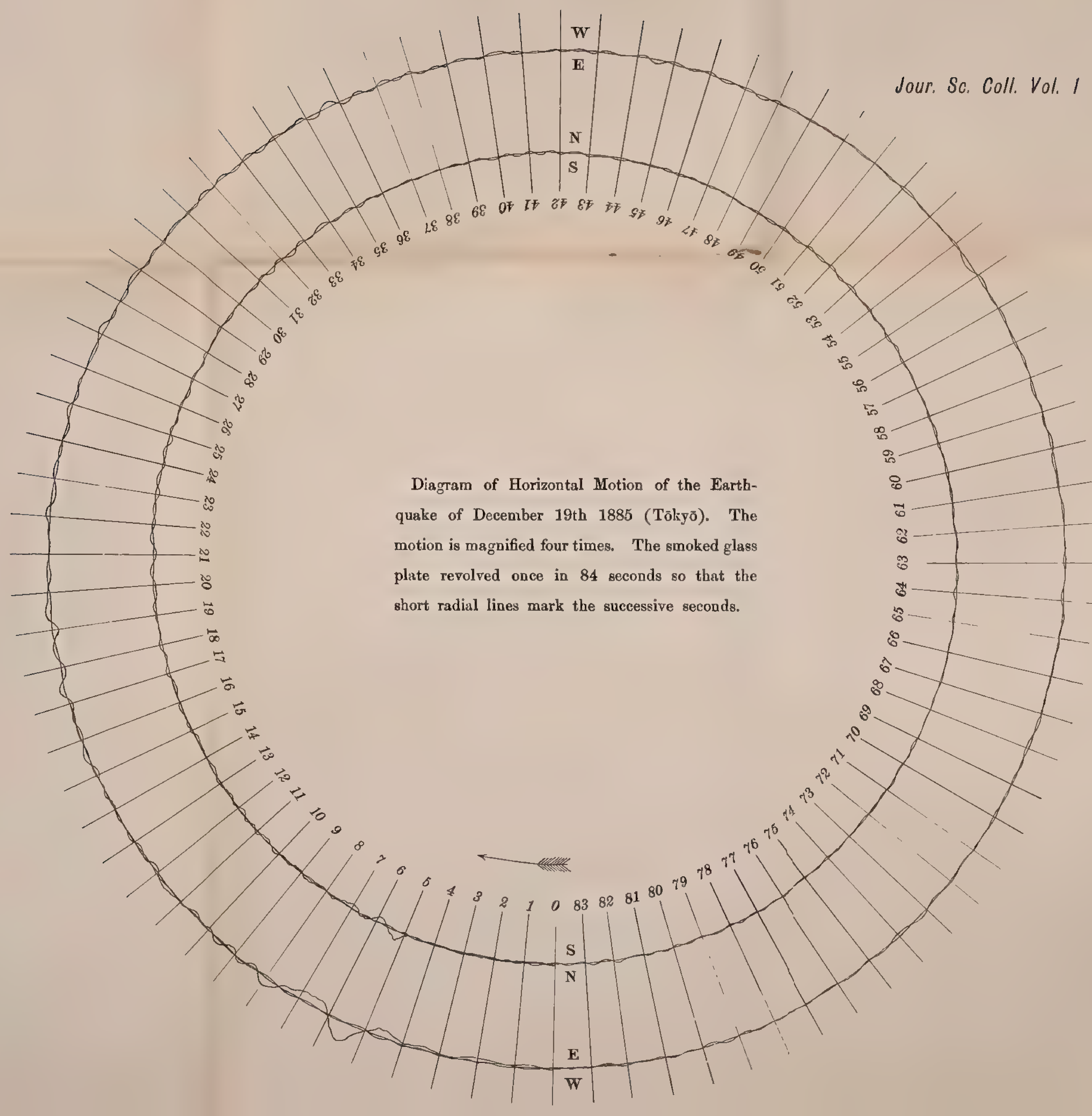

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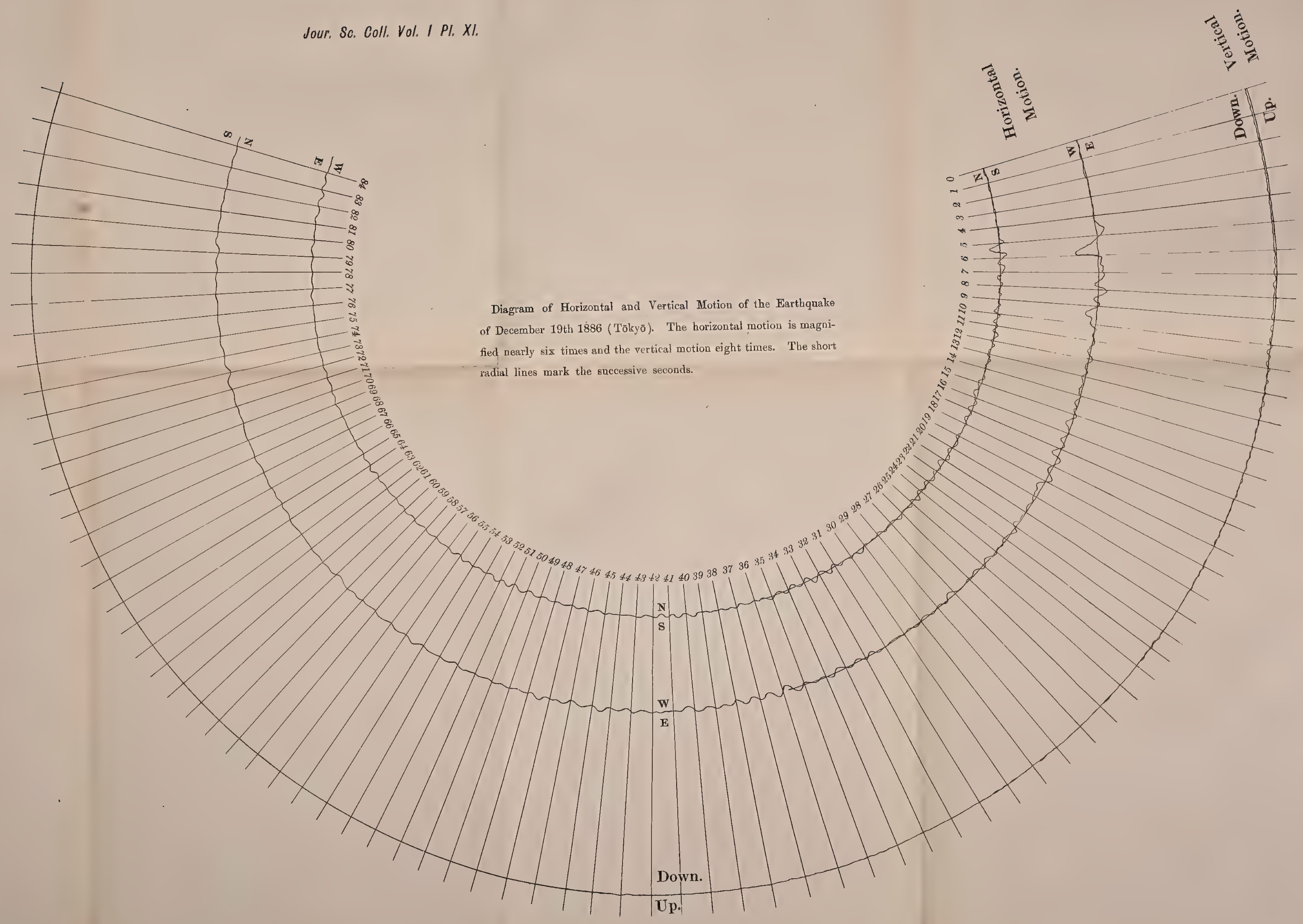

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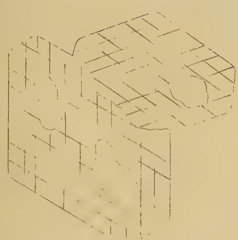

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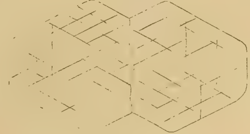

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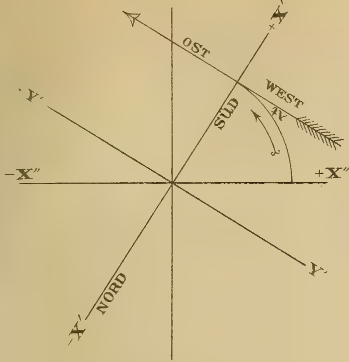

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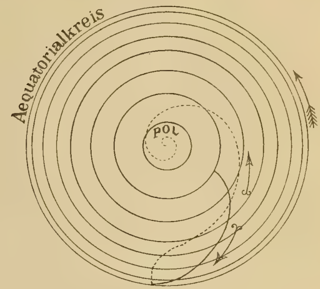

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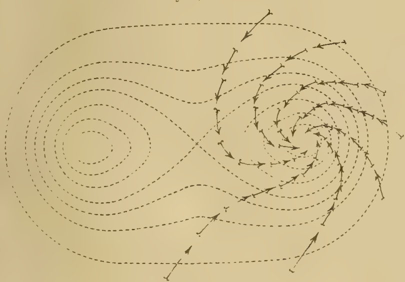

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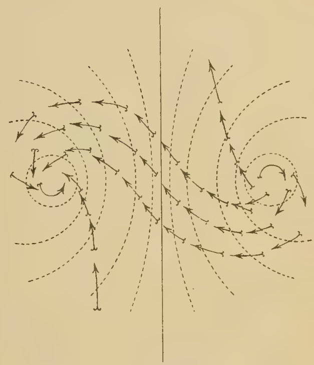

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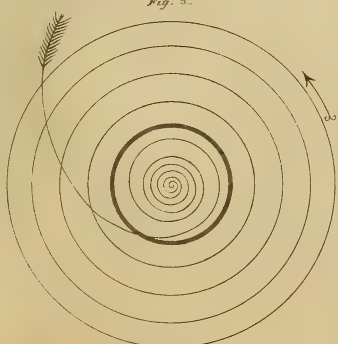

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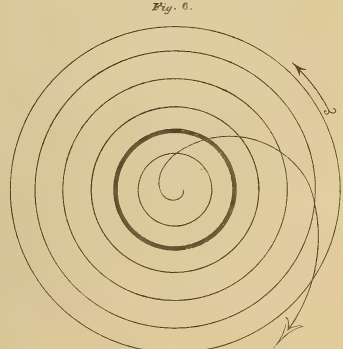


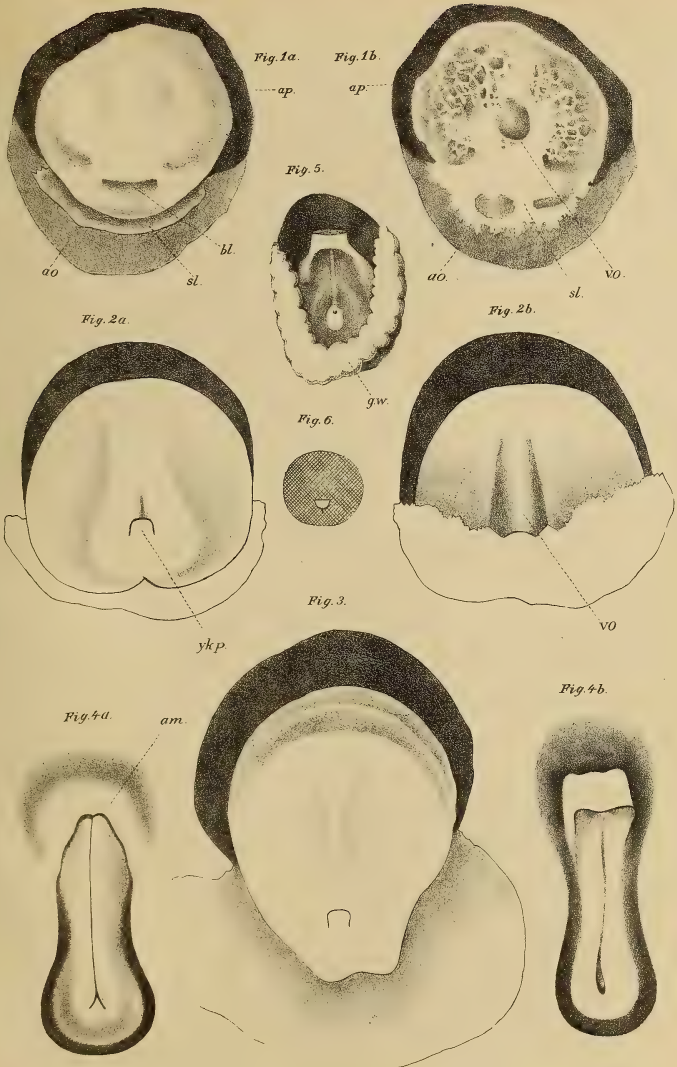


Fig. 1a.
ap.
bl.
ao
sl.
Fig. 1b.
ap.
ao.
sl.
vo.
Fig. 5.
g.w.
Fig. 2a.
ykp.
Fig. 6.
Fig. 2b.
vo
Fig. 3.
Fig. 4a.
am.
Fig. 4b.

Fig. 7.

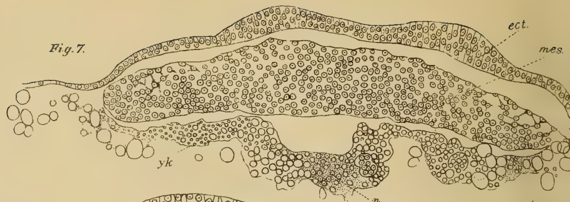
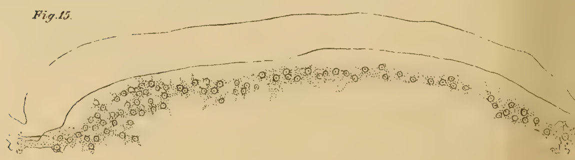


Fig. 8.

Fig. 9.

Fig. 10.

Fig. 15.



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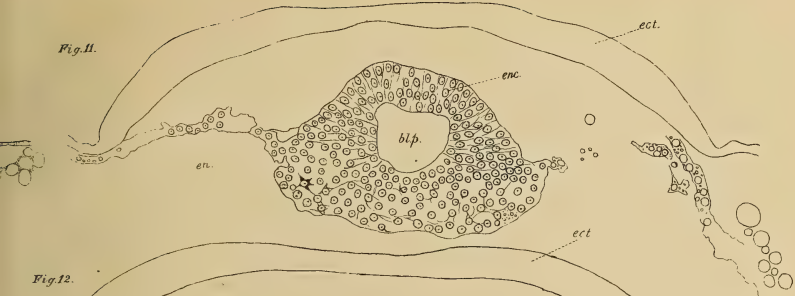

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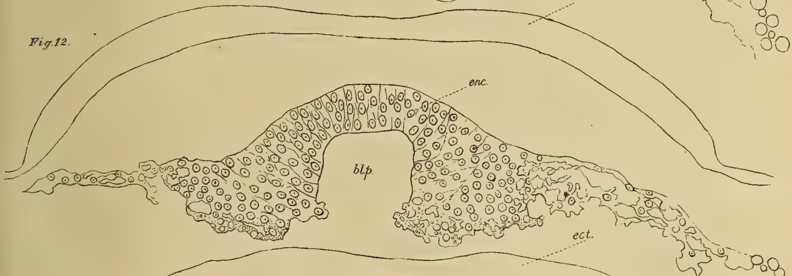

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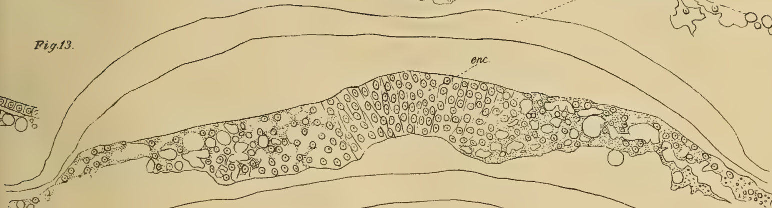


Fig.13.

Fig.14.

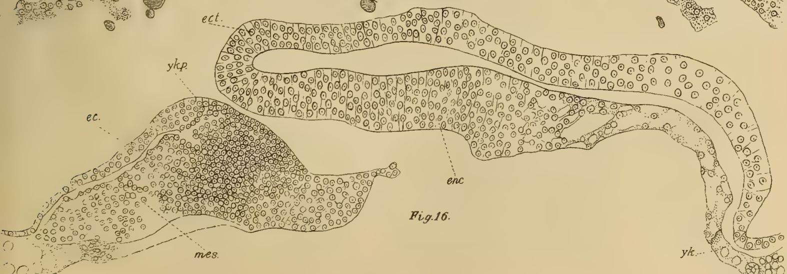


Fig.16.

Fig. 7.

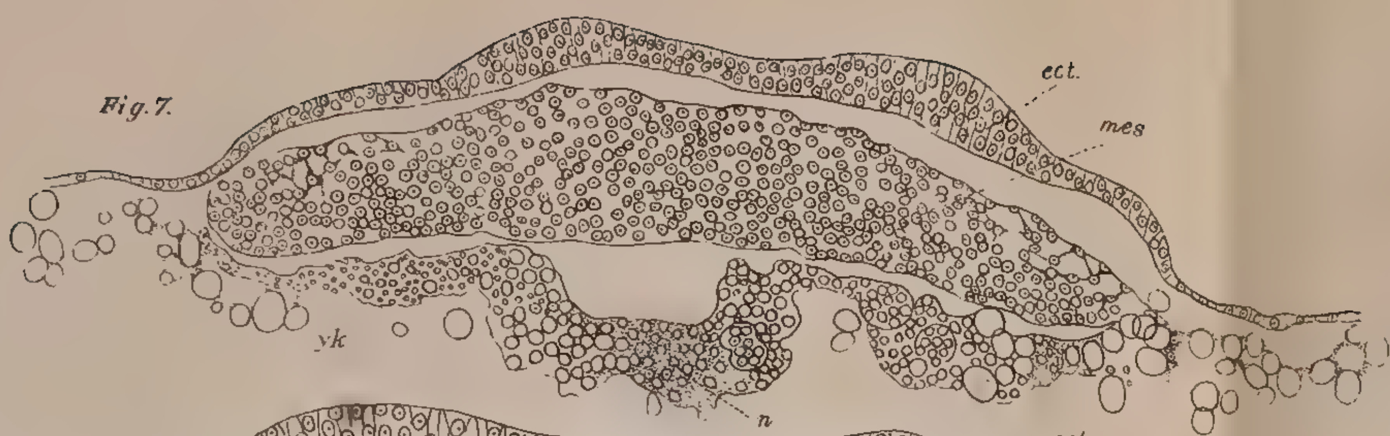


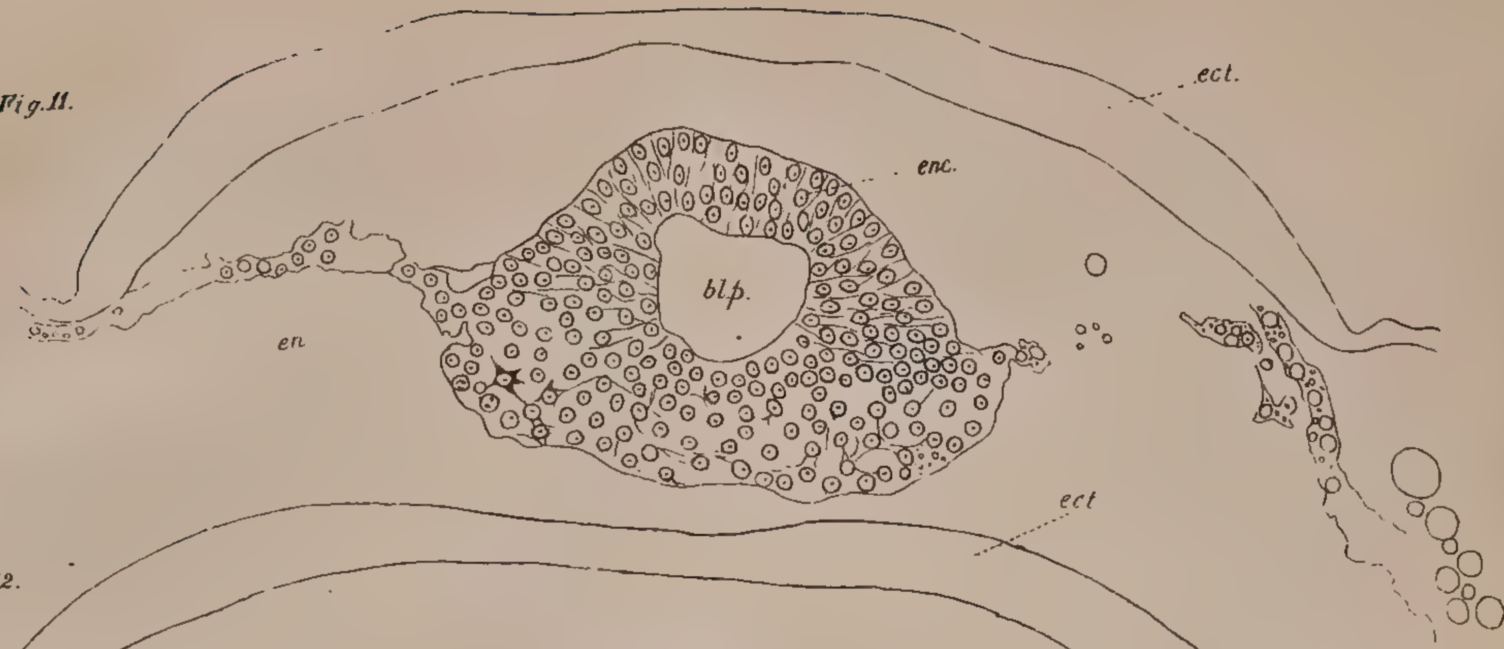
Fig. 8.

Fig. 9.

Fig. 10.

Fig. 15.

Fig. 11.



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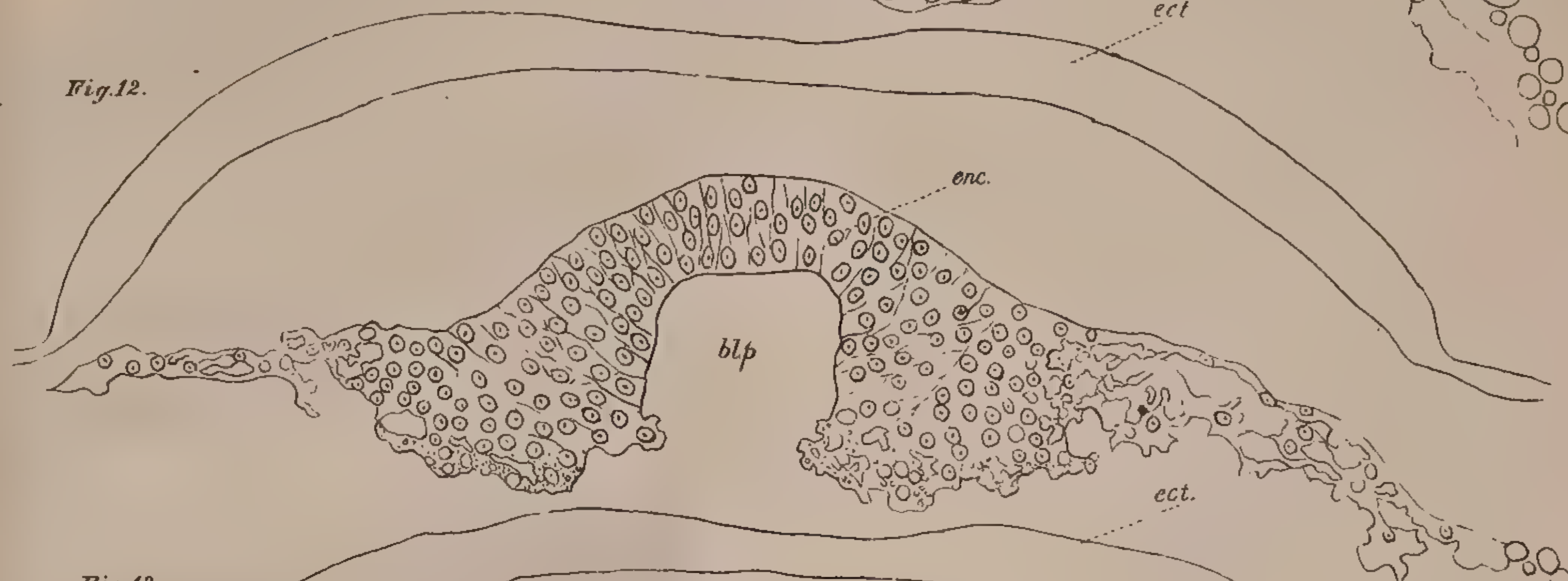

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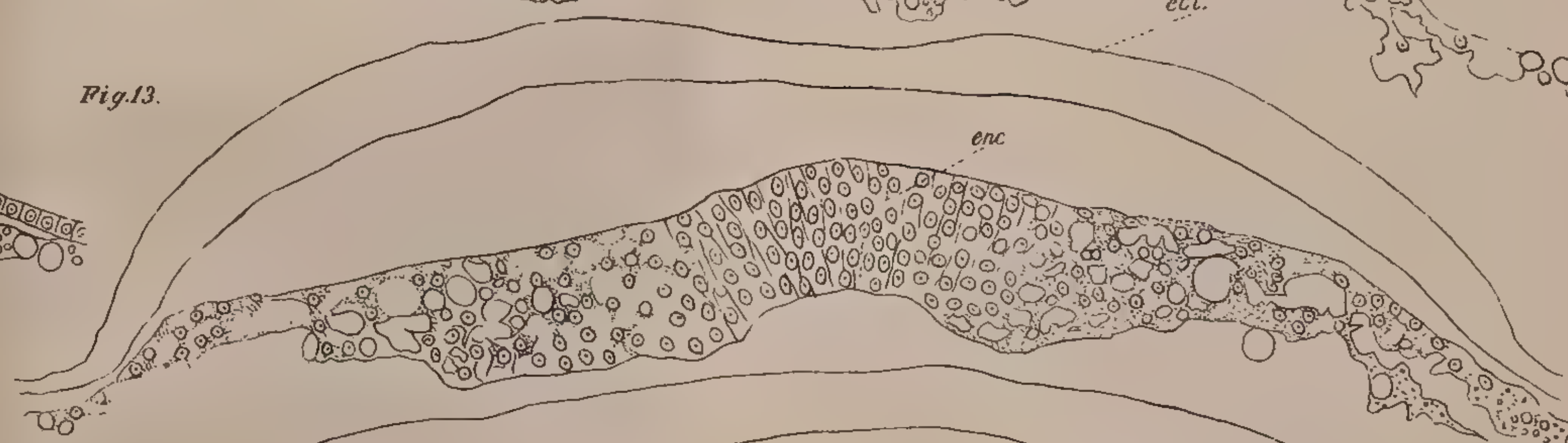

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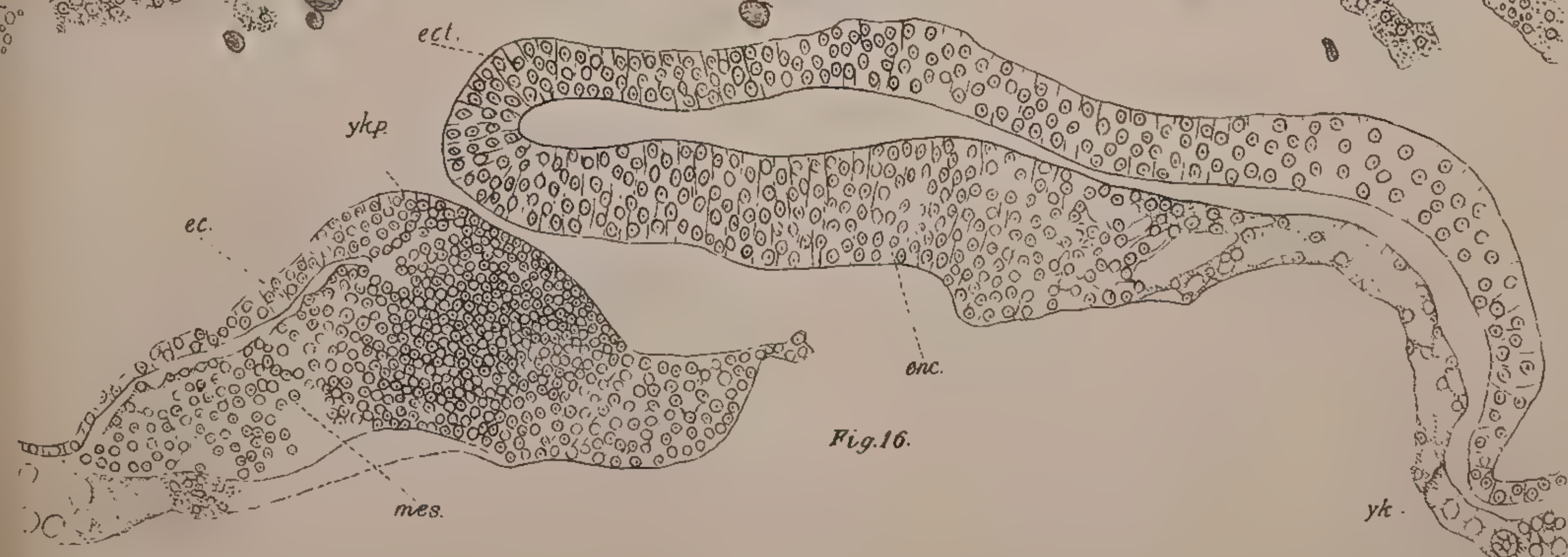
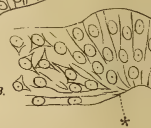


Fig. 16.

Ishikawa del.

Fig. 24.



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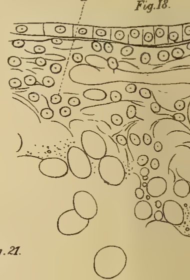

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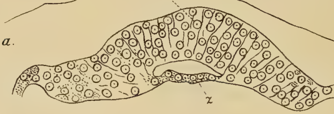
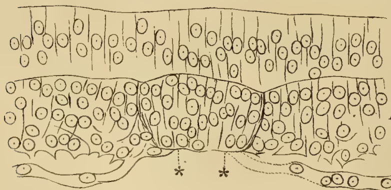


Fig. 14. a.



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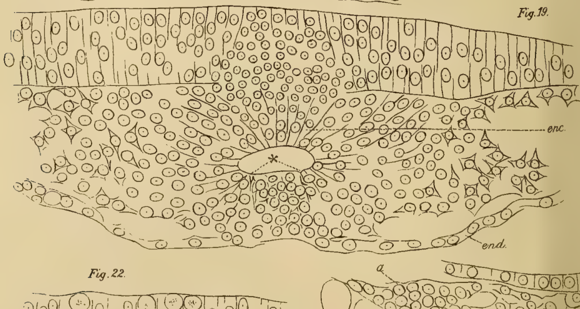


Fig. 19.

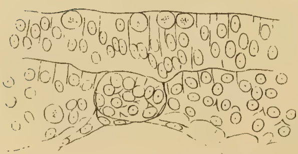
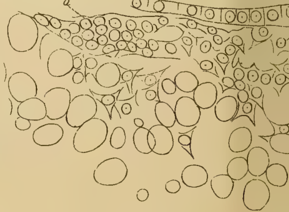
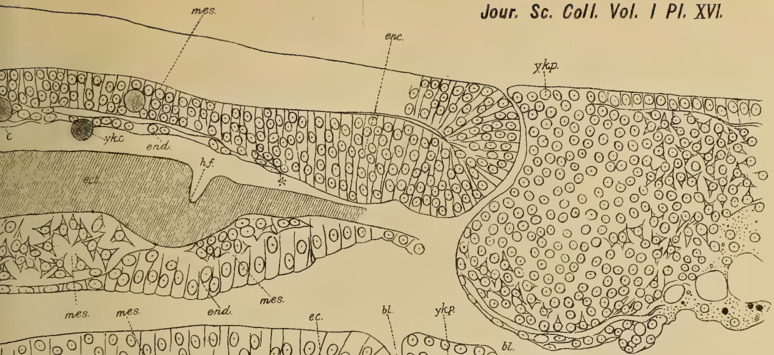


Fig. 22.



Ishikawa del



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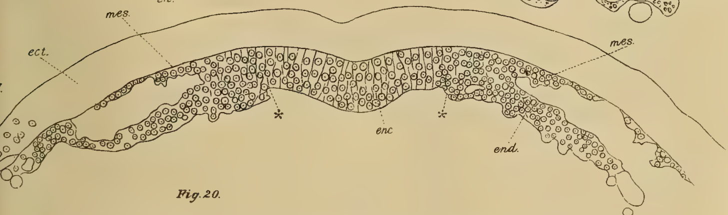
17.



mes.
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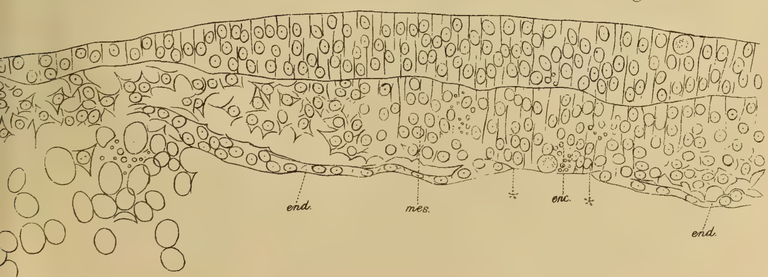
mes.
enc
end.

Fig. 20.



end.
mes.
enc.
end.

Ishikawa del

Fig. 30.

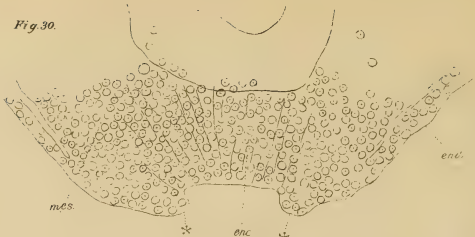


Fig. 31.

Fig. 32.

Fig. 33.

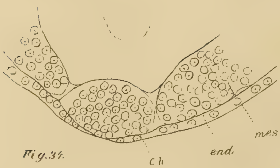


Fig. 34.

Fig. 29.

Ishikawa and Indo del.

5.

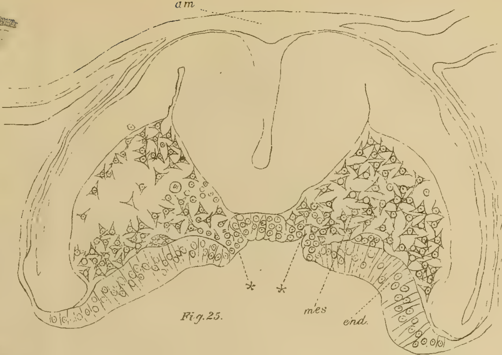


Fig.25.

Fig.26.

Fig.27.

Fig.28.

Fig. 30.

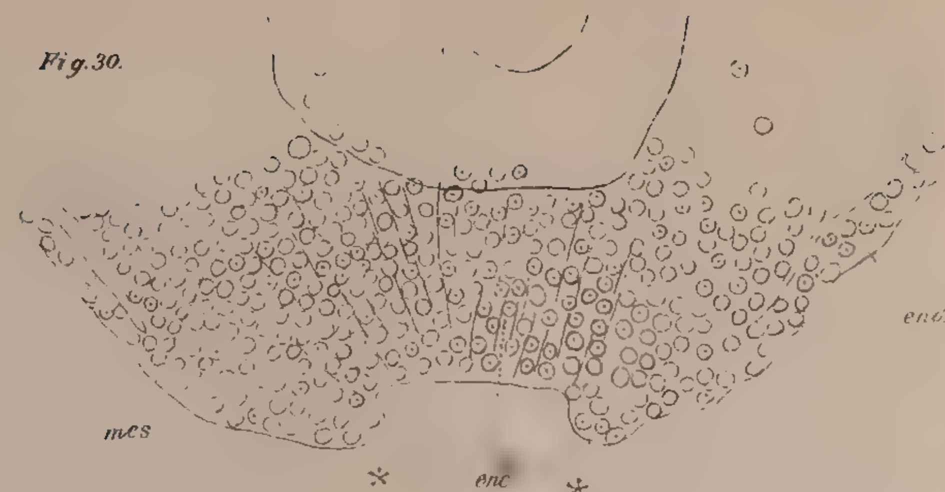
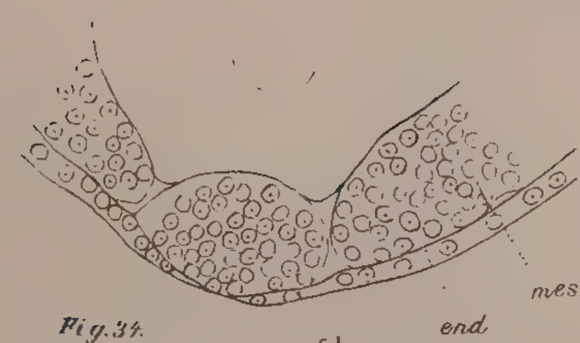


Fig. 31.

Fig. 32

Fig. 33.



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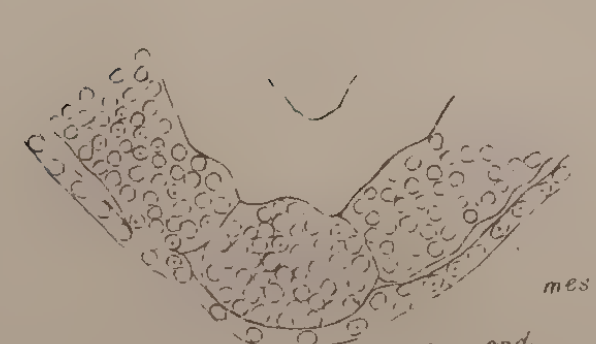


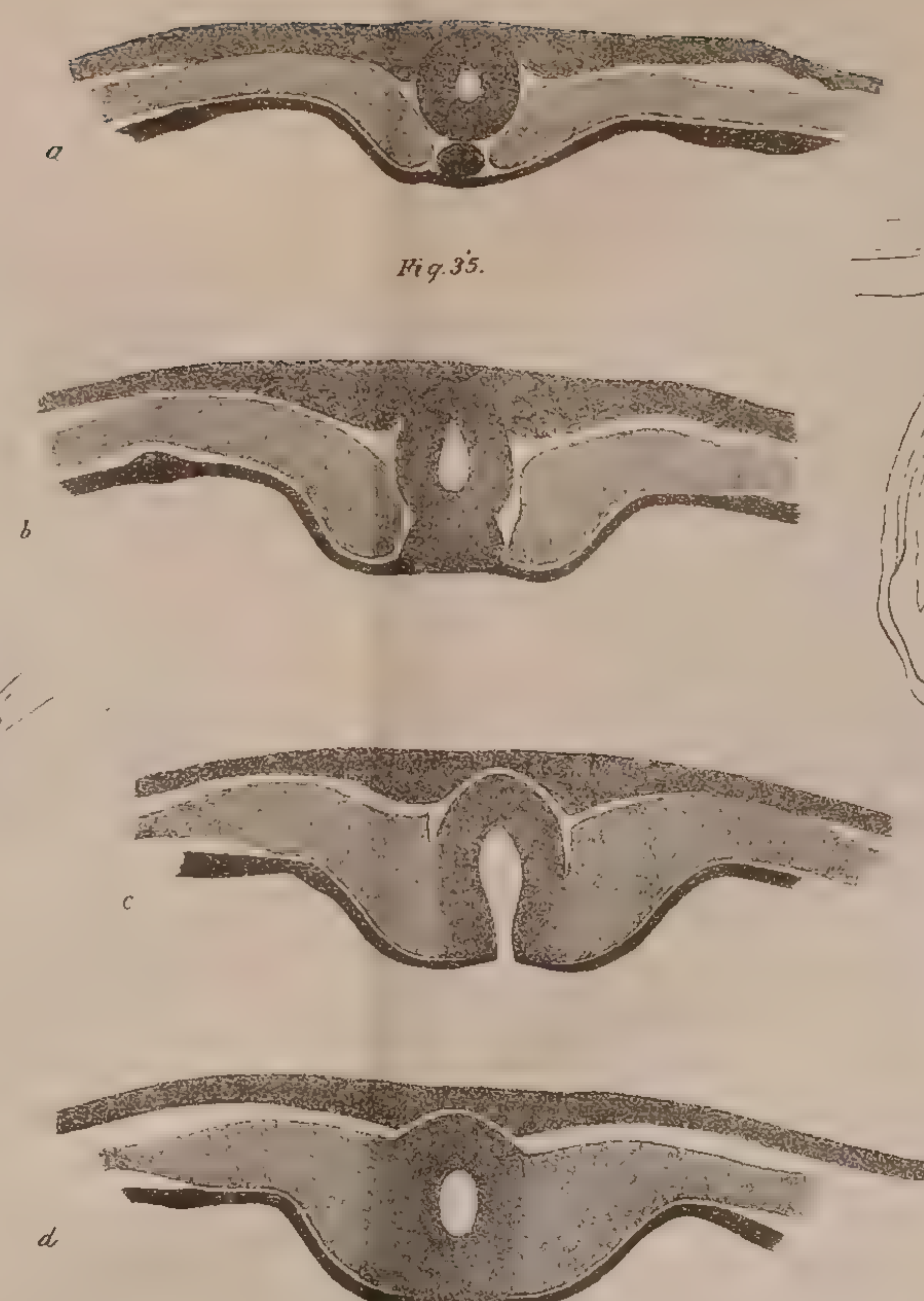
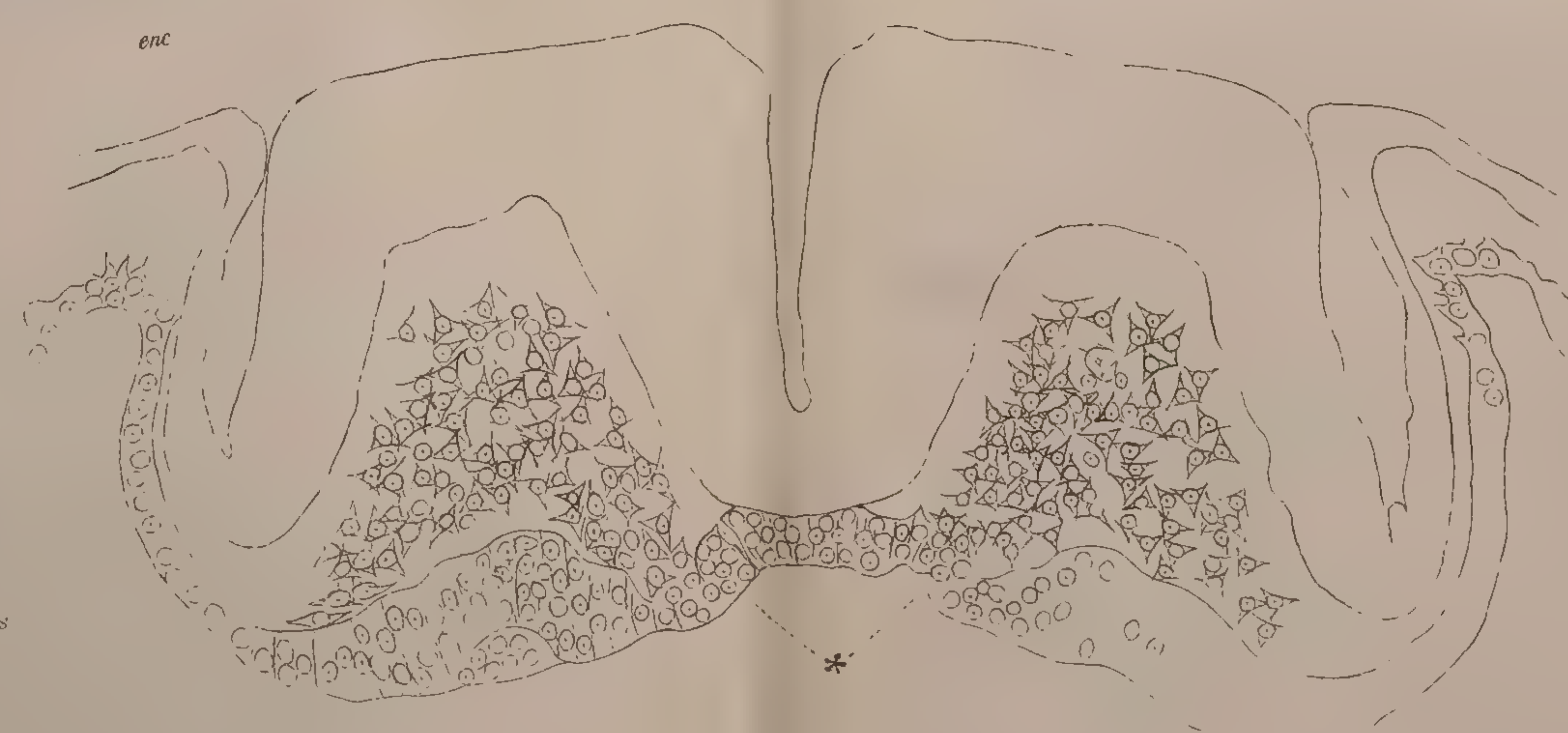


Fig. 35.

Fig. 29.



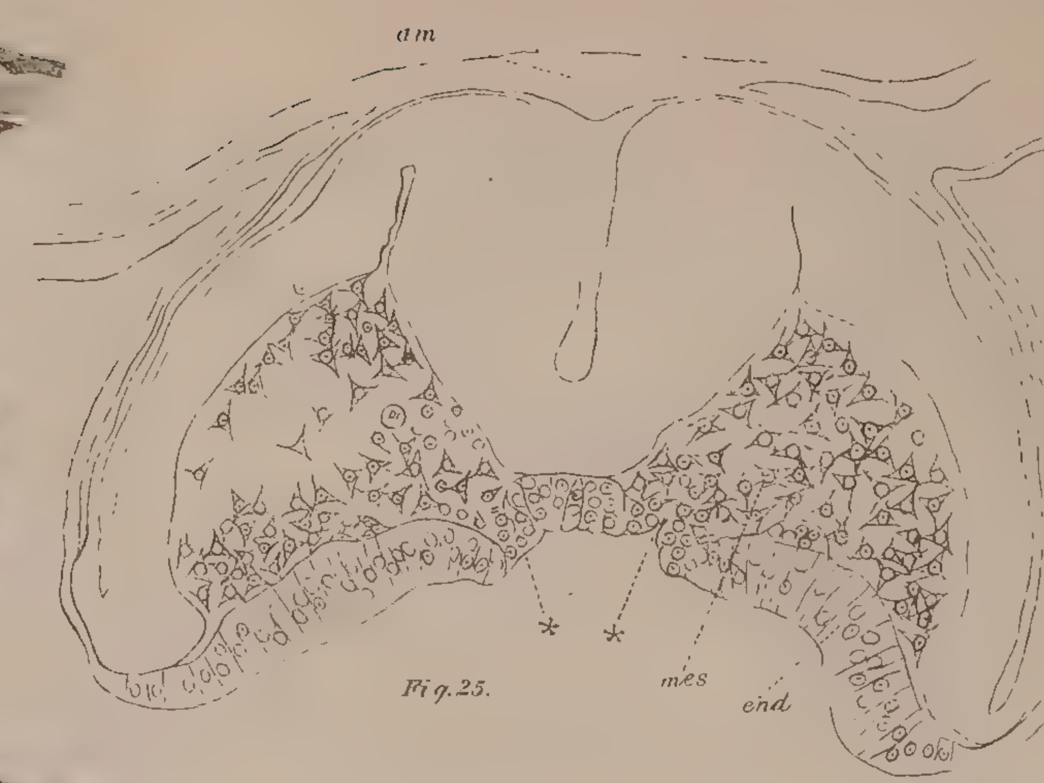


Fig. 25.

Fig. 26.

Fig. 27.

Fig. 28.

Ishikawa and Inda del.


Fig. 5.

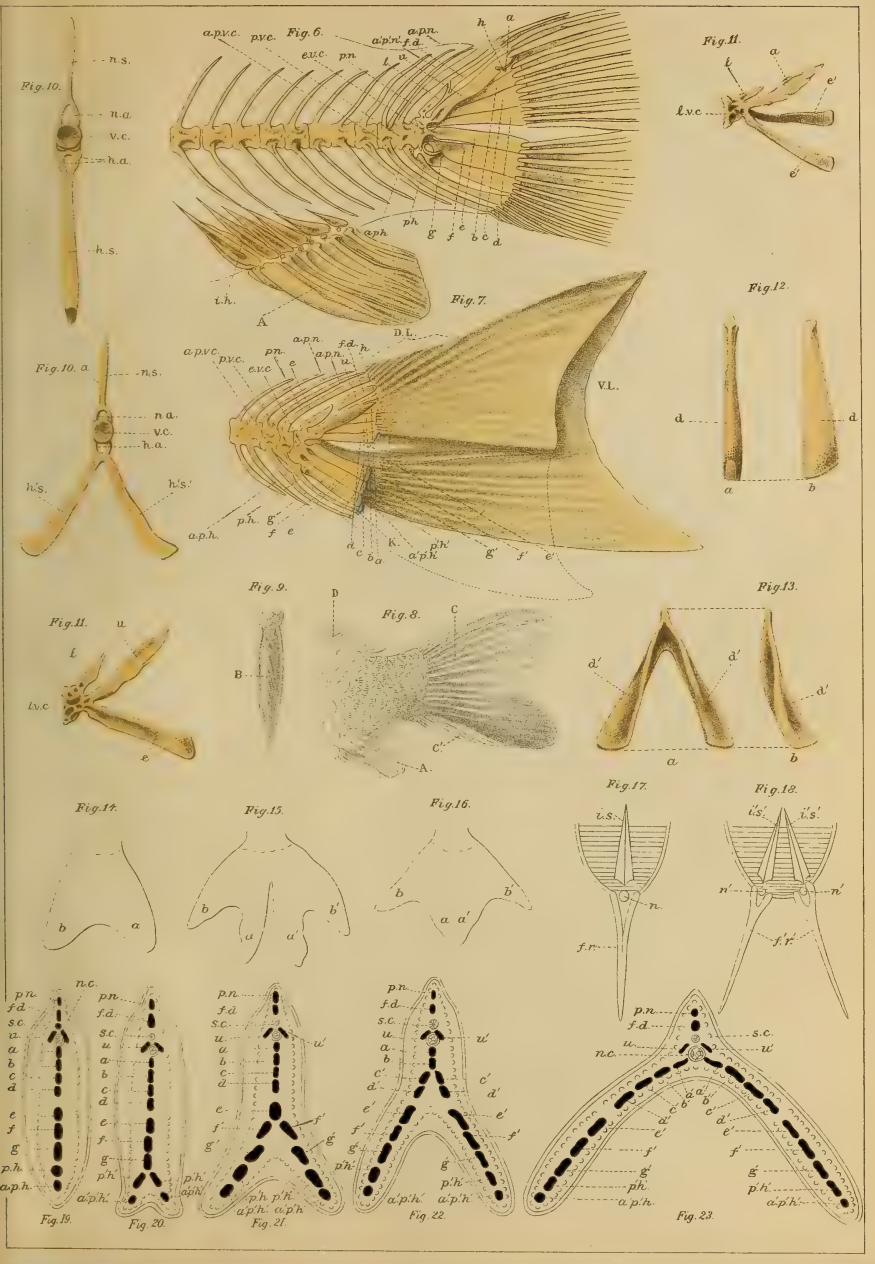


Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
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Fig. 11.
Fig. 12.
Fig. 13.
Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17.
Fig. 18.
Fig. 19.
Fig. 20.
Fig. 21.
Fig. 22.
Fig. 23.
n.s.
n.a.
v.c.
h.a.
h.s.
h'.s.
a.p.v.c.
p.v.c.
e.v.c.
p.n.
a.p.n.
a'.p'.n'.
f.d.
u.
l.
i.h.
a.ph.
ph.
p.h.
a.p.h.
p'.h'.
a'.p'.h'.
D.L.
V.L.
K.
l.v.c.
s.c.
n.c.
i.s.
i'.s'.
n.
n'.
f.r.
f'.r'.

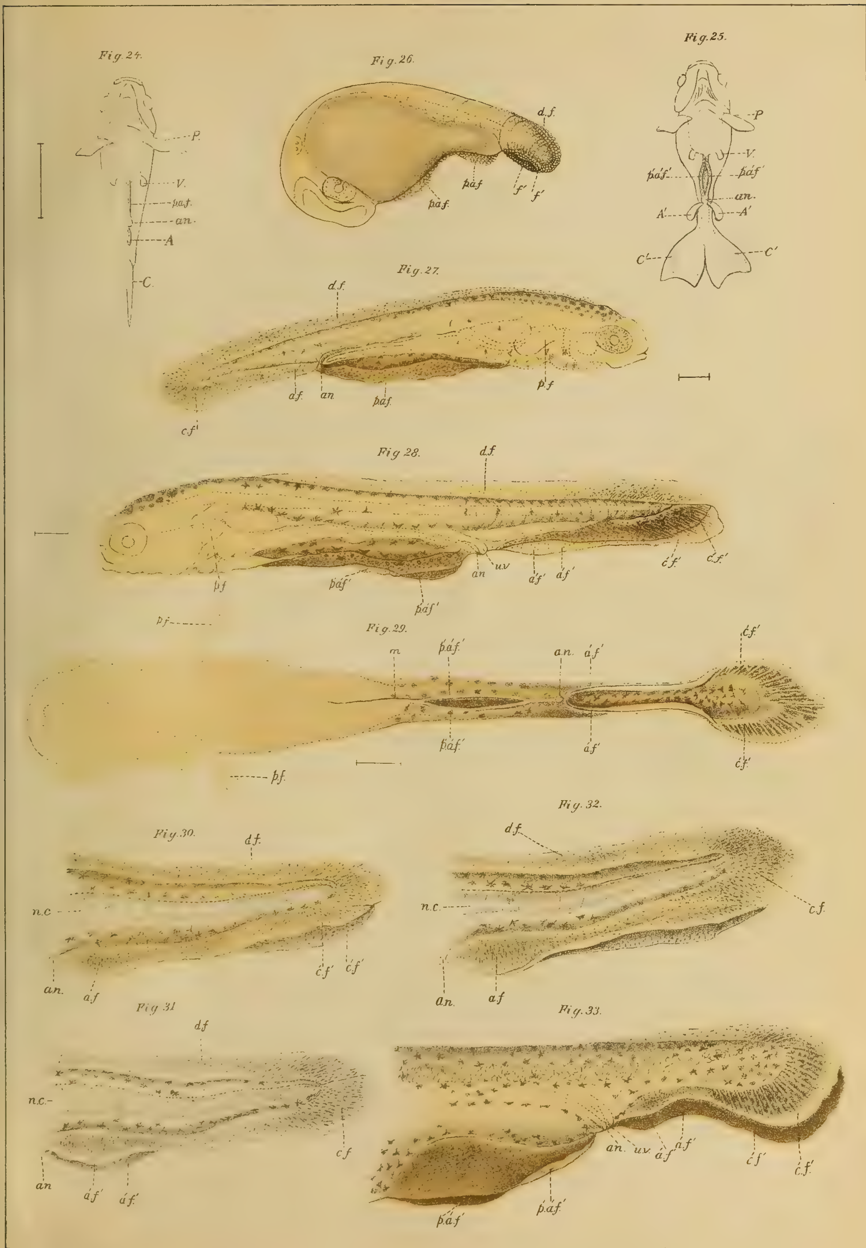
Fig. 24.
P.
V.
p.a.f.
an.
A
C.
Fig. 26.
d.f.
p.a.f
p.a.f.
f'
f'
Fig. 25.
P
V.
p.a.f'.
p.a.f'
an.
A'.
A'
C'.
C'
Fig. 27.
d.f.
a.f.
an
p.a.f.
p.f
c.f'
Fig 28.
d.f.
p.f
p.a.f'
p.a.f'
an
u.v
a.f'
a.f'
c.f'
c.f'
p.f
Fig. 29.
m
p.a.f.'
a.n.
a.f'
c.f'
p.a.f'
a.f'
c.f'
p.f.
Fig. 30.
d.f.
n.c
an.
a.f
c.f'
c.f'
Fig. 32.
d.f.
n.c.
c.f.
an.
a.f
Fig 31
df
n.c.
c.f
an
a.f'
a.f'
Fig. 33.
an.
u.v.
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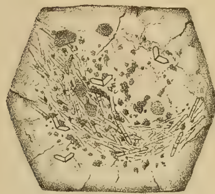
p.a.f'

Fig. 1.

× 19.

Fig. 8



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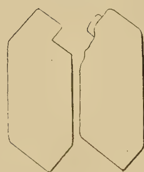

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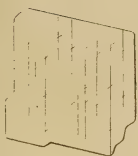

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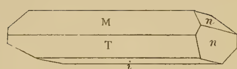
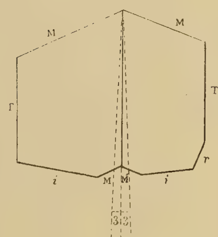
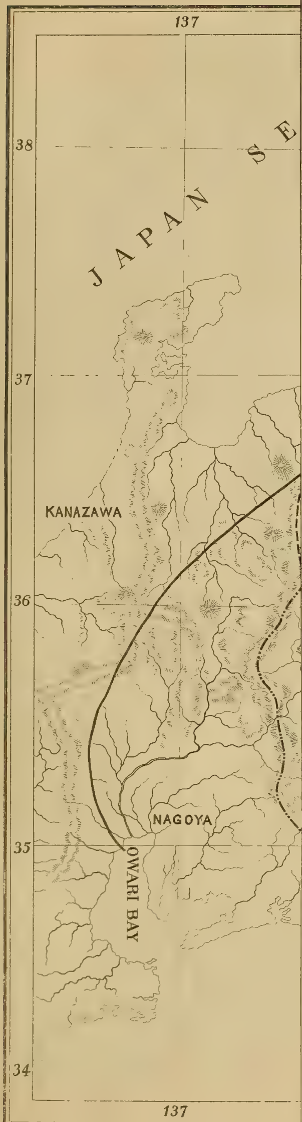


Fig. 7

Fig. 4

Fig. 6.




137
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34
JAPAN SE
KANAZAWA
NAGOYA
OWARI BAY
137

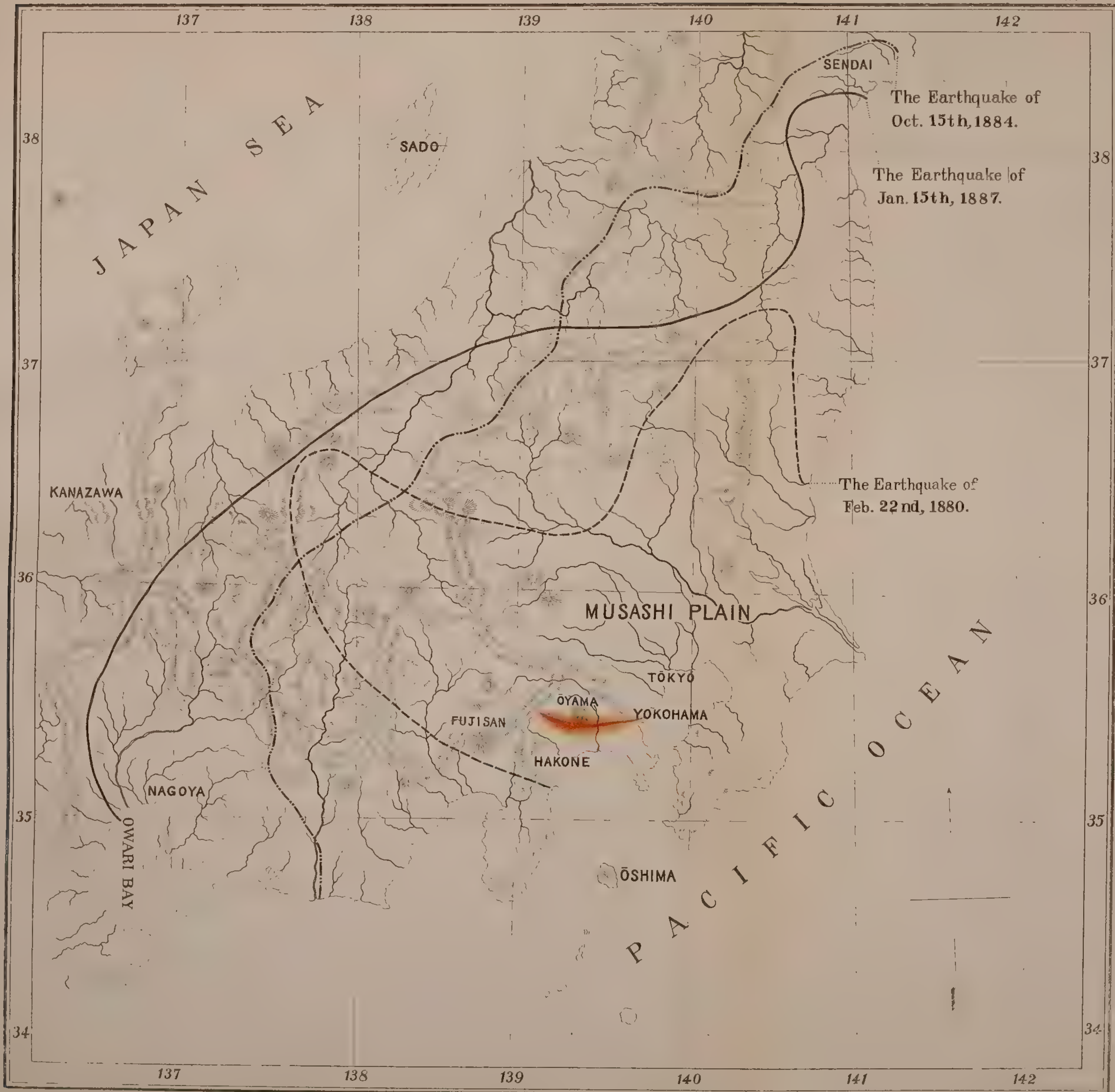
JAPAN SEA
SADO
SENDAI
The Earthquake of
Oct. 15th, 1884.
The Earthquake of
Jan. 15th, 1887.
The Earthquake of
Feb. 22nd, 1880.
KANAZAWA
MUSASHI PLAIN
TŌKYŌ
ŌYAMA
YOKOHAMA
FUJISAN
HAKONE
NAGOYA
OWARI BAY
ŌSHIMA
PACIFIC OCEAN
137
138
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Fig.1

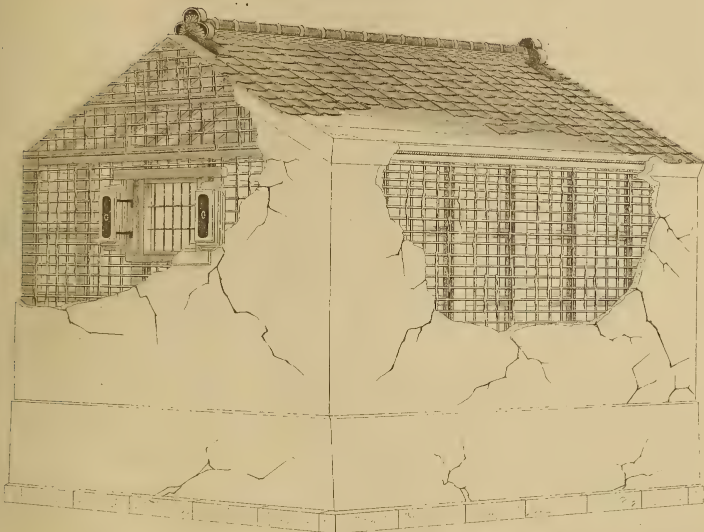


Fig.2

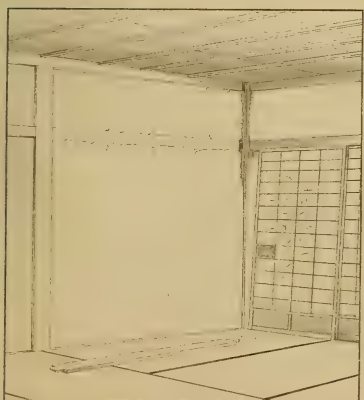
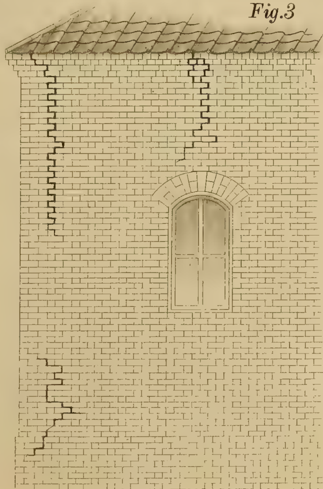
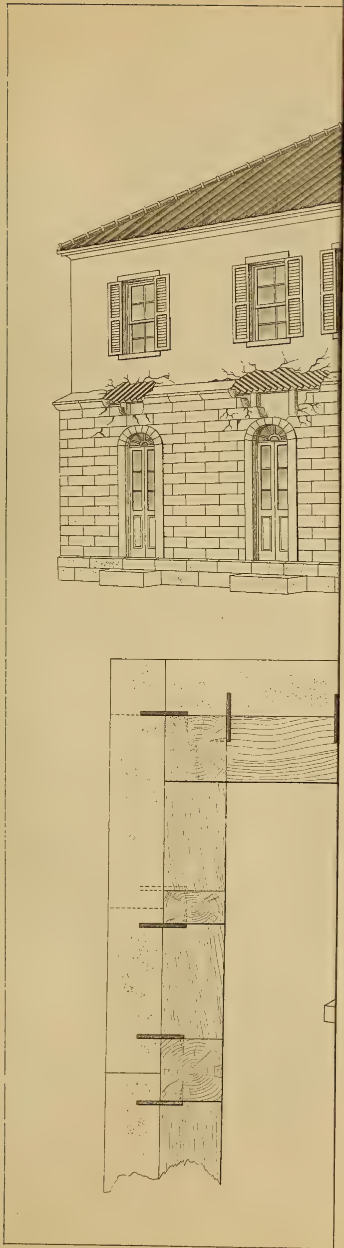
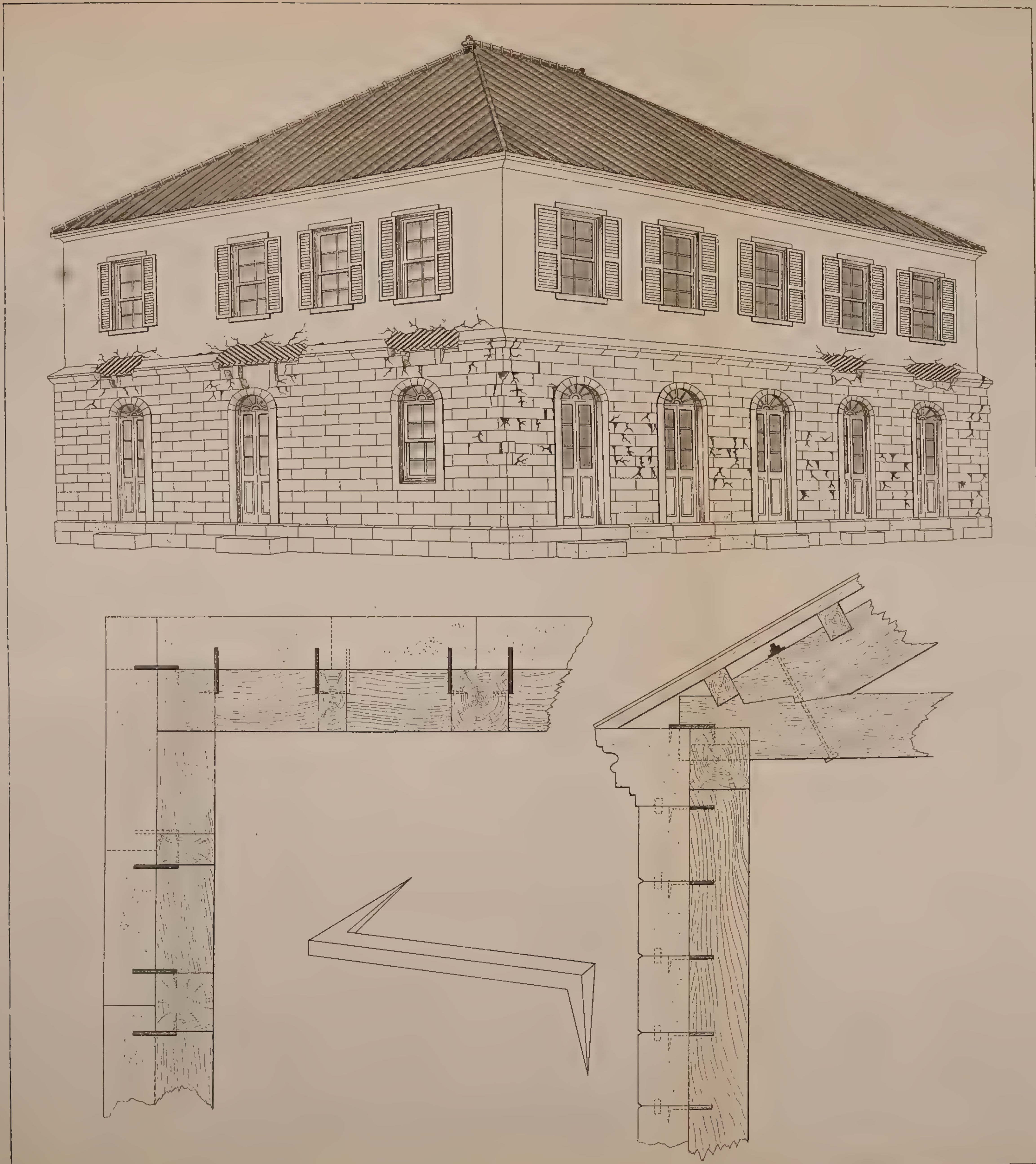
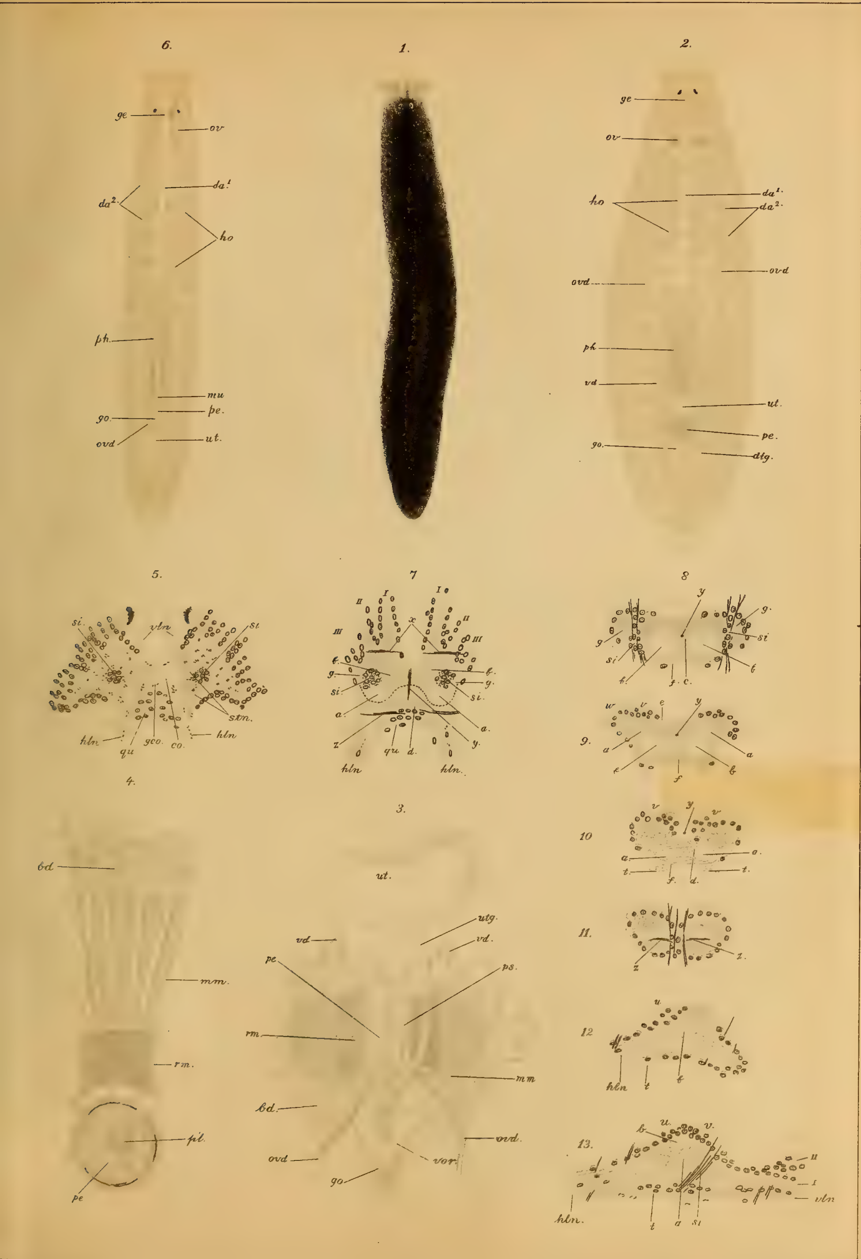


Fig.3








6.
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Iijima del.

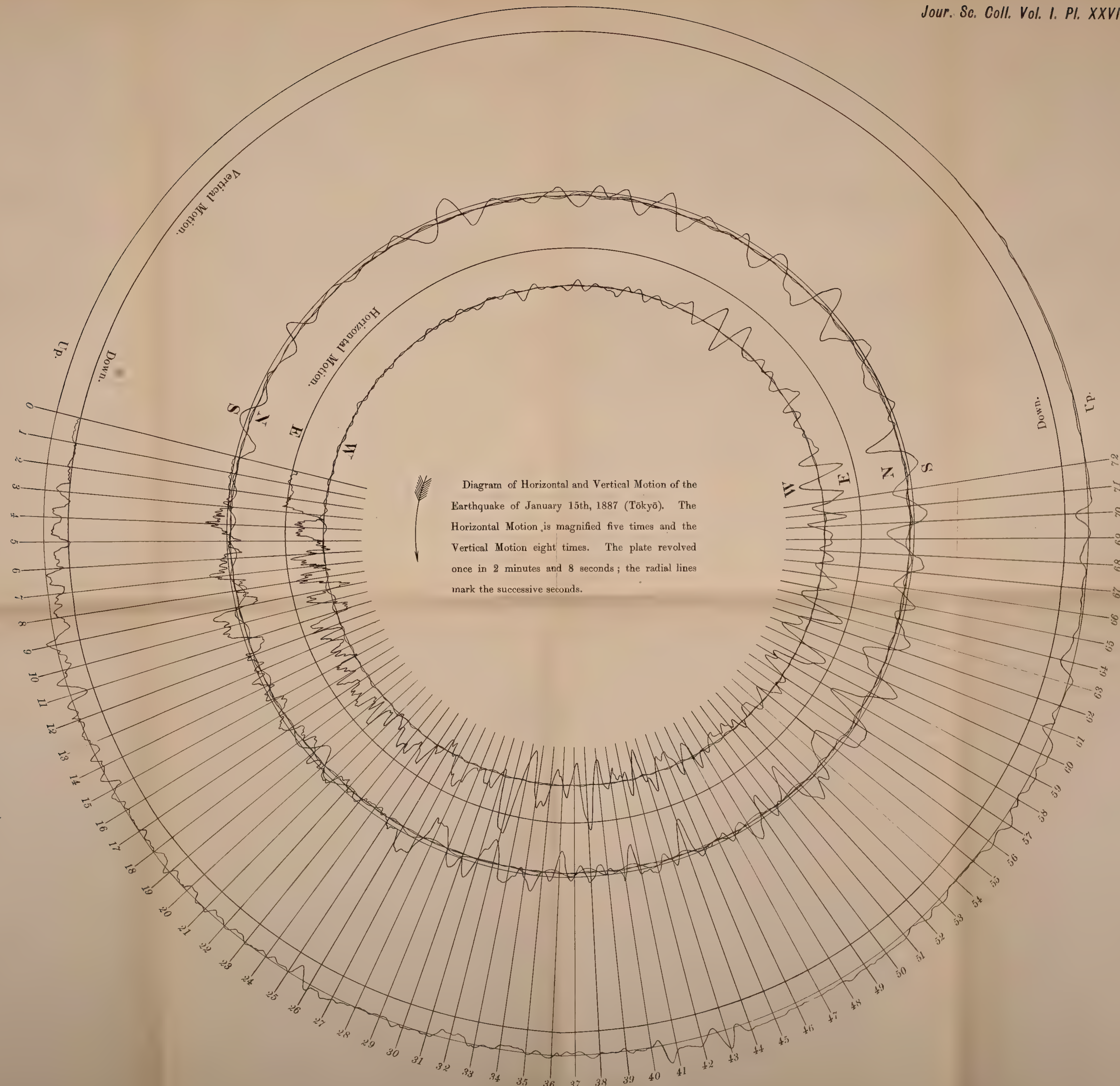


Diagram of Horizontal and Vertical Motion of the Earthquake of January 15th, 1887 (Tōkyō). The Horizontal Motion is magnified five times and the Vertical Motion eight times. The plate revolved once in 2 minutes and 8 seconds; the radial lines mark the successive seconds.

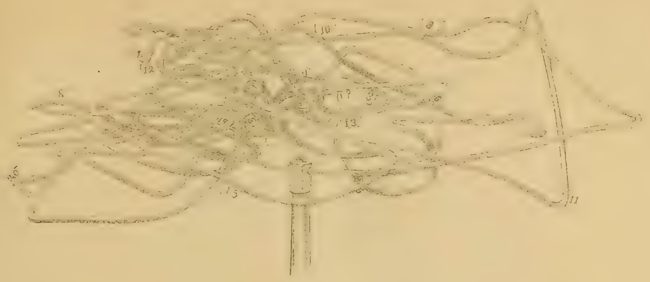


Fig. 1.

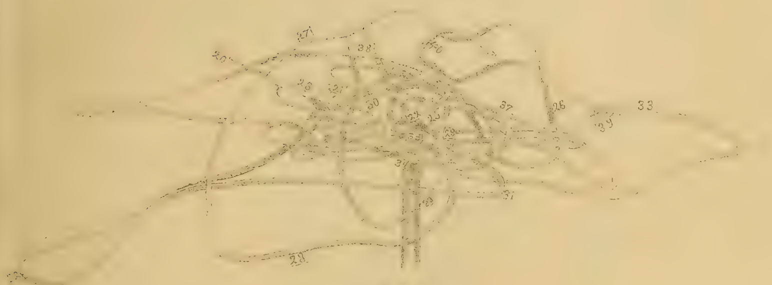


Fig 2.

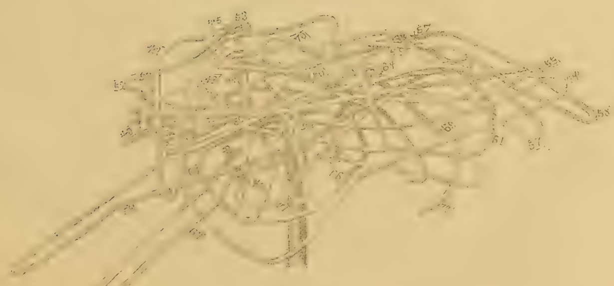
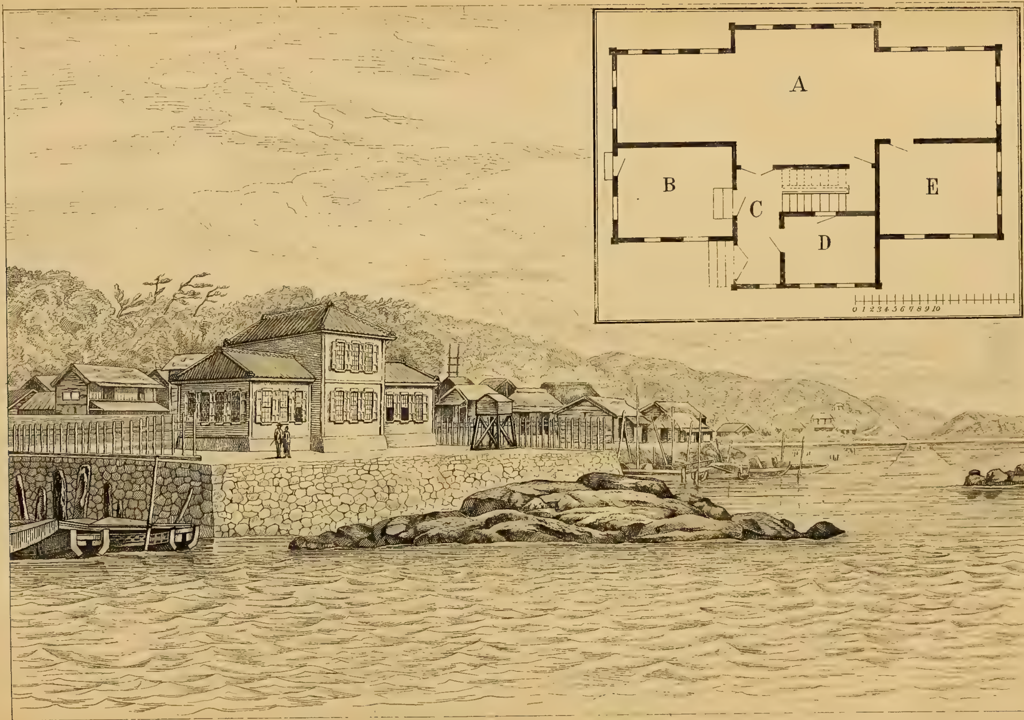
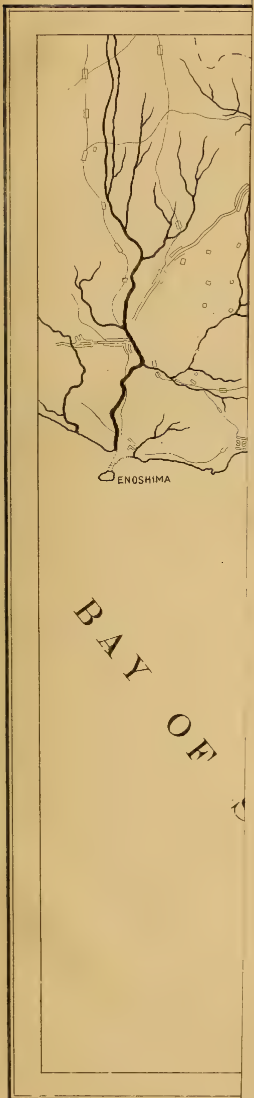


Fig. 3.



THE MARINE BIOLOGICAL STATION OF THE IMPERIAL UNIVERSITY AT MISAKI


ENOSHIMA
BAY OF


YOKOHAMA
BAY OF TOKIO
KANAZAWA
KAMAKURA
NATSUSHIMA
ENOSHIMA
YOKOSUKA
SARUSHIMA
FUTTSUSAKI
KAZUSA
URAGA
KANNONZAKI
MIURA
BAY OF SAGAMI
KOAJIRO
MUROISO
MATSUWA
MISAKI
JOGASHIMA
AWA